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A Finite-Response Master Equation with a Primitive Operator Spectrum Derived from $M_2(\mathbb{C})$

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Abstract

A finite-response operator model is developed in which a discrete logarithmic spectrum is derived from the minimal noncommutative complex operator cell rather than selected from empirical data. Under primitive-factor reduction, diagnostic minimality, typed input-output and source-probe-output roles, and complete coefficient retention, the primitive cell is $M_2(\mathbb{C})$. Its center, traceless self-adjoint subspace, and full self-adjoint part have real dimensions 2, 3, and 4. The primitive response arities are 2 for self-response and 3 for source-probe-output interaction. The resulting complete coefficient modules have real dimensions 4, 9, 16, and 27. Their identification with observable logarithmic windings is a separate conditional step, requiring coefficient-basis-neutral unitary transport and observation through the least nontrivial determinant character. Under that explicitly stated observation realization, a self-adjoint response-number operator generates the associated modes in logarithmic comparison space. Independently, an associative finite-load composition law yields a unique convex saturating response profile. This constitutive law is incorporated into a block master system coupling propagation, ordering, localisation, metric response, interaction, memory, and drift. Local well-posedness is obtained conditionally under explicit symmetric-hyperbolicity assumptions. A standard-library Python implementation verifies composition, covariance, monotonicity, stability, convergence, quotient invariance, and defect sensitivity, and supplies controlled reductions for galaxy dynamics and lensing, turbulent transport, ordered-regional transfer, interaction, merger memory, tensor propagation, and a regular spherical geometry. Empirical studies are used only as external tests of visibility and do not enter the derivation. The uniqueness claim is limited to the stated complex-operator and primitive-response assumptions.

Keywords

Finite-Dimensional C^*-Algebras, Logarithmic Spectra, Nonlinear Response, Symmetric Hyperbolic Systems, Mathematical Physics

1. Introduction and Scope

The authority order is inherited from *The Necessity of Structure*, *The Necessity of Quantum Structure*, and Volumes I - IV of *The Necessity of Natural Physics* [1]-[6]. The present paper does not replace their necessity arguments. It supplies one explicit realization satisfying as many of Volume IV's completion conditions as can presently be closed without importing post-hoc empirical freedom.

Four claim classes are used and may not be merged:

1) A theorem follows from the stated mathematical assumptions within this paper;

2) A conditional theorem follows once separately listed analytic hypotheses are satisfied;

3) A numerical audit verifies a finite frozen computation and nothing outside its support;

4) An empirical contact records a separately published observational test and is not reproduced unless its data, nulls, and code are included here.

The word *master* denotes a block operator relation with common state, domains, sources, coefficients, and descendants. It does not mean that a single scalar equation contains all physical content. The word *standalone* means that a reader can reconstruct and run the proposed realization from this package alone. It does not mean that the books' upstream necessity derivations are re-proved in full. Every theorem stated here is proved from assumptions restated in this paper; the corpus citations supply motivation and provenance rather than hidden lemmas.

Verdict 1 (Exact contribution). *The exact contribution is as follows. The paper* 1) *derives the unique primitive operator cell* $M_2(\mathbb{C})$ *from the terminal complex operator envelope, primitive-factor quotient, and diagnostic minimality*; 2) *derives the carrier ranks* 2, 3, *and* 4 *as the center, traceless self-adjoint contrast space, and complete self-adjoint space of that cell*; 3) *proves that the rank-three Euclidean orientation is induced by* $SU(2)\to SO(3)$ *and is not a hidden spatial-dimension premise*; 4) *derives the only primitive arities,* $r=2$ *for self-response and* $r=3$ *for typed source-probe interaction*; 5) *proves the corpus-relative uniqueness of the primitive carrier signatures*

$\{(d,r)\}=\{(2,2),(3,2),(4,2),(3,3)\}$ *and thereby fixes the complete real coefficient-space dimensions* $\{4,9,16,27\}$; 6) *establishes, as a separate conditional observation corollary, that those dimensions become logarithmic winding numbers when the coefficient modules admit the stated basis-neutral unitary transport and the observable uses the least nontrivial determinant character*; *and* 7) *integrates the resulting registry with the causal, transfer, decoherence, constitutive, PDE, nu-*

merical, and external-tribunal structures developed below. The theorem classifies universal primitive response roles inside the frozen Natural Physics architecture; the winding interpretation belongs to the explicitly declared observation realization. It does not forbid higher-dimensional physical systems, alternative theories outside that architecture, or sector-specific descendants. The causal-cavity realization, route-specific transfer phase, empirical decoherence distribution, and full dynamical completion remain explicit obligations.

2. Derivation of the Primitive Response Registry

The previous edition proved the determinant-winding mechanism after the carrier ranks and the primitive response signatures had been declared. The remaining foundational question was therefore exact:

$$\mathcal{A}_{\text{causal}} + \mathcal{A}_{\text{phase}} + \mathcal{A}_{\text{complete response}} \overset{?}{\Rightarrow} \{(d,r)\} = \{(2,2),(3,2),(4,2),(3,3)\}, \tag{1}$$

with no alternative primitive registry inside the Natural Physics world already selected by the corpus.

The answer is affirmative in the terminal one-world jurisdiction of Volumes III - IV. The theorem is not a classification of every mathematically imaginable theory. It is stronger and more relevant than the earlier branch theorem: once the corpus's unique complex operator envelope, quotient minimality, sort discipline, and complete-response requirement are retained, the carrier ranks and primitive arities are no longer free choices.

2.1. Foundational Assumptions

For the purposes of this paper, C1 - C6 are adopted explicitly as standing axioms. The cited corpus supplies its upstream necessity derivations and provenance, but no unstated result from that corpus is used in the theorem proved here. Accordingly, the registry theorem is conditional on C1 - C6 exactly as written below.

C1. *Quotient primacy and finite diagnostic closure*. A physical distinction is retained only when it survives the complete predictive quotient; a primitive carrier may not contain a removable label, multiplicity copy, superselection spectator, or diagnostically silent direction [1] [3] [5].

C2. *Minimal central phase*. A nontrivial continuous faithful irreducible multiplicity-free real circle action has rank two, and its minimal central unital scalar closure is $\mathbb{C}$, uniquely up to conjugation [2].

C3. *Complex operator selection*. The Volume-I complex-selection tribunal places the relevant finite ordered objects in self-adjoint parts of complex C^*-algebras on its theorem domain; the terminal Volume-III census then merges the surviving real, quaternionic, Jordan, restricted, finite, and singular charts into, or excludes their nonembeddable remainder from, one completion-enlarged complex operator envelope [3] [5].

C4. *One-world inheritance*. Volume IV receives exactly that complex operator envelope. Representations, sectors, normal and singular domains, and finite or infinite completions are internal coordinates of one architecture rather than new primitive worlds [6].

C5. *Typed process and interaction discipline*. Input, output, source, probe, effect, and record roles may not be collapsed across sorts without a predictive-equivalence theorem. Interaction is defined relative to independently addressable source and probe roles, while a higher constituent number may be carried inside a composite source or probe rather than creating a new primitive sort [4] [5].

C6. *No silent surplus*. A proposed primitive parent is certificate-irreducible: deleting, identifying, or factoring one of its roles must destroy a retained diagnostic. Conversely, a construction that factors through already admitted primitive modules is a descendant, not a new parent [3]-[5].

These statements do not contain the integers 4, 9, 16, 27. The dimensions enter only after the terminal operator cell and its typed response modules are classified. C1 - C6 are the complete architecture-level assumption set used in that classification; any rejection of one clause removes the theorem's jurisdiction rather than being repaired by an implicit appeal to the external corpus.

Definition 1 (Operational diagnostic burden and equivalence). *Let* $\mathfrak{B}$ *be a finite-dimensional unital complex* C^**-factor, let* $\tau_{\mathfrak{B}}$ *be its normalized trace, and put*

$$\mathfrak{B}_{\mathrm{sa}}^{0} := \left\{ X \in \mathfrak{B}_{\mathrm{sa}} : \tau_{\mathfrak{B}}\left(X\right) = 0 \right\}.$$

Its diagnostic certificate is the Boolean triple $b\left(\mathfrak{B}\right) = \left(b_{\Phi}, b_{O}, b_{R}\right) \in \{0,1\}^3$, *defined by the following independently checkable tests*:

(D1) $b_{\Phi} = 1$ *exactly when* $Z\left(\mathfrak{B}\right) = \mathbb{C}1_{\mathfrak{B}}$ *and the central action* $\mathrm{e}^{i\theta} \mapsto \mathrm{e}^{i\theta}1_{\mathfrak{B}}$ *is faithful*;

(D2) $b_{O} = 1$ *exactly when* $1_{\mathfrak{B}}$ *is the distinguished Archimedean order unit and* $\tau_{\mathfrak{B}}\left(1_{\mathfrak{B}}\right) = 1$ *fixes its normalization*;

(D3) $b_{R} = 1$ *exactly when* $\mathfrak{B}_{\mathrm{sa}}^{0}$ *contains* X, Y *with* $\left[X, Y\right] \neq 0$ *and the connected inner-automorphism orbit* $U \mapsto UXU^*$ *is nonconstant.*

Two candidate factors carry the same phase, order-unit, and reversible-distinction burdens precisely when both have certificate $(1,1,1)$ *under the same quotient and normalization conventions. This is the equivalence of the three primitive burdens, not an assertion that the two algebras have identical state spaces or every descendant prediction. A candidate is diagnostic-minimal when it has a certificate* $(1,1,1)$ *and no factor of smaller real dimension has that certificate.*

Proposition 1 (Independent audit of diagnostic minimality). *For a complex matrix factor* $M_n\left(\mathbb{C}\right)$,

$$\mathbf{b}\left(M_n\left(\mathbb{C}\right)\right) = (1,1,1) \iff n \geq 2.$$

Since $\dim_{\mathbb{R}} M_n\left(\mathbb{C}\right) = 2n^2$, *the unique least-dimensional factor carrying all three burdens is* $M_2\left(\mathbb{C}\right)$.

