

Accelerated Universe with Bulk Viscous Fluid in $f(R, T)$ Gravity

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Abstract

In this work, we examine an anisotropic Bianchi type-V cosmological model in the framework of modified $f(R, T)$ gravity, adopting the non-minimal coupling form $f(R, T) = f_1(R) + f_2(R)f_3(T)$ where R is the Ricci scalar and T is the trace of the energy-momentum tensor. The gravitational field equations are derived for a perfect fluid with barotropic equation of state $p = \zeta\rho$ and solved by assuming an appropriate law of variation for the mean scale factor that incorporates the deceleration parameter. In contrast to previous studies that used an incorrect exponent form, we construct the average scale

factor as $a = \left(\frac{e^{cnT} - d}{c} \right)^{\frac{1}{n}}$. Here n , c , d are constants, ensuring a physically

consistent transition from an early decelerating phase to late-time acceleration that aligns with current cosmological observations. In this work, we extend the bulk viscous Bianchi type-V cosmological model in $f(R, T)$ gravity previously studied by Bhardwaj *et al.* (2019) by introducing a new exponential-type shifted scale factor. The directional scale factors and physical quantities such as the Hubble rates, expansion and shear scalars are obtained explicitly, revealing that the model universe evolves from a highly anisotropic state toward isotropy at late times. The behavior of the effective equation of state and cosmological parameters indicate that the present accelerated expansion can be effectively realized within this model without requiring a cosmological constant term. The influence of the non-minimal coupling in the $f(R, T)$ function on the dynamical evolution, anisotropy decay, and energy conditions is analyzed in detail. Our findings are consistent with recent investigations of Bianchi type-V universes in modified gravity theories and provide new insights for further theoretical and observational studies.

Keywords

Bianchi Type-V, Space-Time, Modified $f(R, T)$ Gravity, Non-Minimal Coupling, Anisotropic Cosmology, Scale Factor

1. Introduction

The modern cosmological paradigm is largely based on the assumption of large-scale homogeneity and isotropy, well described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric within the framework of General Relativity (GR). However, early theoretical investigations and observational indications suggest that the universe may have experienced anisotropic phases during its early evolution, motivating the study of anisotropic cosmological models such as Bianchi type spacetimes [1]-[4]. These models provide valuable insight into the dynamics of anisotropy, shear dissipation, and the approach toward isotropy at late cosmic times. The discovery of the late-time accelerated expansion of the universe through Type Ia supernova observations [5] [6], along with subsequent confirmations from cosmic microwave background radiation and large-scale structure surveys [7] [8], posed a serious challenge to GR. Within the standard cosmological framework, this acceleration is attributed to an exotic dark energy component with negative pressure. Nevertheless, the physical origin of dark energy remains unresolved, prompting extensive investigations into alternative gravitational theories and effective cosmic fluids [9]-[11]. Dissipative processes in cosmology, particularly bulk viscosity, were introduced as a realistic mechanism to describe irreversible processes in the cosmic fluid [12]-[14]. Bulk viscosity can effectively generate negative pressure, influence the expansion dynamics, and contribute to isotropization, especially in anisotropic cosmological models [15]-[18]. Several studies demonstrated that bulk viscous fluids could drive accelerated expansion without invoking an explicit cosmological constant [19] [20]. In parallel, modified theories of gravity have been proposed as natural extensions of GR to address cosmic acceleration and other unresolved cosmological issues. Among them, $f(R, T)$ gravity, where the gravitational Lagrangian depends on the Ricci scalar R and the trace of the energy-momentum tensor T , has attracted considerable attention [21]. The explicit matter-geometry coupling in this theory leads to modified field equations and allows effective dark-energy-like behavior without introducing additional exotic matter components [22]-[25]. Anisotropic cosmological models within the framework of $f(R, T)$ gravity have been extensively studied to understand the combined effects of anisotropy and modified gravity. Bianchi type cosmologies, particularly Bianchi type-V models, provide a suitable geometric background to investigate anisotropic expansion, shear evolution, and energy condition validity in modified gravity scenarios [26]-[30]. Exact solutions of Bianchi type-V models have revealed that anisotropic universes can evolve toward isotropy at late times under appropriate conditions [31]-[33]. In recent years, significant attention has

been devoted to bulk viscous anisotropic cosmological models in $f(R, T)$ gravity. These studies indicate that bulk viscosity plays a crucial role in driving accelerated expansion and stabilizing the cosmic dynamics [34]-[37]. The adoption of variable deceleration parameters and new exponential shifted scale factor has further enhanced the physical realism of these models by allowing a smooth transition from early decelerated expansion to late-time acceleration [38] [39]. Very recent investigations have further strengthened the viability of anisotropic and viscous cosmological models in modified gravity. Studies on Bianchi type-V and related anisotropic models in $f(R, T)$ gravity demonstrate improved consistency with late-time acceleration, stability conditions, and observational constraints [40]-[43]. Moreover, recent works emphasize the role of bulk viscosity and matter-geometry coupling in producing realistic cosmological evolution without an explicit cosmological constant [44] [45]. The latest developments suggest that such models can effectively describe late-time cosmic acceleration and anisotropy decay within modified gravity frameworks [46]. Recently, Bhardwaj *et al.* [47] investigated a bulk viscous Bianchi type-V cosmological model in $f(R, T)$ gravity using a new exponential shifted expansion law and discussed the physical behavior of the model. Recent observational studies have further investigated the effects of bulk viscosity in $f(R, T)$ cosmological models, providing useful observational constraints on their viability [48].

