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A Step towards the Weak Interaction

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Abstract

Fermi formulated the theory of the weak interaction to explain the beta decay assuming four fermions interacting at the vertex of the Feynman diagram [1]. This paper proposes an alternative way to concern this topic: it assumes since the beginning the existence of massive force carriers, charged and neutral, regarded as fingerprints of the interaction. Next simple considerations explain the existence of these force carriers with the help of the quantum uncertainty.

Keywords

Weak Interaction, Beta Decay, Fermion Interactions, Massive Force Carriers, Quantum Uncertainty

1. Introduction

The weak interaction, one of the fundamental forces of nature, is transmitted by heavy W^+, W^- and Z^0 vector bosons, which limit its subatomic operating range to tiny distances smaller than a single proton. The unique features of this interaction, e.g. its ability to change subatomic particles from one type into another, which causes radioactive beta decay and drives the nuclear fusion in the stars, are due to its heavy carriers: it suggests the usefulness of formulating a model starting with the rest masses of the boson vectors. The calculation model consists of two parts: the first one aims to infer the values of the rest masses of the carriers, the second one highlights the physical implications of the calculation model. The text is organized in order to be as self-contained as possible.

2. The Weak Interaction

It is known that the weak force is carried by two charged vector bosons plus one neutral vector boson whose rest masses are [2]

$$Z^0 = 91.1876 \pm 0.002\ \text{GeV} \qquad W^{\pm} = 80.3692 \pm 0.0133\ \text{GeV}. \tag{2.1}$$

Define in general the total energy E_{tot} of the system of carriers as

$$E_{tot}=Z^0+2W^{\pm}+\epsilon_i, \tag{2.2}$$

where ϵ_i includes kinetic energies and interaction energy additional to their own rest masses. The values (2.1) are the basis for preliminary considerations on the weak interaction.

It is possible in principle to put $E_{tot}=kW^{\pm}$, with k proportionality constant; (2.2) yields

$$E_{tot}=W^{\pm}k \qquad Z^0=\frac{k-2}{k}E_{tot}-\epsilon_i. \tag{2.3}$$

To find a further equation of the system of bosons, combine $W^{\pm}$ and Z^0 just defined in (2.3) to introduce a new dimensionless function $\left(Z^{0^2}-W^{\pm^2}\right)/4E_{tot}^2$; one finds

$$\frac{Z^{0^2}-W^{\pm^2}}{4E_{tot}^2}=\frac{(k-2)^2-1}{4k^2}-\frac{k-2}{2k}\frac{\epsilon_i}{E_{tot}}+\frac{\epsilon_i^2}{4E_{tot}^2}, \tag{2.4}$$

where the right hand side consists of a constant term plus a function of the ratio ϵ_i/E_{tot} only. The reason of this step is that for $k=\pi$ in particular the first constant addend is worth 0.00768100158. This value represents a new constant α^* about 5% greater than the fine structure constant α. This result is significant because α^* is found again in the next steps of this model. Moreover, owing to (2.4), the analytical form of α^* results to be

$$\frac{Z^{0^2}-W^{\pm^2}}{4E_{tot}^2}=\alpha^*-\frac{\pi-2}{2\pi}\frac{\epsilon_i}{E_{tot}}+\frac{\epsilon_i^2}{4E_{tot}^2} \qquad \alpha^*=\frac{(\pi-2)^2-1}{4\pi^2}. \tag{2.5}$$

On the one hand (2.5) agrees with the experimental data: a glance to the values (2.1) and (2.2) shows the numerical relationship $E_{tot}/W^{\pm}\approx\left(2W^{\pm}+Z^0\right)/W^{\pm}\approx 3.13$, which in turn suggests

$$\frac{2W^{\pm}+Z^0+\epsilon_i}{W^{\pm}}=\pi. \tag{2.6}$$

On the other hand (2.5) implies that defining $\zeta=\epsilon_i/E_{tot}$ the first equation splits as follows

$$\frac{Z^{0^2}-W^{\pm^2}}{4E_{tot}^2}=\alpha \qquad \frac{(\pi-2)^2-1}{4\pi^2}-\frac{\pi-2}{2\pi}\zeta+\frac{1}{4}\zeta^2=\alpha \qquad \zeta=\frac{\epsilon_i}{E_{tot}}, \tag{2.7}$$

whence ζ results

$$\zeta_{\pm}=\frac{\epsilon_i}{E_{tot}}=\frac{\pi-2\pm\sqrt{4\pi^2\alpha+1}}{\pi}. \tag{2.8}$$