Proof. Every $M_n\left(\mathbb{C}\right)$ has centre $\mathbb{C}I_n$, a faithful central circle action, and the normalized order unit I_n. For $n = 1$ the traceless self-adjoint space is zero, and (D3) fails. For $n \geq 2$, an embedded Pauli block supplies noncommuting traceless self-adjoint elements and a nonconstant connected unitary-conjugation orbit, so (D3) holds. The function $2n^2$ is strictly increasing for $n \ge 1$, which gives the stated minimum. □

2.2. Primitive Finite Diagnostic Cell

Definition 2 (Primitive response cell). *Let $\mathfrak{A}$ be the terminal complex operator envelope. A primitive finite response cell is a finite-dimensional unital complex C^*-algebra $\mathfrak{A}_0$ admitted as a local diagnostic cell of $\mathfrak{A}$ such that:*

(P1) *$\mathfrak{A}_0$ is noncommutative, because a commutative cell carries no irreducible coherent regional distinction or nontrivial interaction algebra;*

(P2) *Its central projections have already been quotiented into explicit superselection sectors, so the primitive cell is a factor;*

(P3) *It is diagnostic-minimal in the operational sense of Definition* 1*: no factor of smaller real dimension passes the same three diagnostic tests;*

(P4) *No multiplicity copy is counted as a new carrier type.*

Theorem 1 (Minimal operator-cell theorem). *Every primitive finite response cell is $*$-isomorphic to*

$$\boxed{\mathfrak{A}_0 \cong M_2(\mathbb{C}).} \tag{2}$$

Proof. Every finite-dimensional complex C^*-algebra is $*$-isomorphic to a finite direct sum [7]

$$\bigoplus_{k=1}^{m} M_{n_k}(\mathbb{C}). \tag{3}$$

Primitive factor status removes $m > 1$, because each central summand is an independently addressable superselection sector rather than one certificate-irreducible cell. Hence $\mathfrak{A}_0 \cong M_n(\mathbb{C})$ for some n. Noncommutativity excludes $n = 1$. Proposition 1 supplies the operational audit: the three burdens are carried exactly for $n \ge 2$, and their least real-dimensional carrier is uniquely $n = 2$. Any $n > 2$ is a larger representation inside the same complex operator architecture; it may support additional states, sectors, or composite records, but it cannot define a new *primitive* response type while the $n = 2$ cell already realizes the phase, order-unit, and noncommutative distinction burdens. □

Remark 1 (Why the proof is not the Frobenius shortcut). *Frobenius' theorem classifies finite-dimensional associative real division algebras as $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{H}$. That classification is not the correct terminal selector here. The corpus has already selected a central complex scalar line and a complex operator envelope; irreducible quaternionic scalar structure is not a second carried world. The relevant classification is therefore the finite-dimensional complex C^*-algebra classification, followed by primitive factor and diagnostic-minimality reduction. This avoids treating $\mathbb{H}$ simultaneously as a scalar field, a regional carrier, and an observable algebra.*

Proposition 2 (Representation size is not a primitive response type). *A larger matrix algebra $M_n(\mathbb{C})$, $n > 2$, may describe a genuine higher-dimensional sys-*

tem, sector, or composite. Its matrix size does not by itself commission a new universal response parent.

Proof. The registry classifies certificate-irreducible *response roles*, not every representation dimension in which those roles may be realized. The terminal complex envelope already admits arbitrary finite and infinite representations as internal system coordinates. Promoting each n to a new parent would make the registry depend on the chosen system representation and would generate an unbounded catalogue rather than a finite universal grammar. Under quotient primacy and no-surplus closure, a new parent therefore requires a new irreducible typed role or a new invariant response module that does not factor through the existing phase, regional, complete, or interaction roles. Matrix size alone supplies neither certificate. □

2.3. The Ranks Two, Three, and Four from One Cell

Let $\tau(X)=\frac{1}{2}\operatorname{Tr}X$ be the normalized trace on $M_2(\mathbb{C})$. Define typed carrier copies

$$V_\Phi := Z(\mathfrak{A}_0)=\mathbb{C}I_2, \tag{4}$$

$$V_O := \mathbb{R}I_2 \subset (\mathfrak{A}_0)_{\mathrm{sa}}, \tag{5}$$

$$V_R := \{X \in (\mathfrak{A}_0)_{\mathrm{sa}} : \tau(X)=0\}, \tag{6}$$

$$V_C := (\mathfrak{A}_0)_{\mathrm{sa}} = V_O \oplus V_R. \tag{7}$$

The copies are typed: V_Φ is the scalar-amplitude carrier, whereas V_O, V_R, and V_C are self-adjoint diagnostic carriers. Their common real scalar direction is not counted twice as a world degree of freedom.

Theorem 2 (Canonical carrier-rank theorem). *The primitive response cell canonically supplies*

$$\boxed{\dim_{\mathbb{R}} V_\Phi = 2, \quad \dim_{\mathbb{R}} V_R = 3, \quad \dim_{\mathbb{R}} V_C = 4.} \tag{8}$$

Moreover, V_R *carries a canonical oriented Euclidean structure; it is not an arbitrarily postulated real three-space.*

Proof. The center of $M_2(\mathbb{C})$ is $\mathbb{C}I_2$, which is two-dimensional over $\mathbb{R}$. Every self-adjoint 2 × 2 matrix has a unique Pauli decomposition

$$X = x_0 I_2 + x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3, \quad x_\mu \in \mathbb{R}. \tag{9}$$

Therefore, $(\mathfrak{A}_0)_{\mathrm{sa}}$ has real dimension four, its order-unit line has dimension one, and the traceless complement has dimension three. The bilinear form

$$\langle X,Y\rangle_R := \frac{1}{2}\operatorname{Tr}(XY) \tag{10}$$

restricts to a positive Euclidean inner product on V_R. Conjugation by $SU(2)$ preserves the trace and this inner product. In the Pauli basis, its adjoint action is the standard double cover $SU(2)\to SO(3)$ [8]; connectedness fixes positive orientation. Thus, the Euclidean metric and orientation descend from the selected operator cell and its reversible action rather than being primitive spatial assumptions. □

Remark 2 (No inference of three spatial dimensions). *The rank-three carrier* V_R *is the traceless self-adjoint contrast space of the minimal operator cell. It is not the dimension of physical space. Volume III's theorem that the pregeometric premises do not force three spatial dimensions remains untouched. A later geometric realization may identify a regional contrast basis with three spatial directions only on an independently licensed branch.*

Lemma 1 (No independent order parent). *The one-dimensional order-unit line does not generate a nontrivial primitive response parent.*

Proof. A linear response $T: V_O \to V_O$ compatible with unital normalization satisfies $T(I_2) = I_2$. Since every element of V_O is aI_2, linearity gives $T(aI_2) = aI_2$. The response, therefore, contains no free coefficient direction. Allowing $T(I_2) \neq I_2$ changes normalization or the reference scale and belongs to representative choice, not to a quotient-stable physical response. Hence, the rank-one line has trivial determinant transport and supplies no positive parent. □

2.4. Primitive Response Arities

Definition 3 (Primitive response jet). *A primitive response jet is the lowest nonzero quotient-stable derivative of a typed response at its neutral configuration. A higher derivative is a descendant unless an independently commissioned symmetry or selection rule forces every lower derivative to vanish.*

For a carrier V, a self-response is a typed map $R: V^{\text{in}} \to V^{\text{out}}$. Its first nonzero jet has one input slot and one output slot, hence response arity $r = 2$. An interaction is a response of an independently addressed probe to an independently varied source,

$$I: V_R^{\text{src}} \times V_R^{\text{prb}} \to V_R^{\text{out}}, \quad I(s,0) = I(0,p) = 0. \tag{11}$$

Its tangent coupling is the mixed derivative

$$B = D_s D_p I(0,0) \in \text{Hom}_{\mathbb{R}}\left(V_R^{\text{src}} \otimes V_R^{\text{prb}}, V_R^{\text{out}}\right). \tag{12}$$

It has two independently typed inputs and one output, hence $r = 3$.

Theorem 3 (Primitive arity theorem). *Inside the frozen Natural Physics response signature, the only primitive complete-response arities are*

$$\boxed{r = 2 \text{ for self-response}, \quad r = 3 \text{ for interaction.}} \tag{13}$$

Higher constituent number, ancillary extension, or nonlinear saturation does not create another primitive arity.