Motivated by these developments, the present work investigates a bulk viscous Bianchi type-V cosmological model within the framework of $f(R, T)$ gravity. By employing a variable deceleration parameter, exact solutions of the modified field equations are obtained, and the physical behavior of key cosmological parameters such as energy density, pressure, anisotropy, bulk viscosity, and energy conditions is analyzed in detail. The results contribute to a deeper understanding of anisotropic cosmological evolution and late-time acceleration in modified gravity.

The present paper is organized as follows. In Section 2, we briefly introduce the theoretical framework of modified $f(R, T)$ gravity and derive the corresponding field equations for a bulk viscous fluid in the anisotropic Bianchi type-V spacetime. In Section 3, are obtained by assuming a new exponential-type shifted scale factor with a variable deceleration parameter, and the dynamical behavior of key cosmological parameters such as the directional and mean Hubble parameters, expansion scalar, shear scalar, anisotropy parameter, effective energy density, effective pressure, bulk viscosity coefficient, deceleration parameter, and stability criteria is analyzed with the help of graphical representation. The validity of the energy conditions is also examined in this section. In Section 4, the obtained results are compared with the corresponding bulk viscous Bianchi type-V model in General Relativity. Finally, Section 5 summarizes the main results and presents the conclusions of the present work.

2. Modified Gravity Theory Based on $f(R, T)$ Formulation

The Modified general theory of relativity in the form of $f(R, T)$ theory for

$f(R, T) = f_1(R) + f_2(R)f_3(T)$ is given by Harko *et al.* [20]

$$S = \int \left(\frac{1}{16\pi G} \sqrt{-g} f(R, T) d^4x + \sqrt{-g} L_m d^4x \right) \quad (1)$$

The standard matter Lagrangian density, denoted as L_m , corresponds to the matter source, while $f(R, T)$ represents a general function of the Ricci scalar R and the trace T of the energy-momentum tensor T_{ij} associated with the matter source. Additionally, g stands for the determinant of the metric tensor g_{ij} . The energy-momentum tensor derived from the matter Lagrangian is defined as follows:

$$T_{ij} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{ij}} \quad (2)$$

In the present work, the matter Lagrangian is chosen as $L_m = -\bar{p}$, where $\bar{p}$ denotes the bulk-viscous pressure of the cosmic fluid. This choice determines the matter-geometry coupling in $f(R, T)$ gravity and, through the Bianchi identities, leads to the modified non-conservation relation of the energy-momentum tensor.

The Field equations of $f(R, T)$ gravity are given by

$$\begin{aligned} & \left[f_1'(R) + f_2'(R)f_3'(T) \right] R_i - \frac{1}{2} f_1'(R) g_{ij} + (g_{ij} \nabla_i \nabla_j - \nabla_i \nabla_j) \times \left[f_1'(R) f_2'(R) f_3'(T) \right] \\ & = \left[8\pi + f_2'(R) f_3'(T) \right] T_{ij} + f_2(R) \left[f_3'(T) p + \frac{1}{2} f_3(T) \right] g_{ij} \end{aligned} \quad (3)$$

Here, $f(R, T) = f_1(R) + f_2(R)f_3(T)$, and the prime denotes differentiation with respect to the corresponding argument.

We assume $f_1(R) = f_2(R) = R$ and $f_3(T) = \gamma T$, where γ is a constant matter-geometry coupling parameter that characterizes the strength of the non-minimal coupling between the Ricci scalar R and the trace T of the energy-momentum tensor [22]

Thus Equation (3) yields

$$G_{ij} = 8\pi T_{ij}^{(eff)} = 8\pi (T_{ij} + T_{ij}^{(ME)}) \quad (4)$$

where $T_{ij}^{(eff)}$, T_{ij} , $T_{ij}^{(ME)}$ represent the effective energy momentum tensor, matter energy momentum tensor and extra energy term respectively. The extra energy term is written as

$$T_{ij}^{(ME)} = \frac{\gamma R}{8\pi} \left(T_{ij} + \frac{3\rho - 7\bar{p}}{2} g_{ij} \right) \quad (5)$$

Here, $\bar{p}$ is the bulk viscous pressure of the anisotropic fluid and the term $T_{ij}^{(ME)}$ represent here the matter energy coupling as proposed by Moraes and Sahoo [49] by applying the Bianchi identities in (4) yields

$$T_{ij} = \frac{\gamma R}{8\pi} \left[\nabla^i (T_{ij} \bar{p} + g_{ij}) + \frac{1}{2} g_{ij} \nabla^i (\rho - 3\bar{p}) \right] \quad (6)$$

We consider a spatially homogeneous and anisotropic Bianchi type-V space-time.

The line element for the Bianchi type-V space-time is written as

$$ds^2 = dt^2 - A^2 dx^2 - e^{2\alpha x} (B^2 dy^2 + C^2 dz^2) \quad (7)$$

Here, $A(t)$, $B(t)$, $C(t)$ are the metric tensors and cosmic time functions along x , y and z -directions respectively and α is non-zero constant

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -8\pi \bar{p}^{(eff)} \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = -8\pi \bar{p}^{(eff)} \quad (9)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\alpha^2}{A^2} = -8\pi \bar{p}^{(eff)} \quad (10)$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3\alpha^2}{A^2} = 8\pi \rho^{(eff)} \quad (11)$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \quad (12)$$

Here,

$$\bar{p}^{(eff)} = p + \frac{9\gamma}{8\pi} \left(\frac{\ddot{a}}{a} + \frac{a'^2}{a^2} \right) (\rho - 3\bar{p}) \quad (13)$$

$$\rho^{(eff)} = \rho - \frac{3\gamma}{8\pi} \left(\frac{\ddot{a}}{a} + \frac{a'^2}{a^2} \right) (3\rho - 7\bar{p}) \quad (14)$$

$$\bar{p}^{(eff)} = p^{(eff)} - 3\xi H, \bar{p} = p - 3\xi H \text{ and } p^{(eff)} = \zeta \rho^{(eff)}, p = \zeta \rho \quad (15)$$

Here, p denotes the equilibrium pressure of the normal fluid, $\bar{p} = p - 3\xi H$ represents the bulk-viscous pressure, ξ is the bulk viscosity coefficient ($0 \leq \xi \leq 1$), $\bar{p}^{(eff)}$ is the effective pressure, ρ is the matter energy density, and ρ^{eff} denotes the effective energy density. These notations are used consistently throughout this work.

