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A Kinematical Lie Group without Boosts Parameters: Symplectic Realization and Perturbed Hamiltonian Dynamics

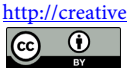
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Abstract

In this paper, we introduce a parameterized kinematical transformation group without boost generators, whose infinitesimal structure yields a physical realization of the class 5 Lie algebra in Ganbouri's classification. By means of a non-trivial Lie algebra 2-cocycle, we then construct a non-central extension of the corresponding Lie algebra. Using the Kirillov coadjoint orbit method, we realize symplectically the extended Lie group and investigate the associated Hamiltonian dynamics. The resulting Hamiltonian explicitly depends on a deformation parameter and the energy, leading to equations of motion that describe a perturbed harmonic oscillator. This results establish a connection between the cohomological properties of the extended kinematical Lie group, its symplectic geometry and the emergence of perturbed Hamiltonian dynamics.

Keywords

Dynamical System, Coadjoint Orbit, Symplectic Realization, Perturbed Hamiltonian Dynamics

1. Introduction

While the idea that symmetry group would determine the system which carries it goes back to Wigner [1], it has been Souriau [2] who put it into precise form. His important theorem says that when a symmetry group G acts transitively on the phase space then the latter is a coadjoint orbit of G endowed with its canonical symplectic structure [2]-[4]. In other words, the classical phase spaces of elementary systems correspond to coadjoint orbits of their symmetry group. The notion of coadjoint orbits (orbits of the action of a Lie group on the dual of its Lie algebra)

is the main ingredient of the orbit method.

Moreover, symplectic geometry is the mathematical counterpart of Hamiltonian formalism in classical mechanics. There are three main sources of symplectic manifold: cotangent bundles, complex algebraic manifolds and coadjoint orbits [5]. In this article, we are interested in this last source of symplectic structures as it will be seen later in this work.

Historically, the orbit method has been proposed in [6] for the description of the unitary dual of nilpotent Lie groups. It turned out that the method not only solves this problem but also gives simple and visual solution to all other principal questions in representation theory [4]. This theory was extended later to the case of solvable groups in [3] and has also paved the way for the geometric quantization which constitutes its natural generalization. The coadjoint orbit method provides then a useful geometric framework for connecting Lie groups symmetries with symplectic structures and classical dynamical systems. Recent works have further developed the use of orbit method in symplectic geometry and geometric approaches to physical systems [7] [8]. There has been also a great interest to study noncommutative phase spaces of kinematical Lie groups by applying the coadjoint orbit method [9]-[13].

In this context, the present work applies the coadjoint orbit method to a noncentral extension of a kinematic Lie group and constructs the corresponding symplectic realization. More precisely, the aim of this paper is the construction of symplectic realization of a kinematical Lie group without boosts parameters whose Lie algebra realize physically the class 5 Lie algebra presenting in [14]. Through these constructions, the maximal coadjoint orbit obtained is a symplectic manifold equipped with a modified symplectic 2-form. This constitutes the geometrical framework of a classical dynamical system associated specifically with a disturbed harmonic osciallator [15]. We have chosen this kinematical transformation group without boost generators because its symplectic realization has not been curried through before. In other papers, we study the cases corresponding to the other classes of Lie algebras of Ganbouri's classification whose symplectic realizations are not already constructed. The obtained results will be compared with these ones and a new classification in terms of symplectic realizations and Hamiltonian dynamics will be established.

The paper is organized as follows. The second section introduces the kinematical transformation group considered in this work and its Lie algebra.

The third section studies the noncentral extension of the above Lie structures which is obtained by use of standard methods [16]-[18]. In the fourth section, we construct symplectic realization which corresponds to this particular Lie group (without boots parameters) by coadjoint orbit method. The expression of the associated Hamiltonian as well as the form of the solutions of the motion equations show that the phase space describes an oscillating universe which is more specifically a disturbed harmonic oscillator.

2. The Kinematical Transformation Group and the Associated Lie Algebra

Let consider the plane transformation group defined by

$$x_0'^i = \mathrm{e}^{vt} R_i^j(t) x_0^i + x^i \tag{1}$$

with

$$R(t) = \begin{pmatrix} \cos\omega t & \sin\omega t \\ -\sin\omega t & \cos\omega t \end{pmatrix} \tag{2}$$

and where ω is a rotation parameter and $i, j = 1, 2$.