Proof. A process necessarily has an input and an output, so self-response begins with two typed slots. Interaction is defined only after an independently addressable source and probe have been commissioned, preserving their distinction, and the output gives three typed slots. Additional physical constituents can be included in a composite source or composite probe without adding another process sort. Sequential and ancillary constructions are compositions or extensions of the same typed maps. Higher Taylor jets describe nonlinear descendants of the primitive map and are already governed here by the finite-response constitutive law. A genuinely new primitive arity would therefore require a new irreducible role not factorable into source, probe, input, output, effect, or record. The terminal one-world signature contains no such uncommissioned role, and the corpus's no-surplus rule forbids adding it merely to manufacture another parent. □

Remark 3 (Why 27 has regional rather than scalar output). *A scalar experimental record is an effect applied after the interaction output; it is not the output carrier itself. Replacing* V_R^{out} *by a scalar record would collapse process and effect sorts, contrary to the regional and process typing theorems. Likewise, taking the full ordered-regional carrier in every source, probe, and output slot produces a reducible* 4^3 *tensor containing fixed order-unit pieces, linear self-response blocks, and the genuine regional interaction block. Only the last is a new primitive interaction coefficient space.*

2.5. Complete Coefficient Modules and Exhaustion

Let the carrier-frame groups be

$$G_\Phi = O(V_\Phi),\quad G_R = SO(V_R),\quad G_C = \{1_{V_O}\} \times SO(V_R),$$

where the disconnected part of $O(V_\Phi)$ implements the conjugation freedom already allowed by C2. For self-response, independent input and output redescriptions act by

$$(g_{\text{out}}, g_{\text{in}}) \cdot T = g_{\text{out}} T g_{\text{in}}^{-1},$$

and for interaction,

$$(g_{\text{out}}, g_{\text{src}}, g_{\text{prb}}) \cdot B = g_{\text{out}} B \left(g_{\text{src}}^{-1} \otimes g_{\text{prb}}^{-1}\right).$$

Proposition 3 (Frame redescription and complete coefficient retention). *Under the independent product actions above, the phase module* $\mathrm{Hom}_{\mathbb{R}}\left(V_\Phi^{\text{in}}, V_\Phi^{\text{out}}\right)$*, the regional module* $\mathrm{Hom}_{\mathbb{R}}\left(V_R^{\text{in}}, V_R^{\text{out}}\right)$*, and the interaction module* $\mathrm{Hom}_{\mathbb{R}}\left(V_R^{\text{src}} \otimes V_R^{\text{prb}}, V_R^{\text{out}}\right)$ *are irreducible real representations of their respective product frame groups. The complete ordered-regional module has the exact invariant typed decomposition.*

$$\mathrm{Hom}_{\mathbb{R}}\left(V_C^{\text{in}}, V_C^{\text{out}}\right) \cong \mathrm{Hom}(V_O, V_O) \oplus \mathrm{Hom}(V_R, V_O) \oplus \mathrm{Hom}(V_O, V_R) \oplus \mathrm{Hom}(V_R, V_R),$$

with dimensions $1+3+3+9=16$. *Complete coefficient retention means retaining this entire irreducible module, or this entire typed direct sum, before any sector-specific constitutive restriction is imposed.*

Proof. The standard real representation of $O(2)$ and the standard real representation of $SO(3)$ are absolutely irreducible. Their external tensor products are therefore irreducible representations of the corresponding direct-product groups, which proves the phase and regional statements; the same argument applied to three factors proves the interaction statement. For $V_C = V_O \oplus V_R$, the distinguished order line is not mixed with V_R, so expanding input and output by this direct sum gives the four displayed invariant blocks and no others.

A symmetric, antisymmetric, trace-free, diagonal, isotropic, rank-restricted, or otherwise physically constrained coefficient space requires additional structure: for example, an identification of input and output frames, a constitutive tensor, a source constraint, or a declared symmetry. Such spaces may be valid descendants, but under C5 - C6, they cannot replace a universal primitive parent before that additional structure is commissioned. Thus, the exclusion is not that proper subspaces are mathematically impossible; it is that they are conditional sector reductions rather than primitive complete-response modules. □

The primitive modules are consequently

$$E_\Phi = \mathrm{Hom}_{\mathbb{R}}\left(V_\Phi^{\mathrm{in}}, V_\Phi^{\mathrm{out}}\right), \quad \dim E_\Phi = 2^2 = 4, \tag{14}$$

$$E_R = \mathrm{Hom}_{\mathbb{R}}\left(V_R^{\mathrm{in}}, V_R^{\mathrm{out}}\right), \quad \dim E_R = 3^2 = 9, \tag{15}$$

$$E_C = \mathrm{Hom}_{\mathbb{R}}\left(V_C^{\mathrm{in}}, V_C^{\mathrm{out}}\right), \quad \dim E_C = 4^2 = 16, \tag{16}$$

$$E_I = \mathrm{Hom}_{\mathbb{R}}\left(V_R^{\mathrm{src}} \otimes V_R^{\mathrm{prb}}, V_R^{\mathrm{out}}\right), \quad \dim E_I = 3^3 = 27. \tag{17}$$

Theorem 4 (Primitive causal-response registry theorem). *Relative to the terminal one-world complex operator envelope and the primitive response definitions above, the primitive complete-response signatures are unique up to typed real-linear isomorphism*:

$$\boxed{\{(d,r)\}_{\mathrm{primitive}} = \{(2,2),(3,2),(4,2),(3,3)\}.} \tag{18}$$

No alternative universal primitive response signature is admissible inside the frozen Natural Physics response jurisdiction. Representation-specific system dimensions remain allowed but are not registry parents by Proposition 2.

Proof. The minimal operator-cell theorem leaves one primitive cell, $M_2(\mathbb{C})$. Its center, traceless self-adjoint part, and complete self-adjoint part give exactly the nontrivial typed carrier ranks 2, 3, and 4; the normalized order-unit line is response-trivial by Lemma 1. The primitive arity theorem leaves only self-response with $r = 2$ and source-probe interaction with $r = 3$. Self-response applies to each of the three nontrivial carriers, giving $(2,2)$, $(3,2)$, and $(4,2)$. The only new interaction block surviving sort decomposition is the regional source-probe-output map, giving $(3,3)$. Proposition 3 fixes the corresponding complete coefficient spaces and classifies every proper invariant or physically constrained subspace as a separately commissioned descendant.

It remains to exclude apparent alternatives. Direct sums and repeated blocks are a superselection or multiplicity structure and fail primitive irreducibility. A higher matrix size belongs to a larger representation of the same operator envelope and fails diagnostic minimality. Real, irreducibly quaternionic, exceptional Jordan, and nonassociative remainders have already been merged into the central-complex envelope, where predictively equivalent or excluded by the terminal census where not. Mixed carrier maps require an independently commissioned type-changing bridge; scalar multiplication, order-unit insertion, trace projection, and inclusion are fixed structural maps rather than new coefficient-complete parents.

A phase bilinear product is already the fixed complex scalar multiplication. A full 4^3 interaction tensor is reducible into order, self-response, observation, and regional-interaction blocks. Higher jets and higher constituent numbers are descendants or composites, not additional primitive sorts. Finally, higher determinant powers are harmonics of a parent character rather than new parent modules. The listed four signatures, therefore, exhaust the primitive response jurisdiction. □

Definition 4 (Coefficient-transport group). *Each real coefficient module* $E_N \in \{E_\Phi, E_R, E_C, E_I\}$, *indexed by its real dimension* $N \in \{4, 9, 16, 27\}$, *carries the Hilbert-Schmidt Euclidean product induced from its typed carrier products. Define its complexified coefficient space*

$$\mathcal{H}_N := E_N \otimes_{\mathbb{R}} \mathbb{C}, \quad \dim_{\mathbb{C}} \mathcal{H}_N = N,$$

with the canonical Hermitian extension of that product. The carrier-frame groups in Proposition 3 *act through a subgroup of* $U(\mathcal{H}_N)$. *The coefficient-transport group used by the global scalar readout is the full basis-neutral group*

$$\mathcal{U}_N := U(\mathcal{H}_N) \cong U(N).$$

This is an observation-level premise: no preferred complex coefficient basis or proper complex coefficient subspace is included in the primitive data. It does not identify the physical carrier-frame group with $U(N)$.

Proposition 4 (Least continuous coefficient character). *Let* $\chi : \mathcal{U}_N \to U(1)$ *be a continuous group character. Then for a unique* $m \in \mathbb{Z}$,

$$\chi(U) = \det(U)^m.$$

On the central phase transport zI_N, *one has* $\chi(zI_N) = z^{mN}$. *If primitive observation applies the no-harmonic-surplus rule and selects the least positive nontrivial character, then* $m = 1$, *so the observable central winding is exactly* N.

Proof. The commutator subgroup of $U(N)$ is $SU(N)$, and every character into the Abelian group $U(1)$ annihilates it. Hence, χ factors through the determinant quotient $U(N)/SU(N) \cong U(1)$. The continuous characters of $U(1)$ are exactly $z \mapsto z^m$, $m \in \mathbb{Z}$. The least positive nontrivial choice is $m = 1$; choices $m > 1$ are determinant harmonics and are descendants rather than new primitive characters. □

Remark 4 (Two distinct covariance statements). *Carrier-frame covariance determines the tensorial coefficient module and the status of its proper subspaces. Coefficient-basis neutrality determines the unitary group on the complexified coefficient space used for scalar character transport. The determinant step relies on the latter and is therefore stated separately rather than inferred from the former.*

Corollary 1 (Conditional primitive β-registry). *If the coefficient module is complexified and transported according to Definition* 4, *and if the observable scalar applies the least-character rule of Proposition* 4, *then*

$$\boxed{\mathrm{Spec}^{+}_{\text{primitive}} = \{2^2, 3^2, 4^2, 3^3\} = \{4, 9, 16, 27\}.} \tag{19}$$

Within this explicitly conditional observation jurisdiction, the resulting wind-

ing registry is not fitted to empirical data or selected from rival numerical lists.

Proof. Theorem 4 fixes the four real coefficient-space dimensions. Definition 4 supplies their N-dimensional Hermitian complexifications, and Proposition 4 proves that the least positive scalar character is the determinant with central winding N. Applying that conditional observation rule to the four dimensions gives the displayed set. □

Proposition 5 (Non-circularity). *None of the selector premises contains the target integers or an equivalent numerical encoding. The numbers arise only after:* 1) *the complex operator cell is classified*; 2) *its center and self-adjoint decomposition are dimension-counted*; 3) *the typed response slots are counted*; *and* 4) *the determinant character is applied.*

Proof. The corpus premises concern quotient identity, central phase, complex operator closure, order unit, primitive factor status, typed source/probe/output roles, and response completeness. Their statements are invariant under basis changes and contain no target parent. The numerical values first appear in Equations (8) and (14) - (17). □

2.6. Ablation and Exact Claim Boundary

The effect of removing each load-bearing selector is summarized in Table 1.

Table 1. Ablation of the registry-uniqueness theorem.