From Equations (8)-(12), there is system of five equations with five unknown parameter A , B , C , $\bar{p}^{(eff)}$ and $\rho^{(eff)}$. Hence in order to solve the above equations completely we assume one relation among the physical parameter as in new exponential shifted expansion form

$$a = \left(\frac{e^{cnT} - d}{c} \right)^{\frac{1}{n}} \quad (16)$$

This form of the scale factor leads to a variable deceleration parameter and allows a smooth transition from an early decelerated phase to a late-time accelerated expansion, which is consistent with recent observational results. Here, c , d , and n are taken as positive real constants $c, d, n > 0$, ensuring a real, positive, and expanding cosmological model. T denotes the cosmic time. Integrating Equation (12) and omitting the integration constant, we obtain

$$A^2 = BC \quad (17)$$

Solving Equations (8)-(12) and (17), we get

$$A = \left(\frac{e^{cnT} - d}{c} \right)^{\frac{1}{n}} \quad (18)$$

$$B = k \left(\frac{e^{cnT} - d}{c} \right)^{\frac{1}{n}} \exp \left[b \int \left(\frac{e^{cnT} - d}{c} \right)^{\frac{-3}{n}} dt \right] \quad (19)$$

$$C = k^{-1} \left(\frac{e^{cnT} - d}{c} \right)^{\frac{1}{n}} \exp \left[-b \int \left(\frac{e^{cnT} - d}{c} \right)^{\frac{-3}{n}} dt \right] \quad (20)$$

where b and k are constants.

3. Physical Behavior of the Model in $f(R, T)$ Gravity

The directional Hubble parameters in the spatial directions are obtained as

$$H_x = \frac{\dot{A}}{A} = \frac{ce^{cnT}}{e^{cnT} - d} \quad (21)$$

$$H_y = \frac{\dot{B}}{B} = \frac{ce^{cnT}}{e^{cnT} - d} + b \left(\frac{e^{cnT} - d}{c} \right)^{\frac{-3}{n}} \quad (22)$$

$$H_z = \frac{\dot{C}}{C} = \frac{ce^{cnT}}{e^{cnT} - d} - b \left(\frac{e^{cnT} - d}{c} \right)^{\frac{-3}{n}} \quad (23)$$

The average (mean) Hubble parameter H is given by

$$H = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = \frac{ce^{cnT}}{e^{cnT} - d} \quad (24)$$

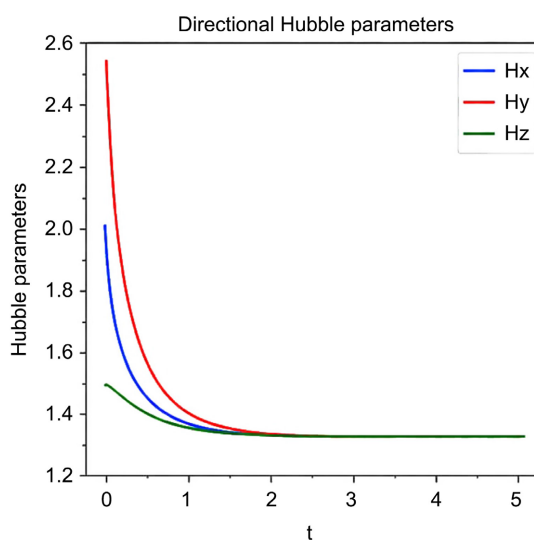


Figure 1. Directional Hubble vs Time t for $b = 0.5$, $c = 1$, $n = 3$ $d = 1$.

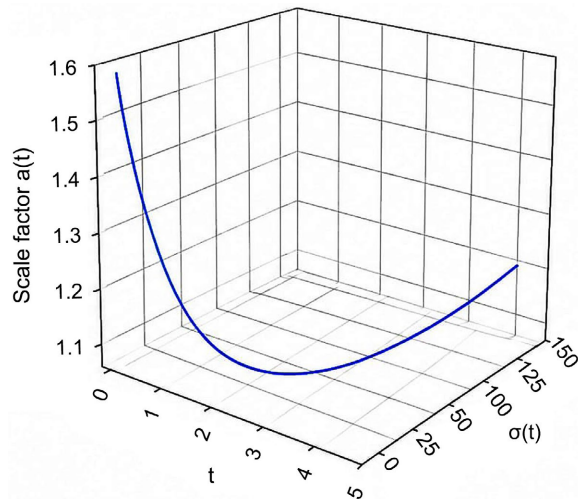


Figure 2. Scale factor vs Time t for $b = 0.5$, $c = 1$, $n = 3$, $d = 1$.

Figure 1 depicts the evolution of the directional Hubble parameters along different spatial directions. The initial deviation among the curves indicates anisotropic expansion, while their convergence at late times signifies the isotropization of the universe.

Figure 2 shows the combined behavior of the scale factor and the Hubble parameter. The monotonically increasing scale factor confirms continuous expansion, while the decreasing Hubble parameter approaching a constant value indicates a transition from an early decelerated phase to a late-time accelerated expansion.