Let now consider the kinematical transformation group G defined by the multiplication law

$$(\boldsymbol{x}, t)(\boldsymbol{x}', t') = \left(\mathrm{e}^{vt} R(t)\boldsymbol{x}' + \boldsymbol{x}, t + t'\right) \tag{3}$$

where $\boldsymbol{x}$ is a space translation vector and t is a time translation parameter. It is a planar Lie group whose associated Lie algebra $\mathcal{G}$ is generated by the left invariant vector fields

$$P_j = \mathrm{e}^{vt}\left(R(t)\right)_j^i \frac{\partial}{\partial x^i}, H = \frac{\partial}{\partial t} \tag{4}$$

which satisfy the following Lie brackets relations

$$\left[H, P_j\right] = P_i \Omega_i^j, \left[P_i, P_j\right] = 0 \tag{5}$$

with $\Omega_i^j = v\delta_i^j + \omega\epsilon_i^j$

i.e.

$$\Omega = \begin{pmatrix} v & -\omega \\ \omega & v \end{pmatrix} \tag{6}$$

It is clear that v, ω like H have the inverse of time as physical dimension.

The Lie structure defined in (5) corresponds to the physic realization of the class 5 Lie algebras of the classification of three-dimensional Lie algebras considered in [14] where the generators as well as the rotation parameter have been renamed.

From (3) follows that G admits the 3-dimentional representation

$$D(\boldsymbol{x}, t) = \begin{pmatrix} \mathrm{e}^{vt}\left(R(t)\right) & \boldsymbol{x} \\ {}^t\mathbf{0} & 1 \end{pmatrix} = \begin{pmatrix} I & \boldsymbol{x} \\ {}^t\mathbf{0} & 1 \end{pmatrix} \times \begin{pmatrix} \mathrm{e}^{vt}\left(R(t)\right) & \mathbf{0} \\ {}^t\mathbf{0} & 1 \end{pmatrix} \tag{7}$$

We can then write the general element of this class of Lie groups as:

$$g = \exp\left(x^i P_i\right)\exp\left(tH\right) \tag{8}$$

In the following sections, G and $\mathcal{G}$ denote the class representation of the above Lie groups and algebras respectively.

3. Noncentral Extension of the Class of Lie Groups *G*

The use of standard methods [3] [4] [16] [17] show that $\mathcal{G}$ admits an extended Lie algebra structure generated by P_i, H and H, for $i = 1, 2$ and defined by

the following Lie brackets:

$$\left[H,P_j\right]=P_i\Omega_i^j,\left[P_i,P_j\right]=e\mathcal{H}\epsilon_{ij},\left[H,\mathcal{H}\right]=2\nu\mathcal{H} \tag{9}$$

The consistency of the extended Lie algebra can be checked by using the Jacobi identities. The non-trivial identity involving the generators $\left(H,P_1,P_2\right)$ is $\left[H,\left[P_1,P_2\right]\right]+\left[P_1,\left[P_2,H\right]\right]+\left[P_2,\left[H,P_1\right]\right]=0$.

Indeed, using $\left[P_1,P_2\right]=e\mathcal{H}$ and $\left[H,\mathcal{H}\right]=2\nu\mathcal{H}$ and both $\left[H,P_1\right]=\nu P_1+\omega P_2$ and $\left[H,P_2\right]=-\omega P_1+\nu P_2$, we obtain $\left[H,\left[P_1,P_2\right]\right]+\left[P_1,\left[P_2,H\right]\right]+\left[P_2,\left[H,P_1\right]\right]=2\nu e\mathcal{H}-\nu e\mathcal{H}-\nu e\mathcal{H}=0$. The other Jacobi identities are satisfied by the commutation relations given in 9. Therefore, the extended bracket define a consistent Lie algebra.

Moreover, $\left[H,\mathcal{H}\right]=2\nu\mathcal{H}$ shows that $\mathcal{H}$ is not central for $\nu\neq 0$. Thus, the extension considered here is a noncentral extension because the additional generator $\mathcal{H}$ does not commute with the time translation generator.

Let g be given by (8). Then the general element $\tilde{g}$ of the connected Lie group $\tilde{G}$ of G can be written as:

$$\tilde{g}=\exp\left(a\mathcal{H}\right)\exp\left(x^iP_i\right)\exp\left(tH\right) \tag{10}$$

where a has the dimension of an area times an electric charge.

By use of the Baker-Campbell-Hausdorff formulae [19] and by identifying $\tilde{g}$ with $\left(a,\boldsymbol{x},t\right)$, we verify that the multiplication law for the connected extended Lie group $\tilde{G}$ is:

$$\left(a,\boldsymbol{x},t\right)\left(a',\boldsymbol{x}',t'\right)=\left(\mathrm{e}^{2\nu t}a'+\frac{e}{2}\mathrm{e}^{\nu t}\boldsymbol{x}\cdot R\left(t\right)J\boldsymbol{x}'+a,\mathrm{e}^{\nu t}R\left(t\right)\boldsymbol{x}'+\boldsymbol{x},t+t'\right) \tag{11}$$

where J is an angular momentum.