Removed Item	Mathematical Consequence	Correct Scientific Verdict
Central minimal phase	Real, split, nilpotent, quaternionic, or multiplicity scalar routes may survive	Rank two and the common coefficient phase are no longer unique
Terminal complex operator envelope	Real, quaternionic, Jordan, GPT, or nonoperator cells re-enter	The operator-cell theorem loses jurisdiction
Primitive factor quotient	Direct sums and superselection centers remain	Additional representation dimensions are not excluded
Diagnostic minimality	$M_n(\mathbb{C})$ for arbitrary $n \geq 2$ remains	Larger carrier ranks become possible, but are nonprimitive in the present theorem
Unit normalization	The scalar order line can be rescaled	A rank-one response is representative-dependent rather than physical
Sort discipline	Process, effect, record, and carrier outputs may be conflated	Spurious mixed dimensions such as 6, 8, 12, 18, 36, or 64 can be manufactured
Primitive-jet rule	Higher derivatives can be promoted to parents	Additional powers are branch-specific and require independent selectors
Complete coefficient retention	Proper constitutive subspaces survive	Parent dimensions may reduce or disappear in a restricted sector
Global determinant character	Module dimensions need not equal observed windings	The carrier theorem survives, but the fixed β-registry does not

Verdict 2 (Exact foundational advance). *The earlier registry theorem was conditional on choosing the ranks* 2, 3, *and* 4 *and on declaring four primitive response modules. The present theorem removes that choice inside the terminal Natural Physics architecture. The ranks are the center, traceless self-adjoint contrast space, and complete self-adjoint diagnostic space of the unique primitive cell* $M_2(\mathbb{C})$; *the arities are fixed by process and interaction sort structure. Accordingly, the primitive carrier-signature registry is a corpus-relative uniqueness theorem. The corresponding* β *-winding registry is its conditional observation corollary under the coefficient-transport and least-character premises; channel-level detection additionally requires nonzero generation, transfer, and observation.*

3. The Response Registry as an Operator Spectrum

3.1. Complete Modules and Projectors

By Theorem 4, the primitive carriers and arities are not free inputs. Their complete coefficient modules are

$$\mathcal{E}_4 = \mathrm{Hom}_{\mathbb{R}}\left(V_\Phi^{\mathrm{in}}, V_\Phi^{\mathrm{out}}\right), \quad \dim \mathcal{E}_4 = 4, \tag{20}$$

$$\mathcal{E}_9 = \mathrm{Hom}_{\mathbb{R}}\left(V_R^{\mathrm{in}}, V_R^{\mathrm{out}}\right), \quad \dim \mathcal{E}_9 = 9, \tag{21}$$

$$\mathcal{E}_{16} = \mathrm{Hom}_{\mathbb{R}}\left(V_C^{\mathrm{in}}, V_C^{\mathrm{out}}\right), \quad \dim \mathcal{E}_{16} = 16, \tag{22}$$

$$\mathcal{E}_{27} = \mathrm{Hom}_{\mathbb{R}}\left(V_R^{\mathrm{src}} \otimes V_R^{\mathrm{prb}}, V_R^{\mathrm{out}}\right), \quad \dim \mathcal{E}_{27} = 27. \tag{23}$$

Set

$$\mathcal{E} = \mathcal{E}_4 \oplus \mathcal{E}_9 \oplus \mathcal{E}_{16} \oplus \mathcal{E}_{27}, \quad I_{\mathcal{E}} = P_4 + P_9 + P_{16} + P_{27}, \quad P_N P_M = \delta_{NM} P_N. \tag{24}$$

Conditional Corollary 1 fixes the least positive central winding of each complete module to its dimension only under the coefficient-transport and least-character premises stated above. The earlier logarithmic-registry paper supplied the conditional determinant mechanism; the present corpus theorem removes the prior freedom in the primitive ranks and signatures [9]. Inside the present realization, this becomes the self-adjoint finite-spectrum operator

$$\boxed{\hat{\mathrm{N}} = \sum_{N \in \{4,9,16,27\}} N P_N, \quad \mathrm{Spec}^+\left(\hat{\mathrm{N}}\right) = \{4, 9, 16, 27\}.} \tag{25}$$

No continuous frequency parameter is fitted.

3.2. Exact Logarithmic Lift

For a response state $\mathrm{Q} = \sum_N \mathrm{Q}_N \in \mathcal{E}$ and a dimensionless logarithmic comparison coordinate x, define

$$\tilde{\mathrm{Q}}(x) = \mathrm{e}^{ix\hat{\mathrm{N}}}\mathrm{Q} = \sum_N \mathrm{e}^{iNx}\mathrm{Q}_N. \tag{26}$$

Then

$$\boxed{\left(\partial_x - i\hat{\mathrm{N}}\right)\tilde{\mathrm{Q}} = 0.} \tag{27}$$

The equation is kinematical in logarithmic comparison space; it does not introduce another time evolution. It states exactly which carriers may occur once a channel admits multiplicative comparison.

Theorem 5 (Support immutability). *Let* $L_j : \mathcal{E}_{\mathbb{C}} \to \mathbb{C}$ *be a fixed bounded observation functional and* $T_j(\hat{\mathrm{N}})$ *a fixed bounded transfer operator. The observable*

$$\mathcal{O}_j(x) = B_j(x) + \Re L_j T_j(\hat{\mathrm{N}}) \mathrm{e}^{ix\hat{\mathrm{N}}} \mathrm{Q} \tag{28}$$

has Fourier support contained in $\{4, 9, 16, 27\}$. *A new nonzero frequency cannot be produced by changing amplitudes, phases, initial response data, or the convex constitutive law; it requires changing the registry, observation coordinate, or transfer operator and therefore defines a new candidate.*

Proof. Using the spectral projectors,

$$\mathcal{O}_j(x) - B_j(x) = \Re \sum_N L_j P_N T_j(N) \mathrm{Q}_N \mathrm{e}^{iNx}.$$

The stated support follows. None of the coefficients changes an eigenvalue of $\hat{\mathrm{N}}$. □

Writing

$$A_{jN} \mathrm{e}^{i\phi_{jN}} := L_j P_N T_j(N) \mathrm{Q}_N, \tag{29}$$

recovers the selective-visibility rule of the registry paper: a parent can be structurally available and observationally absent because generation, projection, or transfer vanishes. Therefore, the empirical absence of 9, 16, or 27 in a channel does not contradict the existence of those modules; it contradicts the theory only when the channel map predicted nonzero detectability before inspection.

4. The Causal Meaning of the Response Number

4.1. Dilation Is the Universal Comparison Clock

The empirical phase is not periodic in X but in the normalized multiplicative coordinate

$$x = \ln\left(\frac{X}{X_0}\right). \tag{30}$$

Consequently $-i\partial_x$ is the generator of scale translation and the response-number eigenvalue is

$$\hat{\mathrm{N}} \tilde{\mathrm{Q}}_N = N \tilde{\mathrm{Q}}_N, \quad \tilde{\mathrm{Q}}_N(x) = \mathrm{e}^{iNx} \mathrm{Q}_N. \tag{31}$$

For a scalar projection, the phase, logarithmic wavelength, multiplicative recurrence ratio, and number of visible cycles are

$$\Theta_N(x) = Nx + \phi_N, \tag{32}$$

$$\lambda_{x,N} = \frac{2\pi}{N}, \tag{33}$$

$$\Lambda_N = \exp\left(\frac{2\pi}{N}\right), \tag{34}$$

$$N_{\text{cyc}} = \frac{N}{2\pi}\ln\left(\frac{X_{\max}}{X_{\min}}\right). \tag{35}$$

Thus, a larger coherent or observed scale range does not by itself select a larger parent. It reveals more cycles of whichever parent is already generated and transmitted. Numerically,

N	$2\pi/N$	$\Lambda_N = e^{2\pi/N}$
4	1.570796	4.810477
9	0.698132	2.009994
16	0.392699	1.480973
27	0.232711	1.262016

These are wavelengths and recurrence ratios in scale space. They should not be confused with ordinary spatial wavelengths unless a route-specific observable map proves that identification.

4.2. Primitive Causal Carrier Count

Let a physical realization possess a coherent active domain with a causal diameter L_{coh}, effective propagation speed v_{eff}, and a collective carrier angular frequency ω_{c}. Its causal crossing time is

$$\tau_{\text{c}} = \frac{L_{\text{coh}}}{v_{\text{eff}}}. \tag{36}$$

A boundary condition contributes a dimensionless phase normalization κ_{bc}; for an elementary standing-wave convention, it is commonly π, while a full-wavelength convention gives 2π. Define the primitive closure count

$$\nu_{\text{c}} = \frac{\omega_{\text{c}}\tau_{\text{c}}}{\kappa_{\text{bc}}} = \frac{\omega_{\text{c}}L_{\text{coh}}}{\kappa_{\text{bc}}v_{\text{eff}}}. \tag{37}$$

Realization Postulate 1 (Causal carrier realization). *On a branch where the physical boundary-value problem identifies the number of independent carrier components with a closed causal mode count, require*

$$\nu_{\text{c}} = d \in \mathbb{N}. \tag{38}$$

This is an optional realization bridge. The response-registry theorem itself selects the ranks $d = 2,3,4$ *structurally and does not depend on every sector possessing a literal cavity.*

The distinction is essential. In a turbulent eddy, for example, the elementary turnover estimate $\omega L/v \sim 1$ cannot explain $N = 4$; in a circular orbit, $\omega a/v = 1$ identically. Equation (37) can therefore use only the collective coherent response frequency and domain licensed by the channel, not an arbitrarily chosen local kinematic frequency and length.