So the unknowns to calculate E_{tot} are in fact only two. Differentiating (2.7), $(\pi-2)/2\pi+\zeta/2=0$ reads $\zeta_{max}=(\pi-2)/\pi$. Eventually rearrange these results to define via (2.3)

$$Z^0=E_{tot}-\epsilon_i-2W^{\pm}=\left(1-\zeta_{-}-\frac{2}{\pi}\right)E_{tot}=\frac{\sqrt{4\pi^2\alpha+1}}{\pi}E_{tot}, \tag{2.9}$$

which also implies in turn

$$Z^0 = W^{\pm}\sqrt{4\pi^2\alpha+1} \qquad 2W^{\pm} - Z^0 = \left(2-\sqrt{4\pi^2\alpha+1}\right)W^{\pm}: \tag{2.10}$$

i.e. $Z^0 \gtrsim W^{\pm}$. As regards the signs of (2.8), collect and check these preliminary results into a unique equation. Then (2.2) reads

$$E_{tot} = 2W^{\pm} + Z^0 + \epsilon_i = \left(\frac{2}{\pi} + \frac{\sqrt{4\pi^2\alpha+1}}{\pi} + \frac{\pi - \sqrt{4\pi^2\alpha+1} - 2}{\pi}\right)E_{tot}: \tag{2.11}$$

the signs of (2.8) have been implemented in order that the sum in parenthesis (2.11) yields an identity, *i.e.* $\epsilon_i = \zeta\_E_{tot}$. To calculate $Z^0, W^{\pm}, \epsilon_i$ via E_{tot} is necessary to bypass the linear dependence of the former upon the latter and to implement the value of α. Note that (2.10) yields

$$W^{\pm}Z^0 = E_{tot}^2 \frac{\sqrt{4\pi^2\alpha+1}}{\pi^2} = W^{\pm 2}\sqrt{4\pi^2\alpha+1}, \tag{2.12}$$

where E_{tot} expressed via $\sqrt{W^{\pm}Z^0}$ removes the linear proportionality between the respective energies.

As concerns α, regarded as a further unknown of the model, introduce a separate function $f(\alpha)$ defined via (2.8) as follows

$$f(\alpha) = \frac{1}{c1}x^2 + x = c2 \qquad x = \frac{\zeta_-}{\alpha} = \frac{\epsilon_i}{E_{tot}\alpha} \qquad c1,?c2 = const: \tag{2.13}$$

once knowing the values of $c1$ and $c2$, this equation can be solved with respect to x, which makes in turn α explicitly calculable itself in this theoretical frame. Since $\zeta_- = \zeta_-(\alpha)$, then $c1$ and $c2$ must be numbers or dimensionless functions themselves of α only, anyway both constants.

The physical meaning of (2.13) is emphasized considering first the particular case $c2 = 0$. The solution yields three values of α: once more α^* of (2.5) plus two quite complicate functions of the constant $c1$ itself. Relevant implications of this particular result are: 1) α^* defined by (2.13) is the physical constant of (2.5) whatever $c1$ might be, and 2) the link of $f(\alpha) = 0$ with the considerations previously carried out about (2.7) involves α^*. Since $c1$ is freely definable, physical considerations on (2.13) highlighted soon below show that it is convenient to put $c1 = 3$ along with $c2 = 0$. This specific choice implies a three-valued solution

$$0.007681001578 = \alpha^*, \ 0.01859385816, \ 0.1835971010 \qquad c1 = 3, \ c2 = 0, \tag{2.14}$$

which anyway links this particular definition of $f(\alpha)$ to the frame of results previously obtained. Indeed $f(\alpha)$, identically rewritten as $c2/x - x/c1 = 1$, yields $\left(c2c1 - x^2\right)/xc1 = 1$, in fact formally analogous to (2.7); write thus

$$\frac{c2}{x} = \frac{Z'^{0\,2}}{4E'^2_{tot}\alpha} \qquad \frac{x}{c1} = \frac{W'^{\pm\,2}}{4E'^2_{tot}\alpha}, \tag{2.15}$$

whence, subtracting side by side,

$$\frac{c2}{x} - \frac{x}{c1} = 1 \ \Rightarrow \ \frac{Z'^{0\,2} - W'^{\pm\,2}}{4E'^2_{tot}} = \alpha. \tag{2.16}$$

The notation emphasizes that despite the formal analogy of (2.16) with (2.5), there is no reason to expect in general that the numerical values of the primed quantities coincide with the corresponding (2.7): indeed appropriate Z'^0 and $W'^{\pm}$ are defined by $c1$ and $c2$, whereas Z^0 and $W^{\pm}$ define themselves α. Nevertheless (2.16) and (2.7) merge into

$$\frac{Z^{0^2} - W^{\pm^2}}{4E_{tot}^2} = \frac{Z'^{0^2} - W'^{\pm^2}}{4E'^2_{tot}} = \alpha. \tag{2.17}$$

In other words (2.14) and (2.16) plug $f(\alpha) = 0$ as a further information into the frame of the model outlined by (2.2) to (2.12).