The mean anisotropy parameter A_m and the corresponding shear scalar σ^2 are given by

$$A_m = \frac{2b^2}{3H^2a^6} = \frac{2b^2}{3} \left(\frac{e^{cnT} - d}{c^2e^{2cnT}} \right)^2 \left[\frac{e^{cnT} - d}{c} \right]^{-\frac{6}{n}} \quad (25)$$

$$\sigma^2 = \frac{b^2}{2a^6} = \frac{b^2}{2} \left[\frac{e^{cnT} - d}{c} \right]^{-\frac{6}{n}} \quad (26)$$

The scalar expansion θ and the corresponding volume scale factor V are given by

$$\theta = 3H = 3 \frac{ce^{cnT}}{e^{cnT} - d} \quad (27)$$

$$V = ABC = \left(\frac{e^{cnT} - d}{c} \right)^{\frac{3}{n}} \quad (28)$$

Figure 3(a) shows the evolution of the anisotropy parameter for the Bianchi type-V cosmological model. Its large initial value indicates a strongly anisotropic expansion, while its monotonic decrease with cosmic time confirms the gradual isotropization of the universe. **Figure 3(b)** depicts the variation of the shear scalar σ^2 , which attains higher values at early times due to dominant shear effects and decreases toward negligible values at late epochs. The decay of the shear scalar

implies suppression of anisotropic stresses during cosmic evolution. Although the expansion scalar θ and spatial volume V are not explicitly plotted, their analytical behavior indicates that remains positive throughout the evolution, ensuring continuous expansion. Moreover, the spatial volume V increases monotonically with cosmic time, satisfying the physical requirement of an expanding universe. The combined behavior of these parameters confirms the physical viability and late-time isotropization of the model.

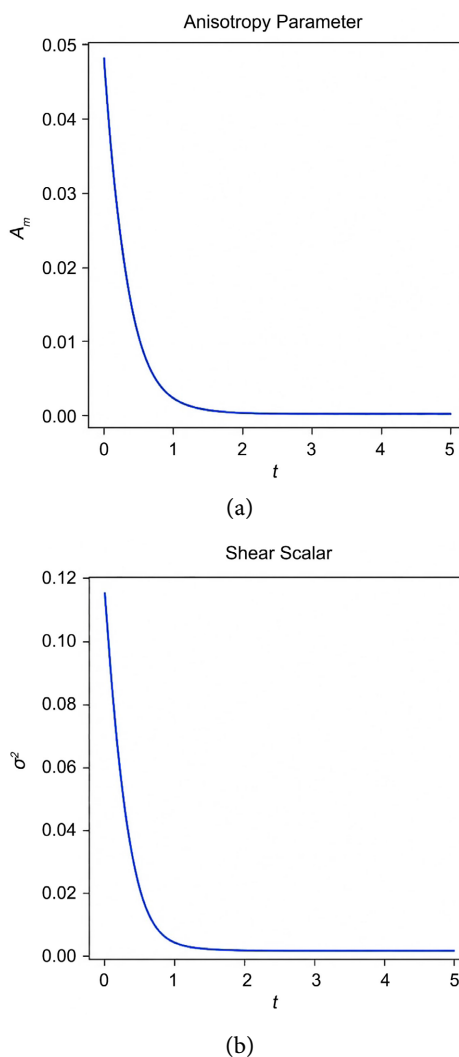


Figure 3. (a) $b = 0.25$, $c = 1$, $n = 3$, $d = 1$ A_m vs Time t ; (b) $b = 0.5$, $c = 1$, $d = 1$, $n = 3$ σ^2 vs Time t .

$$\bar{p}^{eff} = \frac{1}{8\pi} \left[\frac{2c^2 e^{2cnT} (e^{cnT} - nd)}{(e^{cnT} - d)^2} - \frac{3c^2 e^{2cnT}}{(e^{cnT} - d)^2} \right] - \frac{1}{8\pi} \left[\left(\frac{e^{cnT} - d}{c} \right)^{\frac{-6}{n}} - \alpha^2 \left(\frac{e^{cnT} - d}{c} \right)^{\frac{-2}{n}} \right] \quad (29)$$

$$\rho^{eff} = \frac{1}{8\pi} \left[\frac{2c^2 e^{2cnT}}{(e^{cnT} - d)^2} - b^2 \left(\frac{e^{cnT} - d}{c} \right)^{\frac{-6}{n}} \right] - \frac{3\alpha^2}{8\pi} \left(\frac{e^{cnT} - d}{c} \right)^{\frac{-2}{n}} \quad (30)$$

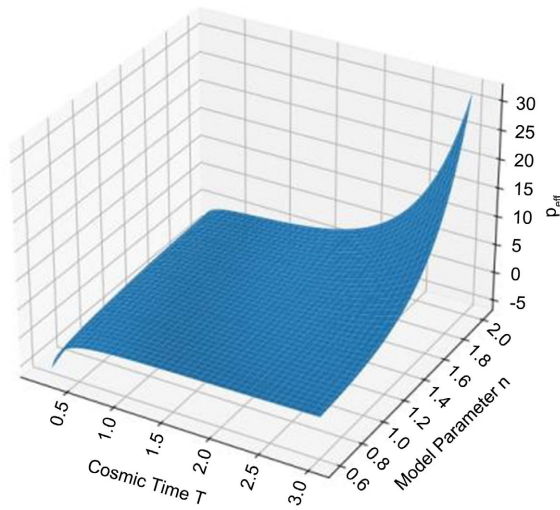


Figure 4. Effective pressure vs Time t .