The additional term in this multiplication law is obtained from the noncommutativity of the translation generators $\left[P_i,P_j\right]=e\mathcal{H}\epsilon_{ij}$. Using the Baker-Campbell-Hausdorff formula, this commutation relation gives the bilinear contribution $\frac{e}{2}\mathrm{e}^{\nu t}\boldsymbol{x}\cdot R\left(t\right)J\boldsymbol{x}'$ in the first component of the group law (11). This term satisfies the cocycle condition, it is the corresponding group 2-cocycle.

4. Symplectic Realization and Motion Equations

We are now going to construct the classical dynamical system associated with the extended Lie group. Following the Souriau's prescription [2], we construct indeed the coadjoint orbit associated to the extension of this Lie group when evolution of time is considered. This coadjoint orbit is identified as an elementary system associated with the extended Lie group. Several results have been obtained by applying the coadjoint orbit method on other Lie groups [9] [10] [12] [13].

The adjoint action $Ad_g\left(\delta\hat{g}\right)=g\left(\delta\hat{g}\right)g^{-1}$ of the quotient group $Q=\tilde{G}/Z\left(\tilde{G}\right)$ on the corresponding extended Lie algebra $\tilde{\mathcal{G}}$ is in this case given by

$$\begin{aligned} &Ad_{(\boldsymbol{x},t)}\left(\mathrm{d}a,\mathrm{d}\boldsymbol{x},\mathrm{d}t\right) \\ &=\left(\mathrm{e}^{2\nu t}\mathrm{d}a+e\mathrm{e}^{\nu t}\boldsymbol{x}\cdot JR\left(t\right)\mathrm{d}\boldsymbol{x}+e\omega\frac{\boldsymbol{x}^2}{2}\mathrm{d}t,\mathrm{e}^{\nu t}R\left(t\right)\mathrm{d}\boldsymbol{x}-\Omega\boldsymbol{x}\mathrm{d}t,\mathrm{d}t\right) \end{aligned} \tag{12}$$

where Ω is given by (6).

If the duality between the extended Lie algebra $\tilde{\mathcal{G}}$ and its dual Lie algebra $\tilde{\mathcal{G}}^*$ is defined by

$$\left\langle\left(B,\boldsymbol{p},E\right),\left(\mathrm{d}a,\mathrm{d}\boldsymbol{x},\mathrm{d}t\right)\right\rangle=B\mathrm{d}a+\boldsymbol{p}\cdot\mathrm{d}\boldsymbol{x}+E\mathrm{d}t \tag{13}$$

then the coadjoint action of the quotient group Q on $\tilde{\mathcal{G}}^*$ is

$$\begin{aligned} Ad^*_{(\boldsymbol{x},t)}\left(B,\boldsymbol{p},E\right)=\Bigg(&\mathrm{e}^{-2\nu t}B,\mathrm{e}^{-\nu t}R\left(t\right)\boldsymbol{p}-e\mathrm{e}^{-2\nu t}BJ\boldsymbol{x}, \\ &E+\mathrm{e}^{-\nu t}\left[\nu\boldsymbol{p}\cdot R\left(t\right)\boldsymbol{x}+\omega\boldsymbol{p}\times R\left(t\right)\boldsymbol{x}+eB\omega\frac{\boldsymbol{x}^2}{2}\right]\Bigg) \end{aligned} \tag{14}$$

where $\vec{p}$ is a linear momentum, E is an energy, B is an action while e is an electric charge.

We set

$$eB=m\omega \tag{15}$$

and we rewrite relation (14) in the following form:

$$\begin{aligned} Ad^*_{(\boldsymbol{x},t)}\left(m,\boldsymbol{p},E\right)=\Bigg(&\mathrm{e}^{-2\nu t}m,\mathrm{e}^{-\nu t}R\left(t\right)\boldsymbol{p}-\mathrm{e}^{-2\nu t}m\omega J\boldsymbol{x}, \\ &E+\mathrm{e}^{-\nu t}\left[\nu\boldsymbol{p}\cdot R\left(t\right)\boldsymbol{x}+\omega\boldsymbol{p}\times R\left(t\right)\boldsymbol{x}+m\frac{\omega^2\boldsymbol{x}^2}{2}\right]\Bigg) \end{aligned} \tag{16}$$

Note that for a fixed element $\alpha\in\mathcal{G}^*$, the coadjoint orbit through α is defined by:

$$O_\alpha=\left\{Ad^*_g\left(\alpha\right),g\in G\right\}$$

It can also be written as

$$O_\alpha=G^{'}G_\alpha$$

where G_α is the stabilizer subgroup of α. This quotient should be distinguished from the quotient group $Q=\tilde{G}/Z\left(\tilde{G}\right)$ used to describe the coadjoint action in Equation (16). The quotient Q describes the group acting on the dual Lie algebra, whereas $G^{'}G_\alpha$ describes the orbit of a particular element α.