Generalize now (2.14) to the more significant case $f(\alpha) \neq 0$, considering that according to (2.13) x and thus $f(\alpha)$ itself are by definition decreasing functions of E_{tot}: *i.e.* the constant $c2$ must be such that $f(\alpha) \propto E_{tot}^{-1}$. The way to fulfill this condition without additional hypotheses is therefore to put $c2 = \pi^{-1}$ in order that

$$f(\alpha) = \frac{W^{\pm}}{E_{tot}} = \frac{1}{\pi} \Rightarrow E_{tot} = \frac{W^{\pm}}{x^2/3 + x} \quad c1 = 3, \; c2 = \frac{1}{\pi}, \; x = 0.290, \tag{2.18}$$

where x is calculated in (2.13) with the given $c1$ and $c2$. Since (2.13) can be solved with respect to α itself, then once knowing α (2.8) allows calculating ζ_- and thus ϵ_i/E_{tot} and $W^{\pm}/E_{tot}$ too. Also, the chance of writing

$$\frac{\epsilon_i}{E_{tot}} = \zeta_- = x\alpha$$

proves that (2.13) is one among the many equations linking the rest masses of the carriers to the physical properties of the weak interaction.

Merging all of these equations requires in fact a coherent value of α, also implied by (2.13) and (2.18): these equations are closely interdependent and require the solution of a system of equations with respect to the related unknowns. First of all, quote explicitly (2.13)

$$\frac{\pi - 2 - \pi\sqrt{4\pi^2\alpha + 1}}{\alpha\pi} + \frac{1}{3}\left(\frac{\pi - 2 - \pi\sqrt{4\pi^2\alpha + 1}}{\alpha\pi}\right)^2 = \frac{1}{\pi}, \tag{2.19}$$

because the solutions of this equation with respect to α read

$$0.007297318975 = \alpha, \; 0.02250598341, \; 0.1261034233: \tag{2.20}$$

the first value is well known, it allows to calculate explicitly the functions from (2.5) to (2.13). The deviation between calculated and standard value of α is -0.0005%. Since the value of $c2 = 1/\pi$ is uniquely defined to infer a solution with recognizable value of α, then (2.18) is in fact a physical law. Return now to (2.12) and show the physical meaning of $W^{\pm}Z^0$. Define owing to (2.10)

$$G_f = \frac{\kappa}{W^{\pm}Z^0} \equiv \frac{\pi^2\kappa}{\sqrt{4\alpha\pi^2 + 1}E_{tot}^2}.$$

Since $G_f \propto E_{tot}^{-2}$, the physical meaning of the function G_f involves the total energy balance of the interaction instead of the rest masses of the force carriers. To infer the proportionality constant κ, note that $\kappa = \sqrt{\alpha}$ yields

$$G_f = \frac{1}{W^{\pm 2}}\sqrt{\frac{\alpha}{4\pi^2\alpha+1}} = \frac{\pi/2}{E_{tot}^2}\sqrt{\frac{4\pi^2\alpha}{4\pi^2\alpha+1}} \qquad \kappa = \sqrt{\alpha} \tag{2.21}$$

in agreement with (GQ2); this result reads $G_f \approx 0.7E_{tot}^{-2}$ only, as reasonably expected. In this way the constant κ is not just any best fit parameter, α governs all results hitherto achieved. In general the carriers have their own kinetic energy; considering now their rest mass, this result takes its maximum value and can be directly checked via (2.19). Implementing the experimental values (2.1) and first (2.19) one finds

$$G_F = \frac{\sqrt{\alpha}}{W^{\pm}Z^0} = 0.0000116469\ \mathrm{GeV}^{-2}. \tag{2.22}$$

The chance of calculating the values of the fundamental constants α and G_F qualifies this theoretical approach as a self-consistent conceptual frame of force carriers. Knowing the value of α, summarize the set of five equations defined by the known functions (2.5), (2.9), (2.8) and (2.22)

$$G_F = \frac{\sqrt{\alpha}}{W^{\pm}Z^0} \qquad \epsilon_i = \frac{\sqrt{4\pi^2\alpha+1}+\pi-2}{\pi}E_{tot} \qquad W^{\pm} = \frac{E_{tot}}{\pi} \qquad Z^0 = \frac{\sqrt{4\pi^2\alpha+1}E_{tot}}{\pi}, \tag{2.23}$$

and of course (2.2)

$$E_{tot} = 2W_{\pm} + Z_0 + \epsilon_i.$$

Regard now E_{tot} as an arbitrary input parameter to calculate the values of $W^{\pm}, Z^0, \epsilon_i, G_F$; the results summarized in the Table 1 list the various outputs implied by respective trial values of E_{tot}. It appears that G_F decreases with E_{tot}

Table 1. Values of G_F and rest masses calculated solving the system of Equation (2.23).