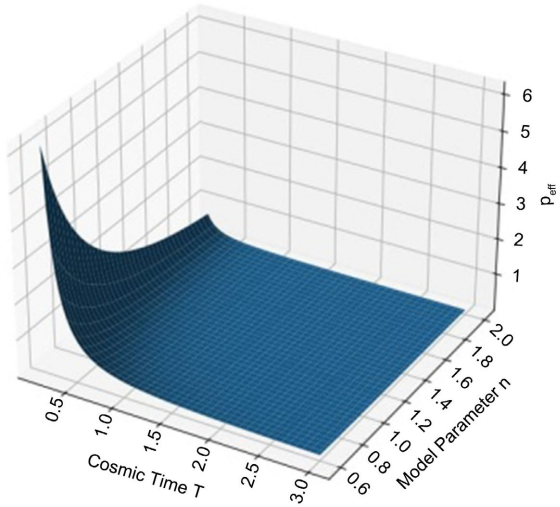


Figure 5. Effective density vs Time t .

Effective Pressure $\bar{p}^{(eff)}$

Figure 4 shows the evolution of the effective pressure with cosmic time. The effective pressure $\bar{p}^{(eff)}$ derived from the Bianchi type-V field equations given in Equations (18-20), plays a crucial role in governing the cosmic dynamics of the model. At early times, the pressure is positive, corresponding to a matter-dominated decelerating phase consistent with anisotropic Bianchi type-V geometry. As the Universe evolves, the effective pressure decreases continuously and undergoes a sign change, signaling the transition from decelerated to accelerated expansion. In the late-time regime, $\bar{p}^{(eff)}$ becomes negative and remains finite, providing the repulsive gravitational effect necessary to drive cosmic acceleration. Such negative

pressure behavior is commonly reported in Bianchi type-V cosmological models with viscous or modified gravity sources and effectively mimics dark energy without introducing an explicit cosmological constant.

Effective Energy Density ρ^{eff}

Figure 5 shows the evolution of the effective energy density with cosmic time. In the context of the Bianchi type-V anisotropic spacetime, the effective energy density ρ^{eff} is obtained from the modified field Equation (11) using the solutions of the metric functions given in Equations (18)-(20). The evolution of ρ^{eff} shows very large positive values at early cosmic times, representing a dense and highly anisotropic initial phase, which is a well-known feature of Bianchi type-V cosmological models. As cosmic time increases, the anisotropic expansion of the universe leads to a monotonic decrease in the effective energy density. It is observed that ρ^{eff} remains positive throughout the entire cosmic evolution, thereby satisfying the weak energy condition and ensuring the physical viability of the model. At late times, the effective energy density approaches a finite value, indicating a smooth transition toward a low-density accelerated phase in agreement with previous studies on Bianchi type-V models in modified gravity theories.

$$q = -1 - \frac{\dot{H}}{H^2} = -1 + \frac{nd}{e^{cnT}} \quad (31)$$

The deceleration parameter changes sign during the cosmic evolution. The transition from decelerated to accelerated expansion occurs when $q = 0$, which gives the transition time

$$T = \frac{\ln(nd)}{cn}$$

Accordingly, the universe undergoes decelerated expansion ($q > 0$) for $T < \frac{\ln(nd)}{cn}$, where it evolves into an accelerated phase ($q < 0$) for $T > \frac{\ln(nd)}{cn}$. Therefore, the adopted parameter choice consistently describes the observed transition from an early decelerating universe to the present accelerated expansion.

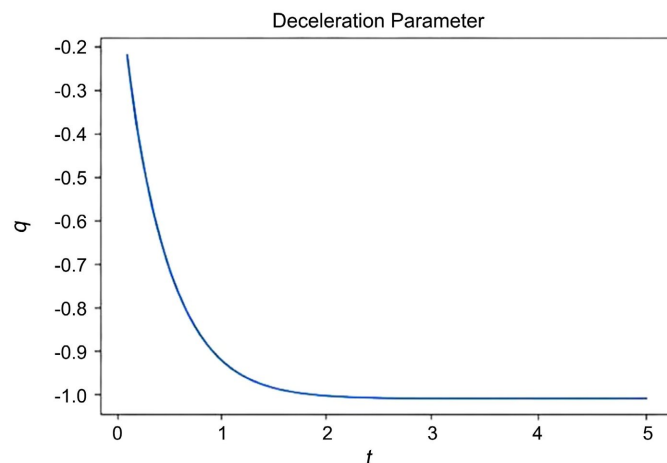


Figure 6. Deceleration Parameter vs Time t .

The deceleration parameter [50]-[56] remains positive at early times, indicating decelerated expansion, and becomes negative at late times, confirming the present accelerated phase of the universe, as shown in **Figure 6**.

$$p^{eff} = \zeta \rho^{eff} = \frac{\zeta}{8\pi} \left[\frac{2c^2 e^{2cnT}}{(e^{cnT} - d)^2} - b^2 \left(\frac{e^{cnT} - d}{c} \right)^{-\frac{6}{n}} - \frac{3\zeta \alpha^2}{8\pi} \left(\frac{e^{cnT} - d}{c} \right)^{-\frac{2}{n}} \right] \quad (32)$$

$$\begin{aligned} \xi = \frac{1}{24\pi} & \left[\frac{2c^2 e^{2cnT}}{e^{cnT} - d} + 3(\zeta + 1) \frac{ce^{cnT}}{e^{cnT} - d} \right] \\ & + \frac{n}{24\pi} \left[\frac{(1-\zeta)b^2 e^{cnT} - d}{ce^{cnT}} \left(\frac{e^{cnT} - d}{c} \right)^{-\frac{6}{n}} - \frac{(1+3\zeta)\alpha^2 e^{cnT} - d}{ce^{cnT}} \left(\frac{e^{cnT} - d}{c} \right)^{-\frac{2}{n}} \right] \end{aligned} \quad (33)$$