4.3. Physical Reading of the Primitive Theorem

The numbers now have one source. The minimal central phase line $V_\Phi = \mathbb{C}I_2$ has real rank two; the traceless self-adjoint contrast space V_R has rank three; and the complete self-adjoint diagnostic space $V_C = \mathbb{R}I_2 \oplus V_R$ has rank four. The first two-slot complete responses therefore carry 2^2, 3^2, and 4^2 coefficients, while the first certificate-irreducible source-probe-output interaction carries 3^3 coefficients. Hence

$$\boxed{\beta = \dim E_d^{(r)} = d^r}, \quad (4, 9, 16, 27) = \left(2^2, 3^2, 4^2, 3^3\right). \tag{39}$$

Parent β	Carrier Rank d	Arity r	Primitive Complete Response
$4 = 2^2$	2	2	central phase input → phase output
$9 = 3^2$	3	2	regional contrast input → regional contrast output
$16 = 4^2$	4	2	complete ordered-regional input → complete ordered-regional output
$27 = 3^3$	3	3	regional source × regional probe → regional output

The rank three in this table is internal operator contrast, not a proof that space has three dimensions. The rank four is one normalized ordering direction plus three regional contrasts, not four spatial wavelengths. The value 27 is not a shifted square: it is the complete coefficient count of the first genuinely typed interaction response. Larger Hilbert or matrix dimensions remain legitimate system representations, but Proposition 2 prevents representation size from being silently promoted into another universal parent.

Combining the optional causal realization with the primitive theorem gives

$$\boxed{\beta = \left(\frac{\omega_{\mathrm{c}} L_{\mathrm{coh}}}{\kappa_{\mathrm{bc}} v_{\mathrm{eff}}}\right)^r} \tag{40}$$

only where Equation (38) is independently established. The linear causal ratio realizes d; response completeness fixes r; the determinant character makes d^r observable as logarithmic winding. None of these three operations may be substituted for another.

4.4. Bridge to Physical Time

Let $X = X(t)$ along a physical history. Differentiating (32) gives the ordinary instantaneous response frequency

$$\Omega_N = \frac{\mathrm{d}\Theta_N}{\mathrm{d}t} = N\frac{\mathrm{d}\ln X}{\mathrm{d}t} = N\frac{\dot{X}}{X}. \tag{41}$$

Hence,

$$\boxed{N = \frac{\Omega_N}{\mathrm{d}(\ln X)/\mathrm{d}t} = \frac{\Omega_N X}{\dot{X}}.} \tag{42}$$

If the comparison variable is a coherent size $X = L$ whose boundary evolves according to $\dot{L} = \eta v_{\text{eff}}$, then

$$\boxed{N = \frac{\Omega_N L}{\eta v_{\text{eff}}}.} \tag{43}$$

This explains the apparent tension in the original intuition. At a fixed parent N, increasing L lowers the physical response frequency as L^{-1} when ηv_{eff} is fixed. At a fixed physical frequency, a larger coherent domain permits a larger response number. The two comparisons hold different quantities fixed and are not contradictory.

4.5. Why All Parents May Coexist

A physical state need not choose one parent. It may carry

$$\mathrm{Q} = \mathrm{Q}_4 \oplus \mathrm{Q}_9 \oplus \mathrm{Q}_{16} \oplus \mathrm{Q}_{27}, \tag{44}$$

and a channel or population may therefore exhibit

$$Z_j(x,\lambda) = \sum_{N\in\{4,9,16,27\}} A_{jN}(\lambda)\mathrm{e}^{i[Nx+\phi_{jN}(\lambda)]}, \tag{45}$$

where λ labels source history, formation channel, geometry, or other latent data. If relative phases are sufficiently decohered across a population, the cross terms vanish under averaging and

$$\left\langle \left|Z_j\right|^2 \right\rangle = \sum_N \int p(\lambda)\left|A_{jN}(\lambda)\right|^2 \mathrm{d}\lambda. \tag{46}$$

Thus, all parent powers can survive in different proportions even when no individual event displays a conspicuous multi-parent alignment. This is the natural operator interpretation of a heterogeneous compact-object population: the parents are simultaneous orthogonal response channels, while their observed proportions measure generation, projection, transfer, and decoherence.

4.6. A Derived Decoherence Hierarchy

Suppose a parent phase is observed with additive logarithmic-coordinate uncertainty δx having zero mean and variance s. For Gaussian phase diffusion,

$$\left\langle \mathrm{e}^{iN\delta x} \right\rangle = \mathrm{e}^{-N^2 s/2}. \tag{47}$$

Therefore,

$$A_{jN}^{\text{obs}} = A_{jN}^{\text{gen}} p_{jN} T_{jN} \exp\left(-\frac{N^2 s}{2}\right). \tag{48}$$

Higher parents are intrinsically more fragile under the same phase uncertainty.

A heterogeneous population need not have one variance. Let s follow a gamma distribution

$$p(s)=\frac{s^{q-1}e^{-s/s_0}}{\Gamma(q)s_0^q},\quad s\geq 0. \tag{49}$$

Averaging (47) gives its Laplace transform,

$$D_q(N)=\int_0^\infty e^{-N^2s/2}p(s)\mathrm{d}s=\left(1+\frac{s_0N^2}{2}\right)^{-q}. \tag{50}$$

At high parent number,

$$D_q(N)\sim N^{-2q}. \tag{51}$$

The earlier binary-black-hole study used the phenomenological hierarchy $a(N)\propto N^{-3/2}$. Equation (50) shows that this law is obtained asymptotically from a gamma mixture with

$$q=\frac{3}{4}. \tag{52}$$

This is a candidate physical explanation, not a retrospective proof. A serious test must derive or freeze the variance distribution from formation modelling, measurement posteriors, and selection effects before evaluating the parent amplitudes.

4.7. Exact Parents and Shifted Observational Contacts

The structural spectrum fixes parent support, but an observation generally sees a complex transfer envelope. Write one projected parent as

$$Z_{jN}(x)=H_{jN}(x)e^{iNx},\quad H_{jN}(x)=A_{jN}(x)e^{i\psi_{jN}(x)}. \tag{53}$$

Its observed phase is $Nx+\psi_{jN}(x)$.

Proposition 6 (Transfer-phase displacement). *On any interval where* H_{jN} *is nonzero and differentiable, the local logarithmic frequency inferred from phase is*

$$\boxed{\beta_{jN}^{\mathrm{eff}}(x)=N+\psi'_{jN}(x)=N+\frac{\mathrm{d}}{\mathrm{d}x}\arg H_{jN}(x).} \tag{54}$$

A constant transfer phase changes only the fitted phase. A frequency displacement requires a nonconstant transfer phase.

Proof. Differentiate $\arg Z_{jN}(x)=Nx+\psi_{jN}(x)$ with respect to x. □

Accordingly, a historical contact near 27.57 is not a fifth exact parent merely because it is numerically stable in a route. It can be a descendant of the exact interaction parent 27 only if an independently specified transfer supplies

$$\psi'_{j,27}(x)\simeq 0.57 \tag{55}$$

over the fitted interval. This statement is falsifiable: once the route's transfer phase is derived or measured and removed, the residual carrier must return to 27. If it does not, the shifted line remains an empirical candidate outside the exact registry.

4.8. Cross-Channel Phase Locking

Repeated detection of the same parent in two coordinates is weaker than demonstrating that the two phases are one physical phase viewed through two quotient maps. Let

$$\Theta_a = N_a \ln\left(X_a / X_{0a}\right) + \phi_a, \tag{56}$$

$$\Theta_b = N_b \ln\left(X_b / X_{0b}\right) + \phi_b. \tag{57}$$

Proposition 7 (Phase-locking scaling law). *If* $\Theta_a - \Theta_b$ *is constant on a common physical history, then*

$$\boxed{X_b = C X_a^{N_a/N_b}} \tag{58}$$

for a constant $C > 0$. *For equal parents,* $X_b \propto X_a$.

Proof. Constancy of the phase difference gives $N_a \ln X_a - N_b \ln X_b = \text{constant}$. Exponentiation yields the result. □

For the earthquake time-radius tribunal, equal 4 parents would imply the stronger prospective relation $r \propto \tau$ only if the temporal and radial phases are demonstrably locked. Marginal power at 4 in both channels does not by itself establish that relation. This is a clean next-generation test because it distinguishes one causal response from two unrelated scale-space projections.

4.9. Generation, Visibility, and Fossil Silence

The exact parent registry is a statement of availability, not universal brightness. Factor the observed complex amplitude as

$$H_{jN}(x) = G_{jN}(x) P_{jN}(x) T_{jN}(x) D_{jN}(x), \tag{59}$$

where G is generated response, P is quotient-level projection, T is physical and instrumental transfer, and D is coherence survival. The observable parent is absent when any load-bearing factor vanishes or when its product lies below the predeclared tribunal sensitivity.

A finite-memory fossil descendant makes the logic explicit. After a source switches off at t_f, let

$$\dot{Q}_N + \tau_N^{-1} Q_N = 0, \quad Q_N(t) = Q_N(t_f) e^{-(t-t_f)/\tau_N}. \tag{60}$$

If propagation and averaging additionally accumulate Gaussian log-phase variance s_N, the visible amplitude obeys

$$\left|A_{jN}^{\text{obs}}\right| \le \left|A_{jN}^{\text{freeze}}\right| \exp\left[-\frac{t-t_f}{\tau_N} - \frac{N^2 s_N}{2}\right]. \tag{61}$$

Thus, higher parents are the first to disappear under a common loss of coherence, and an old fossil may become β-silent without the parent registry ceasing to exist. This is the exact logical role of the BAO tribunal: it tests whether a predeclared fossil route remains silent rather than supplying arbitrary registry power in every smooth residual.

Verdict 3 (Simple physical meaning of β). *The primitive causal count* d *measures how many independent carrier components a coherent closure supports. The parent* $\beta = d^r$ *counts how many independent coefficients are required to* *record the corresponding complete response. Logarithmic comparison turns that coefficient count into a scale-space winding. Observed parent proportions then measure how strongly each complete response is generated, projected, and preserved, not how many ordinary wavelengths fit across the visible object.*

5. The Complete State and Its Domain

Let (M, g_{ab}) be a time-oriented four-dimensional Lorentzian manifold of signature $(-,+,+,+)$. Let T be an ordering scalar with timelike gradient,

$$u_a = -\frac{\nabla_a T}{\sqrt{-g^{bc}\nabla_b T \nabla_c T}}, \quad u^a u_a = -1, \quad h_{ab}^{(u)} = g_{ab} + u_a u_b. \tag{62}$$

Let $\mathcal{R}_{ab}$ be a symmetric type-I response stress possessing a simple future-directed timelike eigenvector v^a,

$$\mathcal{R}_{ab} v^b = -\rho_{\mathcal{R}} v_a, \quad v^a v_a = -1, \quad h_{ab}^{(v)} = g_{ab} + v_a v_b. \tag{63}$$

The simple-eigenvalue condition is essential: at eigenvalue crossings or outside the type-I domain, the present representative is undefined rather than silently continued.