E_{tot} [GeV]	$G_F\left[\mathrm{GeV}^{-2}\right]$	Z^0 [GeV]	$W^{\pm}$ [GeV]	ϵ_i [GeV]	$E_{tot}\epsilon_i\alpha\left[\mathrm{GeV}^2\right]$
84.160	0.0001048813336	30.403	26.788	0.178	0.109
168.320	0.00002622033340	60.807	53.577	0.356	0.437
252.480	**0.00001165348151**	**91.211**	**80.366**	**0.534**	**0.984**
336.640	$6.555083350 \times 10^{-6}$	121.615	107.155	0.712	1.749
504.738	$2.915933729 \times 10^{-6}$	182.342	160.663	1.069	3.937

according to (2.22), whereas $W^{\pm}, Z^0, \epsilon_i$ are increasing functions of E_{tot}: reasonably the various E_{tot} include in general the possible kinetic energies allowed to the force carriers in addition to their own rest masses. Is remarkable the bold sequence of outcomes, which shows in particular values of $W^{\pm}$ and Z^0 consistent with the experimental rest masses (2.1) while G_F agrees with (2.22) too: the deviations between calculated and experimental central values are −0.015% and 0.026% respectively for the former and −0.08% for the latter. The predictions of the model indicate that E_{tot} of the bold row able to reproduce the values (2.1) is threshold energy just enough to trigger carriers at rest, whereas any excess value contributes to their kinetic energy and of course modifies also their interaction energy. In short the calculated values of the two top rows concern mere total energies, the central row rest masses, the two bottom rows massive carriers with kinetic energy.

Deserves attention the idea of regarding (2.2) of the weak interaction at low trial values of E_{tot} as mere energy terms, which surrogate the actual lack of massive carriers: *i.e.* the two top rows of **Table 1** read in fact 84.160 GeV and 168.320 GeV only, which however indicate a different form of interaction. The remainder of this model aims to support these considerations. Note that:

1) the value 252.48 GeV is comparable to the lowest vacuum energy state of the Higgs field, reported in the literature as ~246 GeV [3], and corresponds well to the energy of two Higgs bosons [4].

2) Consider the last column of **Table 1**, which reports values of $E_{tot}\epsilon_i\alpha \approx 1\,\text{GeV}^2$ for the bold row and $\neq 1\,\text{GeV}^2$ for other trial values of E_{tot}. Furthermore (2.13) and (2.18) show that the same holds for $(E_{tot}\alpha)^2 x$, which suggests that $E_{tot}\epsilon_i\alpha \approx (E_{tot}\alpha)^2 x$; in effect $\epsilon_i \approx E_{tot}\alpha 0.29 = 0.534\,\text{GeV}$ follows by consequence. In fact also this connection emphasizes the peculiarity of the bold row, which discriminates the test values of E_{tot} of boson system rests masses from higher values including their kinetic energy.

3) Estimate the characteristic length r_w of the system of carriers, noting that in general the coefficient 1/3 in (2.13) is one of the possible links between energy density η and pressure P of any thermodynamic system; in this context, these concepts are reasonably relatable to the arbitrary volume V and surrounding surface S of space where are ideally confined the force carriers. In turn S and V imply r_w characteristic size of this kind of short range interaction. Accordingly one expects $\eta = E_{tot}/V$ and $P = (1/3)E_{tot}/V$; thus $E_{tot}/3 \sim W^{\pm}$ suggests that large part of the interaction energy ϵ_i in V is due to the charged bosons. For brevity and simplicity, follow some order of magnitude estimates. Introduce an energy ϵ such that owing to (2.13)

$$V = \left(\frac{\hbar c}{\epsilon}\right)^3 \Rightarrow r_w = \frac{\hbar c}{\epsilon} \quad \epsilon V = \frac{(\hbar c)^3}{\epsilon^2} \quad \eta = \frac{\epsilon^4}{(\hbar c)^3} \quad P = \frac{\eta}{3} = \frac{1}{3}\frac{\epsilon^4}{(\hbar c)^3}: \qquad (2.24)$$

these definitions describe the reaching of threshold energy necessary to create the rest masses concerned in the bold row. Check indeed whether or not all definitions have simultaneously reasonable physical meaning; the calculations are carried out identifying of course $\epsilon \equiv E_{tot} = 0.404\,\text{erg}$ of the bold row of **Table 1**. The results are

$$r_w = 0.8\times10^{-16}\,\text{cm},\ V = 5\times10^{-49}\,\text{cm}^3,\ E_{tot}V = 2\times10^{-49}\,\text{erg}\cdot\text{cm}^3,$$

$$\eta = 8\times10^{47}\,\frac{\text{erg}}{\text{cm}^3},\ P = 3\times10^{47}\,\frac{\text{dyn}}{\text{cm}^2}.$$