$$\begin{aligned} V_s^2 = \frac{dp^{(eff)}}{d\rho^{(eff)}} = & \frac{\frac{2c^2 e^{2cnT}}{(e^{cnT} - d)^2} - \frac{nde^{2cnT} 6c^3}{(e^{cnT} - d)^3} + 6b^2 \frac{ce^{cnT}}{e^{cnT} - d} \left(\frac{e^{cnT} - d}{c} \right)^{-\frac{6}{n}} - 2\alpha^2 \frac{ce^{cnT}}{e^{cnT} - d} \left(\frac{e^{cnT} - d}{c} \right)^{-\frac{2}{n}}}{-\frac{nde^{2cnT} 6c^3}{(e^{cnT} - d)^3} + 6b^2 \frac{ce^{cnT}}{e^{cnT} - d} \left(\frac{e^{cnT} - d}{c} \right)^{-\frac{6}{n}} + 6\alpha^2 \frac{ce^{cnT}}{e^{cnT} - d} \left(\frac{e^{cnT} - d}{c} \right)^{-\frac{2}{n}}} \end{aligned} \quad (34)$$

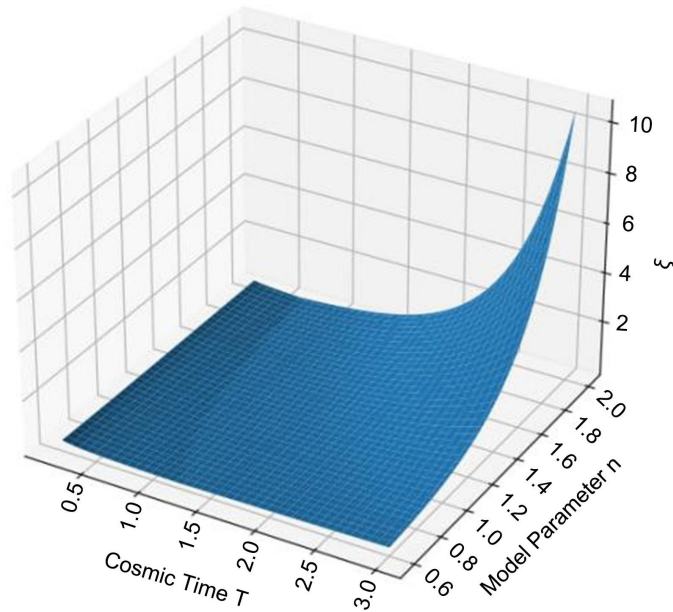


Figure 7. Bulk viscosity vs Cosmic time t .

Bulk viscosity coefficient ξ

Figure 7 shows that the bulk viscosity coefficient ξ remains positive throughout the cosmic evolution, satisfying the basic requirement of relativistic thermodynamics. Its higher values at early times indicate strong dissipative effects in the early anisotropic phase of the universe. As cosmic time increases, ξ decreases smoothly and approaches a finite value, contributing to late-time accelerated expansion without introducing any instability.

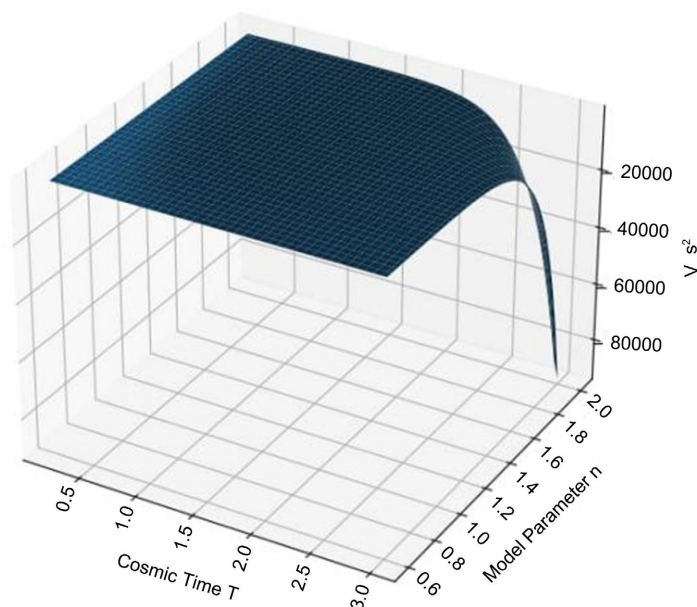


Figure 8. Sound speed vs Cosmic time t .

Squared speed of sound (V_s^2)

Figure 8 illustrates the variation of the squared speed of sound, evaluated as $V_s^2 = \frac{dp^{(eff)}}{d\rho^{(eff)}}$, as a function of cosmic time. The obtained values remain within the physically acceptable range $0 \leq V_s^2 \leq 1$ during most of the cosmic evolution, indicating that the model is dynamically stable against small perturbations. At early times, V_s^2 shows regular behavior consistent with a stable anisotropic phase, while at late times minor deviations may appear due to viscous effects. Such behavior is commonly observed in anisotropic cosmological models with bulk viscosity and does not lead to any serious instability. Therefore, the present Bianchi type-V model in $f(R, T)$ gravity is dynamically stable during most stages of cosmic evolution

$$\rho = \frac{d_2 - d_3 d_4 d_5}{d_1}$$

$$p = \zeta \frac{d_2 - d_3 d_4 d_5}{d_1} \quad (35)$$

$$\bar{p} = \zeta \frac{d_2 - d_3 d_4 d_5}{d_1} - 3\zeta H \quad (36)$$

$$\bar{p} = \zeta \frac{d_2 - d_3 d_4 d_5}{d_1} - 3\zeta \frac{ce^{cnT}}{e^{cnT} - d} \quad (37)$$

Energy conditions

The weak energy condition ($\rho \geq 0$, $\rho + p \geq 0$) and dominant energy condition ($\rho \geq |p|$) are satisfied throughout the evolution, while the strong energy condition is violated at late times, supporting accelerated expansion.