The complete state is

$$\boxed{\mathbf{U} = \left(g_{ab}, T, \Psi, A_a, X, \mathcal{R}_{ab}, \{\mathrm{J}_N, \mathrm{Q}_N, P_N\}_{N\in\{4,9,16,27\}}, \hat{\mathrm{N}}; \mathcal{D}, \mathcal{B}, \mathcal{P}, \mathcal{O}\right),} \tag{64}$$

where Ψ denotes matter fields, A_a a licensed gauge connection, X the global drift coordinate, P_N the registry projectors, $\hat{\mathrm{N}}$ the response-number operator, $\mathcal{D}$ the common domain package, $\mathcal{B}$ boundary/initial data, $\mathcal{P}$ the parameter and provenance ledger, and $\mathcal{O}$ the commissioned family of equation-number-observable maps. The logarithmic comparison coordinate used by an observational channel is not a second physical time; it is a dimensionless coordinate $x_j = \ln\left(X_j / X_{0j}\right)$ defined only when the channel's comparison law is multiplicative. The internal response domain is the product of open unit balls

$$\mathrm{Q}_N \in \mathbb{B}_N := \left\{Q \in \mathcal{E}_N : \|Q\|_N < 1\right\}, \quad N \in \{4, 9, 16, 27\}. \tag{65}$$

The boundary $\|Q\|_N = 1$ is an infinite-source saturation boundary, not an additional finite state.

6. Finite Composition and the Unique Minimal Saturation Law

For $J \in \mathcal{E}_N$, use the root-mean-square norm

$$x_N(J) = \|J\|_N := \sqrt{\frac{\langle J, J\rangle_N}{N}}. \tag{66}$$

The previous papers adopted a useful saturating law but did not explain why that member of the admissible class should be preferred. The following explicit realization axiom removes that arbitrariness without promoting it into a theorem of necessity.

Realization Postulate 2 (Dimension-normalized finite composition). *Two successive nonnegative scalar loads in a complete* N *-component response module compose as*

$$x \oplus_N y = x + y + \frac{xy}{N}. \tag{67}$$

The unsatisfied response fraction $r_N(x)$ *is continuous, positive, multiplicative under this composition, normalized by* $r_N(0) = 1$, *and has a unit initial response slope* $r_N'(0) = -1$.

The composition is associative and isomorphic to ordinary addition through

$$\varphi_N(x) = N \ln(1 + x/N), \quad \varphi_N(x \oplus_N y) = \varphi_N(x) + \varphi_N(y). \tag{68}$$

Theorem 6 (Uniqueness of the finite-response profile). *Under the finite-composition postulate,*

$$\boxed{r_N(x) = \left(1 + \frac{x}{N}\right)^{-N}, \quad \mu_N(x) = 1 - r_N(x) = 1 - \left(1 + \frac{x}{N}\right)^{-N}.} \tag{69}$$

This is the unique continuous solution. Equivalently,

$$\mu_N'(x) = \left[1 - \mu_N(x)\right]^{1+1/N}, \quad \mu_N(0) = 0. \tag{70}$$

Proof. Define $g_N(s) = -\ln r_N\left(\varphi_N^{-1}(s)\right)$. Multiplicativity of r_N and additivity of φ_N give the continuous Cauchy equation $g_N(s+t) = g_N(s) + g_N(t)$, hence $g_N(s) = \lambda s$. The initial-slope condition gives $\lambda = 1$. Therefore, $r_N = \mathrm{e}^{-\varphi_N} = (1 + x/N)^{-N}$. Differentiation gives Equation (70). □

The postulate is falsifiable. A prospectively commissioned response curve requiring an additional shape parameter or violating Equation (67) rejects this minimal realization; the exponent may not be repaired after the data are inspected.

Define the convex potential and constitutive map

$$\mathrm{W}_N(J) = N \int_0^{x_N(J)} \mu_N(s)\,\mathrm{d}s, \tag{71}$$

$$\boxed{\mathrm{C}_N(J) = \nabla_J \mathrm{W}_N(J) = \begin{cases} \dfrac{\mu_N(x_N)}{x_N} J, & x_N > 0, \\ 0, & x_N = 0. \end{cases}} \tag{72}$$

Theorem 7 (Convexity, monotonicity, covariance, and saturation). *For every admitted* N, W_N *is strictly convex. At* $J \neq 0$, *the Jacobian eigenvalues are*

$$\lambda_\parallel = \mu_N'(x_N) = \left(1 + \frac{x_N}{N}\right)^{-N-1}, \quad \lambda_\perp = \frac{\mu_N(x_N)}{x_N}, \tag{73}$$

with $0 < \lambda_\parallel, \lambda_\perp \le 1$. *Consequently* C_N *is strictly monotone, nonexpansive, orthogonally equivariant, and maps* $\mathcal{E}_N$ *diffeomorphically onto the open unit ball* $\mathbb{B}_N$.

Proof. Differentiating Equation (71) and using $\mathrm{d}x_N = \langle J, \mathrm{d}J \rangle_N / (N x_N)$ gives

Equation (72). The Hessian of a radial function splits into the radial and transverse eigenspaces. Positivity follows from Equation (70); nonexpansiveness follows from $\mu_N' \le 1$ and concavity of μ_N with $\mu_N(0) = 0$. Orthogonal covariance follows because the norm and scalar profile are invariant. Finally $\|\mathrm{C}_N(J)\|_N = \mu_N(\|J\|_N)$ increases from 0 to 1. □

Theorem 8 (Spectral-constitutive commutation). *Let* $\mathrm{C} = \oplus_N \mathrm{C}_N$ *and* $\tau(\hat{\mathrm{N}}) = \sum_N \tau_N P_N$. *Then,* $[\mathrm{C}, \mathrm{e}^{ix\hat{\mathrm{N}}}] = 0$ *in the natural complexification and the lifted finite-response equation is*

$$\tau(\hat{\mathrm{N}})\mathfrak{D}_v\tilde{\mathrm{Q}} + \tilde{\mathrm{Q}} = \mathrm{e}^{ix\hat{\mathrm{N}}}\mathrm{C}(\mathrm{J}), \quad (\partial_x - i\hat{\mathrm{N}})\tilde{\mathrm{Q}} = 0. \tag{74}$$

Thus, the registry fixes carrier frequencies while the constitutive law fixes bounded amplitudes and memory.

Proof. Both C and $\tau(\hat{\mathrm{N}})$ preserve every spectral subspace $P_N\mathcal{E}$. Applying $\mathrm{e}^{ix\hat{\mathrm{N}}}$ to the blockwise relaxation equations gives the result. □

The composition and spectral-generator residuals are reported in Table 2.

Table 2. Numerical audit of the finite-composition derivation and spectral generator.

N	Composition Residual	Autonomous-ODE Residual	Generator Residual
4	1.665e−16	6.625e−12	3.403e−11
9	1.665e−16	6.027e−11	4.527e−11
16	9.437e−16	2.386e−11	6.865e−11
27	5.551e−16	9.441e−12	1.458e−10

7. Typed Geometric Source Maps

Choose local oriented orthonormal frames $e_i^{(u)a}$ and $e_i^{(v)a}$ on the u- and v-rest spaces. Their induced derivatives $D_i^{(u)}$ and $D_i^{(v)}$ include the corresponding spatial frame connections, so every expression below transforms in the stated orthogonal representation rather than by componentwise differentiation. Define

$$\bar{a}_i = c^2 e_i^{(u)a} u^b \nabla_b u_a, \quad \bar{a} = \left(\bar{a}_i \bar{a}^i\right)^{1/2}, \tag{75}$$

$$\Theta^i{}_j = e^i_{(u)a} e_j^{(u)b} \nabla_b u^a, \quad B^i{}_j = e^i_{(v)a} e_j^{(v)b} \nabla_b v^a. \tag{76}$$

Let V_2 be oriented and let $\mathrm{e}_4 \in \mathrm{End}(V_2)$ be a fixed unit complex-structure generator. The frozen geometric source maps and the hierarchical interaction source are

$$\mathscr{J}_4[g, T] = \frac{\bar{a}}{a_4}\mathrm{e}_4, \quad a_N = \frac{NcH_\star}{2\pi^2}, \tag{77}$$

$$\left(\mathscr{J}_9[g, \mathcal{R}]\right)^i{}_j = \frac{c}{H_\star} B^i{}_j, \tag{78}$$

$$\mathscr{J}_{16}[g, T, X] = \begin{pmatrix} \dfrac{cu^a\nabla_a X}{H_\star} & \dfrac{\bar{a}_j}{a_4} \\ \dfrac{c\left(D^{(u)}\right)^i X}{H_\star} & \dfrac{c\Theta^i{}_j}{H_\star} \end{pmatrix}, \tag{79}$$

$$\left(\mathscr{I}_{27}[\mathrm{J}_9]\right)^i{}_{jk} = \ell_{27}\left(D_k^{(v)}\mathrm{J}_9\right)^i{}_j, \quad \ell_{27} = \sqrt{27}\,\ell_{\mathrm{P}}. \tag{80}$$

Thus J_9 is first constrained to the response-flow deformation and J_{27} is then constrained to its covariant spatial variation. The hierarchy is first order in the independent variables (v, J_9); eliminating J_9 only after variation recovers the second-derivative geometric expression. A factorized source-probe input $Y^i S_j P_k$ is a rank-one special case, not a replacement for the full 27-component module. The phase source Equation (77) is the licensed one-dimensional embedding used by the stationary $N = 4$ branch; the complete $\mathrm{End}(V_2)$ carrier remains available to general phase-response data and is exercised by the numerical module audit.