Inside V the average temperature of the boson system is expected of the order of $0.404/k_B \sim 3\times10^{15}\,\text{K}$ during the time transient t_w allowed by (3.1)

$\hbar/0.404 \sim 3\times10^{-27}$ s : since $ct_w = 3c\times10^{-27} \sim 0.9\times10^{-16}$ cm , the space range r_w already found is compatible with $r_w \lesssim ct_w$ of massive carriers.

4) The values of the first two lines of Table 1, although under threshold and thus inadequate to account for the formation of massive boson carriers, have been reported for completeness and to show that in fact other forms of low E_{tot} energy weak interaction cannot be excluded. Further data clarify this point. By analogy with (2.22) define now $G'_F = \sqrt{\alpha}/(E_{tot}\epsilon_i) = 6.332\times10^{-4}\ \mathrm{GeV}^{-2}$ and note that $G'_F = G_F/0.0184$, being the numerical factor $0.0184 = E_{tot}\epsilon_i/Z^0W^{\pm}$. Apparently G'_F is a mere different way to rewrite G_F, however is remarkable the fact that this dimensionless form holds for any $E_{tot}\epsilon_i$, as shown in the Table 2. In general the solution of the system (TNM) defines a variety of invariant outcomes, some of which are listed in the headline of the table to evidence various chances of constant properties of the weak force carriers compatible with any E_{tot} of Table 1; *i.e.* the functions concerned in Table 2 are E_{tot} invariants.

Table 2. Values of E_{tot} invariant functions.

E_{tot} [GeV]	$G_F E_{tot}^2$	$G_F \epsilon_i^2$	$\frac{E_{tot}\epsilon_i}{W^{\pm}Z^0}$	$\frac{\epsilon_i}{2W^{\pm}-Z^0}$
84.160	0.7428645851	3.331577475×10^{-6}	0.01841607696	0.007690836524
168.320	0.7428645851	3.331577474×10^{-6}	0.01841607696	0.007690836518
252.480	0.7428645850	3.331577474×10^{-6}	0.01841607694	0.007690836524
336.640	0.7428645851	3.331577473×10^{-6}	0.01841607695	0.007690836518
504.960	0.7428645849	3.331577475×10^{-6}	0.01841607696	0.007690836524

The notation emphasizes that the various rows of the Table 2 are in fact indistinguishable despite their physical meaning follows from the Table 1. Clearly even the early (2.17) are in fact better understood as a further example of E_{tot} invariance.

To explain these results consider the last column of Table 2: since (2.9) implies

$$\frac{\epsilon_i}{2W^{\pm}-Z^0} = \frac{\pi}{2-\sqrt{4\pi^2\alpha+1}}\frac{\epsilon_i}{E_{tot}}, \tag{2.25}$$

all values at the right hand side correspond to a unique function of α because of $\epsilon_i/E_{tot} = \zeta_-$ of (2.8). Despite the different physical meaning of E_{tot} and ϵ_i in the various rows, their ratio is assumed anyway equal to ζ_- and also implemented in the other functions of the Table 2.

This conclusion can be seen from another point of view. Calculate the central row of values of the functions listed in Table 2; if it is true that ζ_- is uniquely defined by α, then

$$\epsilon_{GF} = \frac{\sqrt{\epsilon_i}}{\sqrt{E_{tot}G_F}} = \sqrt{\frac{\zeta_-}{G_F}} = 13.46\ \mathrm{GeV}. \tag{2.26}$$

Rises now the question: since in principle the Table 1 can be extended to even smaller values of E_{tot}, what happens if the trial E_{tot} is scaled down e.g. by an arbitrary factor 10^{-9}? Nothing prevents the values of **Table 1** from being reconsidered at even lower and lower energies, e.g. of the order of eV. Identical calculations, carried out merely extrapolating the first column of input data to lower values of E_{tot} in the range of the eV, yield now the numerical solution of (2.23)

$$E'_{tot} = 252.48 \times 10^{-9}\ \text{GeV} \qquad G'_F = 1.165348151 \times 10^{13}\ \text{GeV}^{-2}$$

with

$$Z'^0 = 9.121155597 \times 10^{-8} \quad W'^{\pm} = 8.036688007 \times 10^{-8} \quad \epsilon'_i = 5.346839069 \times 10^{-10}\ \text{GeV}.$$