In such a case, the relevant Poisson brackets define the Lie algebra isomorphic to the initial one. Therefore, computing the Kirillov 2-form corresponding to the Lie algebra structure (9) in the basis $\left(H,P_1,P_2,\mathcal{H}\right)$ gives rise:

$$K=\begin{pmatrix} 0 & \nu p_1+\omega p_2 & \nu p_2-\omega p_1 & 2\nu B \\ -\nu p_1-\omega p_2 & 0 & eB & 0 \\ -\nu p_2+\omega p_1 & -eB & 0 & 0 \\ -2\nu B & 0 & 0 & 0 \end{pmatrix} \tag{17}$$

For $\alpha \in \mathcal{G}^*, X, Y \in \mathcal{G}$, it has been shown that

$$K_\alpha(X,Y) = \langle \alpha, [X,Y] \rangle$$

where K_α is of the form

$$K_{ij}(\alpha) = -\alpha_k C_{ij}^k \tag{18}$$

the Kirillov 2-form [20] on $\mathcal{G}^*$. Its kernel is the infinitesimal stabilizer of α. Hence, $\dim O_\alpha = \dim(\mathcal{G}^*) - \dim(\mathrm{Ker}(K_\alpha)) = \mathrm{rang}(K_\alpha)$. For the Kirillov matrix 17, we obtain $\det K = 4e^2\nu^2 B^4$. Thus, for $e \neq 0$, $\nu \neq 0$, $B \neq 0$, the matrix K, has maximal rank four. Therefore, $\dim O_\alpha = 4$. In this case, the Kirillov form is non-degenerate and its inverse can be used to construct the symplectic two-form on the coadjoint orbit. This agrees with the dimension of the group manifold considered in the paper. Hence, in this particular case, the coadjoint orbit has the same dimension as the corresponding Lie group. Then, the Lie group considered here provides a realization of four-dimensional coadjoint orbit in the regular case.

The inverse matrix

$$K^{-1} = \begin{pmatrix} 0 & 0 & 0 & -\dfrac{1}{2\nu B} \\ 0 & 0 & -\dfrac{1}{eB} & \dfrac{\nu p_2 - \omega p_1}{2e\nu B^2} \\ 0 & \dfrac{1}{eB} & 0 & -\dfrac{\nu p_1 + \omega p_2}{2e\nu B^2} \\ \dfrac{1}{2\nu B} & -\dfrac{\nu p_2 - \omega p_1}{2e\nu B^2} & \dfrac{\nu p_1 + \omega p_2}{2e\nu B^2} & 0 \end{pmatrix} \tag{19}$$

The coadjoint orbit of the corresponding extended Lie group on the dual of its Lie algebra is then four-dimensional. It is a symplectic manifold endowed with the symplectic 2-form given (a priori) by:

$$\begin{aligned} \sigma = {} & \frac{1}{2\nu B} \mathrm{d}B \wedge \mathrm{d}E + \frac{1}{eB} \mathrm{d}p_2 \wedge \mathrm{d}p_1 + \frac{\nu p_2 - \omega p_1}{2e\nu B^2} \mathrm{d}p_1 \wedge \mathrm{d}B \\ & - \frac{\nu p_1 + \omega p_2}{2e\nu B^2} \mathrm{d}p_2 \wedge \mathrm{d}B \end{aligned} \tag{20}$$

But from the coadjoint action (14) follows that $\mathrm{d}B = -2\nu B \mathrm{d}t$.

Moreover, there must exist generators (namely momenta and boosts) which yield the basic canonical variables through the coadjoint orbit method. Thus to define the canonical (Darboux's) coordinates on the orbit in this particular case (where there is no boots), let us use new coordinates (p,q) such that $p_1 = eBq$ and $p_2 = p$ where dimensional analysis has been taking into account.