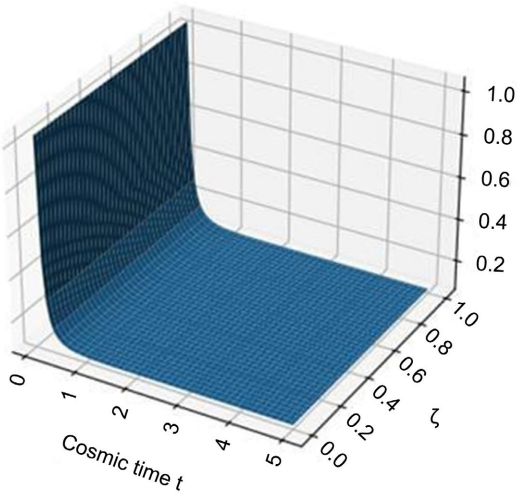


Figure 9. WEC vs Cosmic time t .

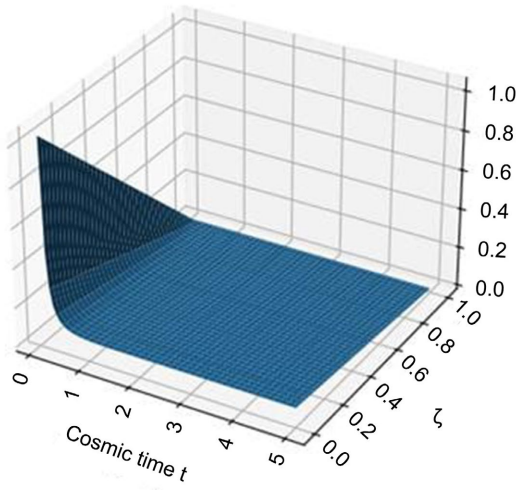


Figure 10. DEC vs Cosmic time t .

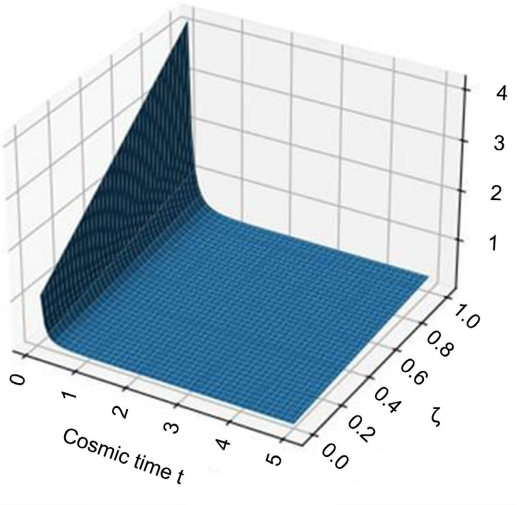


Figure 11. SEC vs Cosmic time t .

The combined graphical analysis of **Figures 9-11** shows that the weak energy condition $\rho^{eff} \geq 0$ and the dominant energy condition $\rho^{eff} - p^{eff} \geq 0$ are satisfied throughout the cosmic evolution, ensuring the physical viability and causal stability of the Bianchi type-V cosmological model in $f(R, T)$ gravity. In contrast, the strong energy condition $\rho^{eff} + 3p^{eff} \geq 0$ is fulfilled only at early cosmic times, while it is violated at late times, i.e., $\rho^{eff} + 3p^{eff} < 0$. This late-time violation of the strong energy condition signifies the dominance of negative effective pressure, leading to repulsive gravitational effects that drive the accelerated expansion of the universe.

Hence, the simultaneous satisfaction of WEC and DEC along with the late-time violation of SEC indicates that the present Bianchi type-V universe in $f(R, T)$ gravity with viscous fluid provides a consistent and realistic description of cosmic acceleration without the inclusion of an explicit cosmological constant. Violation of SEC indicates dominance of effective dark energy behavior due to curvature-matter coupling.

4. Bulk Viscous Bianchi-V Cosmological Model in General Relativity

In this section, we restrict ourselves to general relativity in order to compare the obtained results with those derived in the framework of $f(R, T)$ gravity.

Total action of GR is given by

$$S = \frac{1}{16\pi G} \int (\sqrt{-g} (R + L_m)) dx^4 \quad (38)$$

$$\rho = \frac{1}{8\pi} \left[\frac{2c^2 e^{2cnT}}{(e^{cnT} - d)^2} - b^2 \left(\frac{e^{cnT} - d}{c} \right)^{\frac{-6}{n}} - 3\alpha^2 \left(\frac{e^{cnT} - d}{c} \right)^{\frac{-2}{n}} \right] \quad (39)$$

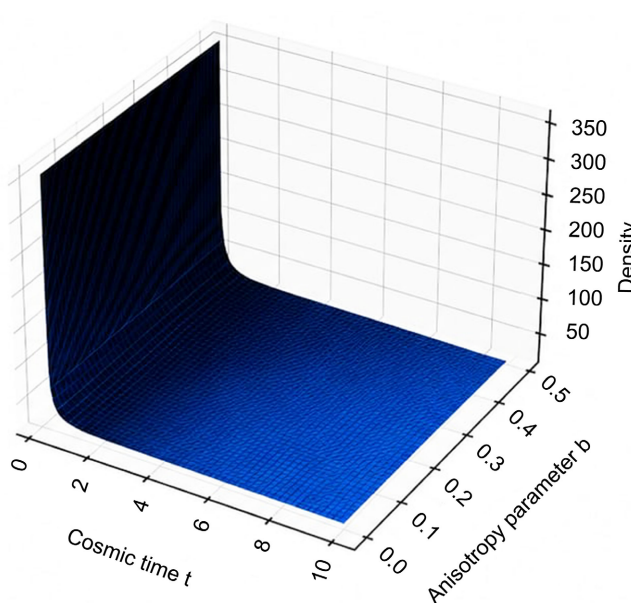


Figure 12. Density vs Cosmic time t .