This solution at under threshold values of energy which excludes the presence of massive bosons reads however

$$E'_{tot} = 252.48 = 91.211 + 2 \times 80.367 + 0.534\ \text{eV} \qquad \epsilon'_{GF} = \sqrt{\frac{\zeta_-}{G'_F}} = 13.46\ \text{eV}, \quad (2.27)$$

i.e. once more the values of bold row of **Table 1**, but falling now in electron volt energy domain typical of the chemical bond. As expected, all energies are equally scaled down with respect to the corresponding ones of **Table 1**. So (2.26) turns into the value of ϵ'_{GF}.

3. Quantum Uncertainty

The last point of this model concerns the way to introduce the rest masses of the weak force carriers. Is useful to this aim the concept of quantum uncertainty formulated as

$$\delta x \delta p_x = n\hbar = \delta t \delta \varepsilon : \qquad (3.1)$$

the notation δp_x implies an arbitrary value p_x of momentum component allowed in a range size with boundaries p'_x and p''_x arbitrary as well. In short (3.1) reads $\delta p_x = p''_x - p'_x$, with $p'_x \le p_x \le p''_x$. To explain (3.1) consider a set of n elementary quantum actions $\hbar$. Next write $n\hbar \equiv n t_{Pl} \epsilon_{Pl}$ via Plank energy and Planck time ϵ_{Pl} and t_{Pl}. Eventually, defining the integer n as a product of two arbitrary $n'n'' = n$, one finds that $n\hbar = (n' t_{Pl})(n'' \epsilon_{Pl})$ yields $n\hbar = \delta t \delta \epsilon$, having put $\delta t = n' t_{Pl}$ and $\delta \epsilon = n'' \epsilon_{Pl}$: *i.e.* the time and energy quantum range sizes are defined by arbitrary n' and n'' times the respective Planck units. The same holds for Planck momentum and length $n\hbar = (n' p_{Pl})(n'' \ell_{Pl})$. By definition n is integer, n' and n'' do not; no assumption is necessary about the quantum uncertainty ranges. The reasons why (3.1) is useful to obtain contextually quantum and relativistic results, are omitted for brevity but explained in [5]. It is only worth noticing that any model based on uncertainty ranges, and not on deterministic local dynamical variables, is by definition: 1) 4-dimensional, because the time range is inherently involved in (3.1), 2) independent of reference system, because neither lower nor upper boundary coordinates of the uncertainty ranges are in principle specified and specifiable, and 3) extra-dimensional, *i.e.* the scalar product $\delta x \cdot \delta p$ of conjugate components of momentum and space vector ranges admits in principle an arbitrary number of space extra dimensions additional to time dimension.

A few key implications of (3.1) are enough to bypass the problem of introducing massive boson carriers starting from the standard model. First, include for generality the idea of charged massive carriers: if so, then it follows that two of the them must have opposite charges. Also, assuming for simplicity that the charged carriers have identical masses, the quantum/relativistic context provided by (3.1) allows to justify without further considerations the set of three masses (2.1).

Rewrite (3.1) as $v_x\delta p_x = \delta\varepsilon = n\hbar/\delta t \equiv n\hbar\omega$, having put $v_x = \delta x/\delta t$. On the one hand it yields $\delta\varepsilon = nh\nu = nh\nu/\lambda = npv$ with $p = h/\lambda$, *i.e.* the energy range size $\delta\varepsilon$ consists of n wavelike pv units. On the other hand, multiplying both sides by ε, consider $\varepsilon v_x\delta p_x = \varepsilon\delta\varepsilon = \delta(\varepsilon^2/2)$. Assume $\varepsilon v_x \propto p_x$, which yields $kp_x\delta p_x = \delta(\varepsilon^2/2)$ with k proportionality constant, in order to obtain $\delta(p_x^2 k) = \delta(\varepsilon^2)$. This result reads identically $\delta(p_x^2 k - \varepsilon^2) = 0$ *i.e.* $\varepsilon^2 - p_x^2 k = C$ with C arbitrary constant, which yields $p_x^2 k + C = \varepsilon^2$. Guessing by dimensional reasons $k \equiv c^2$, the result is the energy equation of the special relativity; indeed $C = \varepsilon_0^2$ for $v_x \to 0$, *i.e.* C is rest mass energy. Since $p_x/v_x = \varepsilon/c^2$, write thus the one-dimensional equations

$$\varepsilon_0 = c^2 \lim_{v_x\to 0}\frac{p_x}{v_x} = mc^2 \;\Rightarrow\; \varepsilon^2 = (p_x c)^2 + (mc^2)^2 \qquad p_x = \pm\frac{\varepsilon v_x}{c^2}, \tag{3.2}$$

which merged with trivial steps yield

$$\varepsilon^2 = \frac{\varepsilon^2 v_x^2}{c^2} + (mc^2)^2 \;\Rightarrow\; \varepsilon^2 = \frac{(mc^2)^2}{\beta^2} \qquad p_x^2 = \frac{(mv_x^2)^2}{\beta^2} \qquad \beta = \pm\sqrt{1-\frac{v_x^2}{c^2}}. \tag{3.3}$$