The maps contain no new continuous parameter. They may nevertheless fail physically: if the required oriented frames, derivatives, source/probe roles, or quotient actions are absent, the corresponding sector is undefined. Numerical activation of a module does not prove that a specific observed system realizes its source map.

8. Positive Curvature Gate and Finite Response

The Lorentzian Kretschmann contraction is not a positive norm on arbitrary spacetimes. Define instead

$$\gamma^{ab} = g^{ab} + 2v^a v^b, \quad \mathcal{C}_v = R_{abcd}R_{efgh}\gamma^{ae}\gamma^{bf}\gamma^{cg}\gamma^{dh} \geq 0. \tag{81}$$

For each registry parent let

$$\ell_N = \sqrt{N}\,\ell_{\mathrm{P}}, \quad \boxed{\tau_N^{-2} = H_\star^2 + \left(\frac{c}{\ell_N}\right)^2 \mu_N \left(\ell_N^4 \mathcal{C}_v\right)^2.} \tag{82}$$

Thus, $\tau_N \to H_\star^{-1}$ at sub-Planckian response curvature and $\tau_N \to \ell_N / c$ when $\ell_N^4 \mathcal{C}_v \gg 1$.

Let $\mathfrak{D}_v$ denote the representation-covariant material derivative along v^a. The four internal response equations are

$$\boxed{\tau_N \mathfrak{D}_v \mathrm{Q}_N + \mathrm{Q}_N = \mathrm{C}_N\left(\mathrm{J}_N\right), \quad N \in \{4, 9, 16, 27\}.} \tag{83}$$

For a constant target,

$$\mathrm{Q}_N(\lambda) = \mathrm{C}_N\left(\mathrm{J}_N\right) + \mathrm{e}^{-(\lambda-\lambda_0)/\tau_N}\left[\mathrm{Q}_N(\lambda_0) - \mathrm{C}_N\left(\mathrm{J}_N\right)\right]. \tag{84}$$

The squared distance to equilibrium decays exactly as $\mathrm{e}^{-2\Delta\lambda/\tau_N}$.

9. Equilibrium Generator and Spacetime Response

The $N = 4$ metric embedding is retained from the corrected phase action. With $a = \sqrt{a_a a^a}$, $a^a = u^b \nabla_b u^a$, and $Z_N = 1 + c^2 a / (N a_N)$, define

$$I_N(Z) = \frac{Z^{2-N} - 1}{2-N} - \frac{Z^{1-N} - 1}{1-N}, \tag{85}$$

$$f_N(a) = \Lambda_\star - \frac{2N^2 a_N^2}{c^4} I_N(Z_N). \tag{86}$$

The phase action is

$$S_4[g, T, \Psi] = \frac{c^3}{16\pi G} \int \sqrt{-g}\left[R - 2 f_4(a)\right] \mathrm{d}^4 x + S_m[g, \Psi]. \tag{87}$$

It gives

$$f_N'(a) = -2a\left(1 + \frac{\bar{a}}{N a_N}\right)^{-N}, \quad 1 + \frac{f_N'(a)}{2a} = \mu_N(\bar{a}/a_N). \tag{88}$$

For $N = 9, 16, 27$, let W_N^* be the convex conjugate of Equation (71). Introduce auxiliary multipliers Λ_N and the first-order registry functional

$$S_{\mathrm{reg}} = \frac{c^3}{8\pi G} \int \sqrt{-g}\left(\frac{H_\star}{c}\right)^2 \sum_{N \in \{9,16,27\}} \left[\mathrm{W}_N^*(\mathrm{Q}_N) - \langle \mathrm{Q}_N, \mathrm{J}_N \rangle_N + \langle \Lambda_N, \mathrm{J}_N - \mathscr{J}_N \rangle_N\right] \mathrm{d}^4 x. \tag{89}$$

Variation with respect to Q_N gives $\mathrm{Q}_N = \mathrm{C}_N(\mathrm{J}_N)$; variation with respect to Λ_N gives the three source constraints. Because $\mathscr{J}_{27}$ depends on the independent J_9, the multiplier equations are hierarchical: $\Lambda_{27} = \mathrm{Q}_{27}$, while the J_9 equation contains the formal adjoint of $\ell_{27} D^{(v)}$ acting on Λ_{27}. No false claim that every multiplier equals its response is required. The equilibrium response stress is defined by the single variational source

$$\mathcal{R}_{ab}^{\mathrm{eq}} := -\frac{2}{\sqrt{-g}} \frac{\delta\left(S_4 - S_{\mathrm{EH}} - S_m + S_{\mathrm{reg}}\right)}{\delta g^{ab}}. \tag{90}$$

Diffeomorphism invariance gives $\nabla^a \mathcal{R}_{ab}^{\mathrm{eq}} = 0$ when the ordering, auxiliary, frame, and matter equations hold and when boundary fluxes are included in the declared variational domain. The hierarchical auxiliary form keeps Equation (80) first order before elimination. It does not by itself prove that the fully gauge-fixed coupled principal symbol is strongly hyperbolic; that remains an explicit completion test.

Generated response need not remain in instantaneous equilibrium. Decompose

$$S_{ab} = \Delta_{ab}{}^{cd} \mathcal{R}_{cd}, \quad \Delta_{ab}{}^{cd} = h_{(a}^{(v)c} h_{b)}^{(v)d}, \tag{91}$$

and evolve

$$\nabla^a \mathcal{R}_{ab} = 0, \tag{92}$$

$$\tau_{27} \Delta_{ab}{}^{cd} \mathcal{L}_v S_{cd} + S_{ab} - S_{ab}^{\mathrm{eq}} = 0, \quad S_{ab}^{\mathrm{eq}} = \Delta_{ab}{}^{cd} \mathcal{R}_{cd}^{\mathrm{eq}}. \tag{93}$$

There are four conservation equations and six spatial-stress equations for the ten components of a symmetric stress.

Theorem 9 (Response equation-count closure). *On the simple type-I domain, Equations* (92) *and* (93) *provide ten nonduplicated equations for the ten components of* $\mathcal{R}_{ab}$. *The eigenflow* v^a *is algebraically determined by* $\mathcal{R}_{ab}$ *and introduces no additional propagating degree of freedom.*

This theorem establishes component counting and Bianchi compatibility. It is not a proof of strong hyperbolicity or global nonlinear stability of the gauge-fixed gravity-ordering-response system.

10. Conditional Local Cauchy Closure

A covariant equation is not predictive merely because its indices are correct. A local initial-value theorem is required. The full unrestricted theorem depends on the matter and ordering completions, so the strongest defensible result is conditional and explicit.

Theorem 10 (Conditional local well-posedness). *Fix a spacelike Cauchy surface and suppose*:

(H1) *The metric equations are imposed in generalized harmonic gauge and reduced to the standard first-order symmetric-hyperbolic Einstein block*;

(H2) *The matter and ordering equations admit first-order symmetric-hyperbolic reductions with positive symmetrizers on the declared state domain*;

(H3) *After retaining* J_9 *and the auxiliary multipliers as independent variables*, *every source map is* C^1 *and contains at most first derivatives of the reduced state*;

(H4) $0 < \tau_{\min} \le \tau_N \le \tau_{\max} < \infty$;

(H5) $\mathcal{R}_{ab}$ *remains type I with a simple timelike eigenvalue separated by a uniform gap*, *so* $v^a(\mathcal{R})$ *is* C^1 *and uniformly timelike*;

(H6) *The projected spacetime-response block possesses a positive spatial symmetrizer and subluminal characteristic advection relative to the chosen Cauchy foliation*.

Then, *for Sobolev data* $\mathbf{U}_0 \in H^s$ *with* $s > 5/2$ *satisfying the constraints*, *the gauge-fixed master equation admits a unique local solution*

$$\mathbf{U} \in C\left(\left[0, t_*\right], H^s\right) \cap C^1\left(\left[0, t_*\right], H^{s-1}\right), \tag{94}$$

which depends continuously on the initial data and remains inside the regular domain until one of the stated domain bounds fails.

Proof. Under (H1) - (H6), the reduced equations have the quasilinear first-order form

$$A^0(\mathbf{U})\partial_t \mathbf{U} + A^i(\mathbf{U})\partial_i \mathbf{U} = F(\mathbf{U}), \tag{95}$$

where the direct sum of the metric, matter, ordering, registry-advection, and spatial-response symmetrizers is positive definite and symmetrizes every principal matrix. The constitutive maps are C^1 and locally Lipschitz by strict convexity; the algebraic eigenflow map is C^1 by the simple-eigenvalue hypothesis. Standard quasilinear symmetric-hyperbolic existence and uniqueness then apply [10] [11]. Constraint propagation follows from the harmonic-gauge subsidiary system and the common stress-conservation identity while the solution remains in the declared domain. □

This theorem does not establish that every possible matter action or singular extension satisfies (H1) - (H6). The executable audits one frozen principal block. Its characteristic speeds in units of c are metric ±1, ordering ±0.82, matter ±0.61, and response advection 0.23; the recorded symmetrizer is positive, and all frozen speeds are real and causal. This is a diagnostic witness of the theorem's hypotheses, not a replacement for a model-specific analytic proof.

11. The Single Block Master Equation

Let $\mathcal{D}_a = \nabla_a - iqA_a/\hbar$ on a licensed Abelian scalar branch and $\Pi_a = \nabla_a S - qA_a$. Define

$$\mathfrak{G}^{ab} = -\Omega^2 u^a u^b + c^2 e^{-2X} h^{(u)ab}, \quad \hat{\mathcal{O}}\psi = -\hbar^2 \mathcal{D}_a\left(\mathfrak{G}^{ab}\mathcal{D}_b\psi\right) + \hbar^2\omega_0^2\psi, \tag{96}$$

and impose the drift identity

$$cu^a\nabla_a X = \frac{c}{3}\nabla_a u^a. \tag{97}$$

On FLRW, this gives $X = \ln A$ after one normalization datum.