We then derive that $\mathrm{d}p_1 = eB\mathrm{d}q - 2e\nu Bq\mathrm{d}t$. We also verify that

$$Bq\,\mathrm{d}q = \mathrm{d}\left(B\frac{q^2}{2}\right) + \nu q^2 B\,\mathrm{d}t \tag{21}$$

and

$$\frac{p\,\mathrm{d}p}{B} = \mathrm{d}\left(\frac{p^2}{2B}\right) + \nu\frac{p^2}{B}\mathrm{d}t \tag{22}$$

The symplectic 2-form becomes then

$$\sigma = \mathrm{d}p \wedge \mathrm{d}q + \mathrm{d}\left(E + \frac{p^2}{2m} + \frac{q^2}{2C} - \nu pq \right) \wedge \mathrm{d}t \tag{23}$$

Or equivalently

$$\sigma = \mathrm{d}p \wedge \mathrm{d}q + \mathrm{d}H\left(p,q\right) \wedge \mathrm{d}t \tag{24}$$

where $H\left(p,q\right)$ is the Hamiltonian function (energy) and $C = \frac{1}{m\omega^2}$ is the compliance [21], that is the inverse of the usual Hooke constant, p and q being the canonical coordinates.

The Hamiltonian H can be written as:

$$H = H_0 + E - \nu pq \tag{25}$$

where

$$H_0 = \frac{p^2}{2m} + \frac{m\omega^2 q^2}{2} \tag{26}$$

is the Hamiltonian of the simple harmonic oscillator while E has an energy as physical dimension. The term $\left(-\nu pq\right)$ therefore gives a modification of the usual harmonic oscillator Hamiltonian.

The corresponding Hamiltonian equations of motion are then

$$\frac{\mathrm{d}q}{\mathrm{d}t} = -\nu q + \frac{p}{m} \tag{27}$$

and

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -m\omega^2 q + \nu p \tag{28}$$

These Hamiltonian equations of motion (27) and (28) can be viewed as autonomous systems, the variable E being kept in the Hamiltonian function (25). The solutions of this Hamiltonian system which are also called integrals of the motion are of the form:

$$q\left(t\right) = \mathrm{e}^{\nu t}\left(a\cos\omega t + b\sin\omega t\right) \tag{29}$$

and

$$p\left(t\right) = eB\mathrm{e}^{\nu t}\left(b\cos\omega t - a\sin\omega t\right) \tag{30}$$

where relation (15) have been used and where both integration constants a and b are determined by the initial conditions $q\left(0\right)$ and $\dot{q}\left(0\right)$. Thus, the motion keeps the oscillation frequency ω, its amplitude is multiplied by the factor $\mathrm{e}^{\nu t}$. For ν greater than 0, the amplitude increases, whereas for ν less than 0, it decreases. for $\nu = 0$, the usual harmonic oscillator is recovered. Therefore, the system can be considered as a perturbed harmonic oscillator [15] with the parameter ν controlling the time evolution of the oscillation amplitude.

From relation (27), we find that the massive particle (with mass m) is submitted to the following force:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = m\frac{\mathrm{d}^2 q}{\mathrm{d}t^2} + m\nu\frac{\mathrm{d}q}{\mathrm{d}t} \tag{31}$$

Then, the kinematical Lie group whose Lie algebra corresponds to the physical realization of the fifth class of the Ganbouri's classification [14] is a symmetry group of a disturbed harmonic oscillator governed by the modified Newton law [17] given by relation (31).

The resulting dynamical system can be viewed as an effective model for an oscillator with amplification or attenuation. The parameter ν controls the evolution of the amplitude through the factor $\mathrm{e}^{\nu t}$. Such structure may be relevant in physical systems where gain, loss, amplification, or attenuation mechanisms are present. However, the present work does not establish a direct correspondance with a specific experimental system. A detailed comparison with a particular physical realization is left for future work.

5. Conclusions

Models associated with a given symmetry group can be conveniently constructed by using Souriau's method. His theorem says in fact that when a symmetry goup G acts transitively on a phase space, then the later is a coadjoint orbit of G equipped with a canonical symplectic form [2]-[4]. These ideas were later extended to many groups and depending on the symmetry group considered, more general symplectic structures (so called modified symplectic ones) have been constructed [9]-[13].

In this work, we use the Souriau's prescription to construct a coadjoint orbit of noncentrally extension of the kinematical Lie group whose Lie algebra corresponds to the physical realization of the fifth class of the Ganbouri's classification. It is a four-dimensional symplectic manifold (equipped with a modified symplectic 2-form) which discribes a disturbed harmonic oscillator governed by a modified Newton law [17].

The resulting Hamiltonian describes a perturbed harmonic oscillator in which the parameter ν controls the time evolution of the oscillator amplitude. The usual harmonic oscillator is covered for $\nu = 0$. This type of effective dynamics may be relevant to system involving gain, loss, amplification or attenuation. A direct comparison with specific physical realizations would require additional model-dependent analysis and is left for future work.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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