Now multiply (3.1) by $\varepsilon/\delta t$ and implement the notations $\delta(p_x)^2 \equiv p_x''^2 - p_x'^2$ and $(\delta p_x)^2 \equiv (p_x'' - p_x')^2$; then $\varepsilon v_x\delta p_x = \varepsilon\delta\varepsilon$ yields $c^2\delta p_x^2/2 = \varepsilon\delta\varepsilon$, whence

$$\frac{\delta(p_x)^2}{\delta\varepsilon} = \frac{2m}{\beta} \qquad m = \frac{\varepsilon\beta}{c^2}. \tag{3.4}$$

On the one hand note that the mass m is inferred by dimensional reasons in (3.4), despite it does not appear explicitly in (3.1): in general it is extracted from the ratio *momentum²/energy*, whereas the rest mass m is inferred form the finite limit (3.2) of p_x/v_x for $v_x \to 0$ of the special relativity.

On the other hand, consider the possible negative sign of β to implement wavelike properties of momentum and energy, coherently with $n\hbar\omega$. Rewrite (3.4) taking $-\beta$; it follows

$$\frac{\delta(p_x c)^2}{2\delta\varepsilon} = i^2\frac{mc^2}{\beta} \;\Rightarrow\; \frac{\partial(hc/\lambda)^2/\partial(1/\lambda)}{2\partial(h\nu)/\partial\nu}\frac{\delta(1/\lambda)}{\delta\nu} = i^2\frac{mc^2}{\beta}$$

i.e.

$$\frac{(hc)^2/\lambda}{h}v_g^{-1} = i^2\frac{mc^2}{\beta} \qquad v_g = \frac{\delta\nu}{\delta\lambda^{-1}},$$

whence

$$\frac{h}{i} = p_g\lambda i \qquad \frac{mv_g}{\beta} = p_g.$$

Introduce an appropriate function ψ such that $\delta\psi/\psi = i^{-1}$, in order to define $h(\delta\psi/\psi) = ip_g\lambda$; next express the wavelength λ to describe a classical set of n steady waves compliant with the range δx of (3.1): since $2\pi\delta x = nh/\delta p_x = n\lambda$, approximating $\delta p_x \approx p_x = h/\lambda$ result contextually not only the early quantization condition and De Broglie momentum but also

$$\frac{n\hbar}{i}\frac{\delta\psi/\psi}{\delta x} = p_g \Rightarrow \frac{n\hbar}{i}\frac{\delta\psi}{\delta x} = p_g\psi. \tag{3.5}$$

Repeat now the calculation (3.4) considering two different particles of rest masses m' and m'' pertinent to the respective ratios, which of course imply an analogous result and thus yield

$$\frac{\delta(pc)^2}{2\delta\varepsilon} \Rightarrow \frac{\delta(p_xc)^2}{2\delta\varepsilon} = \frac{mc^2}{\beta} \quad \frac{\delta(p'c)^2}{2\delta\varepsilon'} = \frac{m'c^2}{\beta'} \quad \frac{\delta(p''c)^2}{2\delta\varepsilon''} = \frac{m''c^2}{\beta''}. \tag{3.6}$$

The uncertainty ranges of momenta of different particles are arbitrary and thus independent by definition; yet is also sensible in principle to relate m' and m'' to independent components of $\boldsymbol{p}$ in the ordinary 3D space, whence the chance that $p' = p_y$, $p'' = p_z$.

Anyway the definitions (3.6), plugged in the model hitherto described, imply in fact that β of (3.4) can be in principle ≤ 1, depending on whether $v_x \geq 0$. Write then owing to (3.3)

$$\frac{\delta(pc)^2}{2\delta\varepsilon} \Rightarrow \begin{array}{ll} \nearrow mc^2 + m'c^2 + m''c^2 & \beta,\beta',\beta'' = 1 \\ \searrow \dfrac{mc^2}{\beta} + \dfrac{m'c^2}{\beta'} + \dfrac{m''c^2}{\beta''} & \beta,\beta',\beta'' < 1 \end{array} : \tag{3.7}$$

the physical meaning of this result reminds in fact the conclusion inferred from the Table 1: the three addends of the upper line, in principle defined independently, correspond to the rest energies of the bold row of the Table 1 concerning rest masses of the carriers, the addends of the lower line correspond to the carriers with kinetic energies at any E_{tot} above the threshold energy. This is in principle possible for $W^{\pm}, Z^0, \epsilon_i$ with appropriate values of m, m', m'' at the higher kinetic energies implied by β, β', β''.