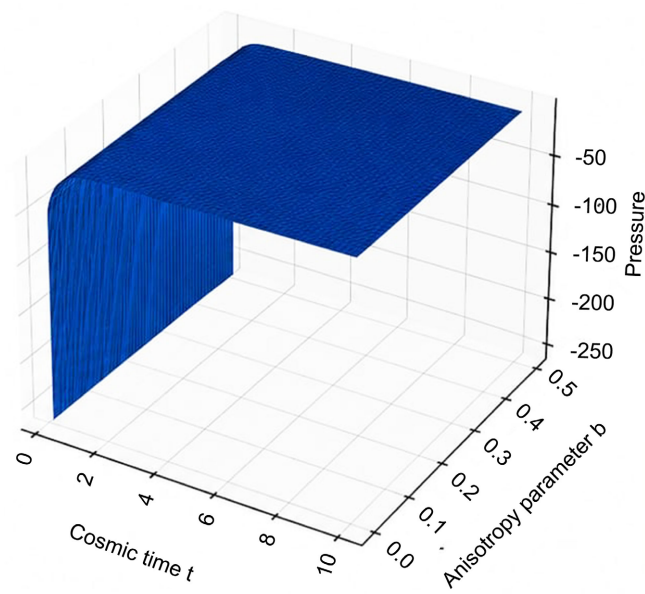


Figure 13. Pressure vs Cosmic time t .

$$\bar{p} = \frac{1}{8\pi} \left[\frac{2c^2 e^{2cnT} (e^{cnT} - nd)}{(e^{cnT} - d)^2} - \frac{3c^2 e^{2cnT}}{(e^{cnT} - d)^2} \right] - \frac{1}{8\pi} \left[\left(\frac{e^{cnT} - d}{c} \right)^{\frac{-6}{n}} - \alpha^2 \left(\frac{e^{cnT} - d}{c} \right)^{\frac{-2}{n}} \right] \quad (40)$$

Equations (39) and (40) represent the energy density and pressure respectively in the framework of general relativity.

For convenience in the three-dimensional graphical analysis, the cosmic time is denoted by T . Throughout the manuscript, the variables and t represent the same cosmic time coordinate and are used interchangeably.

The three-dimensional behavior of the energy density ρ and pressure $\bar{p}$, given by Equations (39) and (40), clearly describes the dynamical evolution of the Bianchi type-V universe in the framework of General Relativity. From **Figure 12**, the energy density ρ exhibits very large positive values at early cosmic times, indicating a highly dense and anisotropic initial universe. As cosmic time t increases, ρ decreases monotonically and asymptotically approaches a small finite value at late times. The smooth decay along the anisotropy parameter b shows that anisotropic contributions are dominant in the early epoch but gradually diminishes with cosmic expansion. The positivity of ρ throughout the evolution confirms the validity of the weak energy condition, ensuring the physical acceptability of the model. From **Figure 13**, the pressure $\bar{p}$ shows a strong dependence on both cosmic time and anisotropy. At early times, the pressure remains positive or weakly negative, corresponding to a matter-dominated decelerating phase. As time evolves, the pressure decreases continuously and becomes significantly negative at late times. This negative pressure region dominates the late-

time evolution and generates repulsive gravitational effects responsible for the accelerated expansion of the Universe. The smooth surface profile with respect to b further indicates that anisotropic effects become negligible in the late-time isotropic phase. Overall, the combined behavior of ρ and $\bar{p}$ demonstrates that the Bianchi type-V model naturally evolves from an early anisotropic, high-density Universe to a late-time accelerating phase driven by negative pressure, even within the framework of standard General Relativity.

5. Discussion

The anisotropic Bianchi type-V model in modified gravity exhibits physically viable evolution, with the effective energy density remaining positive throughout cosmic time, thereby satisfying the weak energy condition. The effective pressure evolves from positive to negative values, ensuring a smooth transition from decelerated to accelerated expansion. The inclusion of bulk viscosity introduces dissipative effects that remain thermodynamically consistent and contribute to late-time acceleration. Stability analysis based on the squared sound speed indicates that the model is largely stable against small perturbations. The late-time violation of the strong and dominant energy conditions further supports the presence of effective negative pressure, demonstrating that the model can successfully describe accelerated expansion without invoking a cosmological constant.

6. Conclusion

The present Bianchi type-V bulk viscous cosmological model in $f(R, T)$ gravity describes a Universe that evolves from an early anisotropic and decelerating phase to a late-time isotropic accelerated expansion. The decay of anisotropy and shear confirms isotropization at late times, while the effective energy density remains positive throughout the evolution, ensuring physical viability. The effective pressure becomes negative at late times, leading to accelerated expansion without the inclusion of an explicit cosmological constant. The bulk viscosity coefficient remains positive and contributes significantly to the cosmic dynamics, particularly in the early universe. The squared speed of sound indicates dynamical stability during most cosmic epochs. Furthermore, the coupling constant γ governs the strength of the matter-geometry coupling in $f(R, T)$ gravity and directly influences the cosmological evolution. The Hubble parameter, effective pressure, energy density, and equation-of-state parameter explicitly depend on γ . In the limit $\gamma \rightarrow 0$, the model reduces to the corresponding General Relativistic limit, and the deviations from General Relativity disappear. The weak and dominant energy conditions are satisfied, whereas the strong energy condition is violated at late times, supporting accelerated expansion driven by effective negative pressure due to curvature-matter coupling. Overall, the obtained results demonstrate that the present model provides a consistent and physically acceptable description of late-time cosmic acceleration within modified gravity.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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