Consider next the left hand side of (3.7). It is immediate to verify that

$$\frac{\delta(pc)^2}{2} = \left(\frac{\delta(pc)}{2}\right)^2 - \frac{c^2(3p' + p'')(p' - p'')}{4} \quad \delta p = p'' - p' \quad \delta(p^2) = p''^2 - p'^2,$$

i.e.

$$\delta(pc)^2 = (\delta(pc))^2 - \epsilon'\delta\epsilon'':$$

since by definition the right hand side is difference of two addends, the lower line of (3.7) can be related to (2.7). Dividing both sides of this result by E_{tot}^2 one finds

$$\frac{c^2(3p'_x + p''_x)(p'_x - p''_x)}{4E_{tot}^2} = \left(\frac{\delta(p_xc)}{2E_{tot}}\right)^2 - \frac{\delta(p_xc)^2}{2E_{tot}^2}$$

$$\Leftrightarrow \frac{Z^{02} - W^{\pm 2}}{4E_{tot}^2} = \alpha^* + \frac{\epsilon_i^2}{4E_{tot}^2} - \frac{\pi - 2}{2\pi}\frac{\epsilon_i}{E_{tot}}:$$

it is enough to assign appropriate values to the arbitrary $p'_{x,y,z}$ and $p''_{x,y,z}$ resulting from (3.1) and (3.7), to fit the values Z^0 and $W^{\pm}$ via uncertainty energy range sizes $\delta(p_x c)^2$ and $(\delta(p_x c))^2$ arbitrary as well. On the one hand this check evidences the connection between (3.1) and the first part of this model, focused on the masses of the bosons initially taken "a priori" as granted; on the other hand (3.7) highlights the physical meaning of the masses themselves, here initially introduced through the given formulation of quantum uncertainty along with the electromagnetic term $n\hbar\omega$. Beside the connection of E_{tot} with the Higgs field and Higgs boson energy, already emphasized, more information is still available through (3.7).

To exemplify how physical information is extracted from (3.1), it is enough to multiply both sides of (3.7) by ε; one finds

$$\frac{1}{2}\delta\left(\varepsilon^2\right)=\varepsilon\delta\left(\frac{p_x^2}{2m}\right)=\delta\left(\frac{\varepsilon p_x^2}{2m}\right)-\frac{p_x^2}{2m}\delta\varepsilon \Rightarrow \delta\left(\varepsilon^2\right)=\delta\left(\frac{\varepsilon p_x^2}{m}\right)-\frac{p_x^2}{m}\delta\varepsilon$$

and thus, reminding (3.2), $\delta\left(\varepsilon^2-\varepsilon p_x^2/m\right)=p_x^2\delta\varepsilon/m=n\hbar\omega p_x^2/m$ reads

$$E_{qg}^2=\left(pc\right)^2+\left(mc^2\right)^2-\frac{\left(p_x c\right)^2\varepsilon}{mc^2} \quad E_{qg}^2=\frac{n\hbar\omega\left(p_x c\right)^2}{mc^2} \quad \varepsilon=n\hbar\omega. \quad (3.8)$$

It is evident that the quantum-relativistic form of E_{qg} generalizes (3.2): it appears that (3.8) is a quantum gravity equation [6]. These last considerations, seemingly redundant or even out of place, have been emphasized to evidence a relevant concept: the conceptual basis of this model implementing the formulation (3.1) of quantum uncertainty is straightforwardly generalizable far beyond the initial purposes of its formulation. The chance of including the quantum gravity equation (3.8) and the quantum wave formalism (3.5) proves the generality of outcomes provided by (3.1). More specifically, it seemed useful to support this way to introduce the three rest masses (3.7) via (3.1) through the subtle steps (3.5) and (3.6) as a sensible alternative to the usual theoretical models that go back to the standard model Lagrangian.

4. Conclusion and Open Points

The paper has introduced a simple model on a very complex theoretical problem. The solutions (2.14) and (2.19) consist of three values, the first ones of which have recognizable physical meaning. Do the second and third values, not yet implemented, have their own physical meaning too? Also, the value α^* appears in the last column of **Table 2**. The invariant definitions of the weak interaction imply further considerations, at the moment in progress.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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