The complete regular-domain representative is one block equation,

$$\boxed{\mathfrak{M}[\mathbf{U};x] = \begin{pmatrix} \hat{\mathcal{O}}\psi \\ G_{ab} - \kappa_g\left(T^m_{ab} + \mathcal{R}_{ab}\right) \\ \delta S_m/\delta\Psi \\ \delta\left(S_4 + S_{\text{reg}}\right)/\delta T \\ cu^a\nabla_a X - \frac{c}{3}\nabla_a u^a \\ \left\{\mathrm{J}_N - \mathscr{J}_N\right\}_{N=4,9,16,27} \\ \left\{\tau_N \mathfrak{D}_v \mathrm{Q}_N + \mathrm{Q}_N - \mathrm{C}_N\left(\mathrm{J}_N\right)\right\}_{N=4,9,16,27} \\ \left(\partial_x - i\hat{\mathrm{N}}\right)\tilde{\mathrm{Q}} \\ \nabla^a\mathcal{R}_{ab} \\ \Delta_{ab}{}^{cd}\left[\tau_{27}\mathcal{L}_v S_{cd} + S_{cd} - S^{\text{eq}}_{cd}\right] \end{pmatrix} = 0.} \tag{98}$$

The spectral row does not add physical time evolution; it closes the exact logarithmic carrier structure of every commissioned observable channel. The cosmological term is counted once through the equilibrium functional; no second $+\Lambda g_{ab}$ is appended to the metric row.

Under the eikonal ansatz, the scalar characteristic descendant is

$$\boxed{\Omega^2\left(u^a\Pi_a\right)^2 - c^2 \mathrm{e}^{-2X} h^{(u)ab}\Pi_a\Pi_b - \hbar^2\omega_0^2 = 0.} \tag{99}$$

It is the five-regime shell of the books, but it is not the complete law by itself.

12. Five Mandatory Regimes and Their Common Overlaps

The five controlled descendants and their failure boundaries are set out in Table 3.

The same Ω, c, X, ω_0, gauge-covariant momentum, and source package occur in all overlaps. Setting gap, potential, or drift variables to their limiting values commutes algebraically; no parameter is retuned between regimes. The executable verifies the combined shell at machine precision and separately verifies gauge compensation.

Table 3. Controlled descents of one master package.

Regime	Controlled Limit	Closed Equation/Observable	Failure Boundary
Propagation	$\omega_0 = 0$, regular principal block	Characteristic cone of Equation (96); gapless phase speed	No hyperbolic principal block or no signal observable
Ordering	Timelike ∇T, one positive energy sheet	u^a, clock/phase ordering, no second primitive time	Null/spacelike ∇T, sheet crossing, multiple independent orderings
Localization	$\omega_0 > 0$ on the same shell	Rest gap $\hbar\omega_0$, $m = \hbar\omega_0 / c^2$ where licensed	No persistent gapped sector or uncommissioned mass relabelling
Universal coupling	Stationary $N = 4$ equilibrium with localization source	Nonlinear Poisson equation, one metric, equal lensing potentials	Source/response mismatch, nonuniversal matter coupling, failed domain
Global drift	Homogeneous metric branch with Equation (97)	$X = \ln A$, $1 + z = e^{X_o - X_e}$	Arbitrary imposed history or local potential relabelled as cosmology

13. Equation-Number-Observable Commissioning

Warning 1 (Algebra-to-observation bridge). *The algebraic registry theorem fixes available coefficient-module dimensions; it does not, by itself, assert an observed logarithmic winding. A channel-level claim* $\beta = N$ *requires, before inspecting the tested residuals:* 1) *a positive comparison variable* X_j *with the multiplicative coordinate* $x_j = \ln\left(X_j / X_{0j}\right)$; 2) *a nonzero generated component* $P_N\mathrm{Q}$; *and* 3) *specified transfer and observation maps* T_j *and* L_j *for which* $L_j T_j P_N \mathrm{Q} \neq 0$. *If any condition fails, the parent remains algebraically available, but the channel makes no detection claim.*

For every empirical or numerical channel j, the operational record is

$$\mathfrak{C}_j = \left(X_j, X_{0j}, x_j, B_j, L_j, T_j, \Sigma_j, \mathcal{N}_j, \tau_j, V_j\right), \tag{100}$$

where $x_j = \ln\left(X_j / X_{0j}\right)$, B_j is the frozen nonoscillatory baseline, L_j the physical projection, T_j the transfer operator, Σ_j the uncertainty/covariance record, $\mathcal{N}_j$ the null family, τ_j the statistic, and V_j the verdict map. The master equation supplies Q; the channel record supplies the operational descent through Equation (28). Neither may be fitted by changing the other after residual inspection.

A fixed-frequency regression has the form

$$Y_j(x) = B_j(x) + \sum_{N \in \{4,9,16,27\}} \left[A_{jN} \cos(Nx) + B_{jN} \sin(Nx)\right] + \epsilon_j. \tag{101}$$

Only parents predicted visible by the frozen $L_j T_j$ map are confirmatory. A scan is a different statistic and must be calibrated by applying the identical scan to every null surrogate. The accompanying solver contains a reusable fixed-frequency engine and a synthetic active/fossil unit test; it does not manufacture empirical evidence.

The resulting active/fossil protocol check is reported in Table 4.

Table 4. Synthetic protocol unit test. The active signal was generated with parents 4 and 9; the fossil control contains neither. Values are amplitudes and fractional baseline-RSS improvements.

N	Active Amplitude	Active Improvement	Fossil Improvement
4	1.205098e−01	0.770944	0.000074
9	6.605294e−02	0.231542	0.002165
16	1.437112e−03	0.000110	0.000173
27	1.383044e−03	0.000102	0.000011

14. Covariant $N = 4$ Phase Branch, Galaxies, and Lensing

The metric variation of Equation (87) gives

$$\mathcal{R}_{ab}^{\mathrm{eq},(4)} = -\frac{c^4}{8\pi G}\left[f_4 g_{ab} + 2u_a u_b \nabla_c \left(\chi_4 a^c \right) - 2\chi_4 a_a a_b \right], \quad \chi_4 = \frac{f_4'(a)}{2a}. \tag{102}$$

The ordering equation follows from variation with respect to T. On the stationary order-aligned weak-field branch,

$$\mathrm{d}s^2 = -c^2\left(1+\frac{2\Phi}{c^2}\right)\mathrm{d}t^2 + \left(1-\frac{2\Psi}{c^2}\right)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j + O\left(c^{-4}\right), \tag{103}$$

with $\Psi = \Phi$ and

$$\boxed{\nabla \cdot \left[\mu_4\left(\frac{|\nabla\Phi|}{a_4} \right)\nabla\Phi \right] = 4\pi G \rho_b.} \tag{104}$$

For spherical symmetry,

$$g\mu_4\left(g/a_4\right) = g_b, \quad V_c\left(r\right) = \sqrt{rg\left(r\right)}. \tag{105}$$

The function $g \mapsto g\mu_N\left(g/a_N\right)$ is strictly increasing, so every $g_b \ge 0$ has one nonnegative solution.

The embedded 232-point, seven-galaxy audit is summarized in Table 5.

Table 5. Frozen registry control on the same galaxy subset. These are not four simultaneous fits; they are parent-selection controls with no nuisance-parameter or covariance fit.

N	a_N (m·s^{-2})	RMSE (km·s^{-1})	MAE (km·s^{-1})	R^2
4	1.326968e−10	10.729	8.014	0.9608
9	2.985677e−10	21.631	18.575	0.8406
16	5.307870e−10	38.382	35.001	0.4980
27	8.957031e−10	57.147	52.601	-0.1129

The $N = 4$ control has RMSE 10.729 km·s^{-1} versus 49.164 km·s^{-1} for the baryon-only velocities on this selected subset. This is retrospective internal contact, not a prospective population-level model comparison with distance, inclination, mass-to-light, covariance, or selection uncertainties [12].

Because $\Phi = \Psi$, the stationary lensing potential equals Φ. For a spherical source,

$$\alpha_4(b) = \frac{4b}{c^2}\int_b^\infty \frac{g_4(r)}{\sqrt{r^2 - b^2}}\mathrm{d}r. \tag{106}$$

For the frozen $10^{11} M_\odot$ point-source example at $b = 10\ \text{kpc}$, $z_L = 0.3$, $z_S = 1.5$, the solver obtains

$$\alpha_4 = 0.809617\,\text{arcsec}, \quad \theta_E = 0.862765\,\text{arcsec}, \tag{107}$$

compared with $0.394837\,\text{arcsec}$ for baryons-only GR. The calculation is conditional on the phase equilibrium branch and a separately constrained baryonic mass map.

15. Physical Commissioning of the 9, 16, and 27 Modules

Numerical operation of a vector space is not a physical sector. A sector is commissioned only when a typed source, response equation, projection, and observable are all explicit. The following benchmarks are controlled descendants, not claims that the corresponding observational datasets have been reanalyzed here.

15.1. $N = 9$: Regional Turbulent Transport

For a local incompressible flow with characteristic velocity U and length L, define the complete regional source

$$(\mathrm{J}_9)^i{}_j = \frac{L}{U}\nabla_j u^i, \quad \operatorname{tr}\mathrm{J}_9 = 0, \tag{108}$$

and $\mathrm{Q}_9 = \mathrm{C}_N(\mathrm{J}_9)$ in equilibrium. The symmetric and antisymmetric parts provide frame-invariant dissipation and enstrophy proxies,

$$\mathcal{D}_9 = 2\left\|\operatorname{sym}\mathrm{Q}_9\right\|_F^2, \quad \mathcal{Z}_9 = 2\left\|\operatorname{anti}\mathrm{Q}_9\right\|_F^2. \tag{109}$$

A log-wavenumber channel uses $x = \ln(k/k_0)$ and a frozen linear projection of $\mathrm{e}^{ix\hat{\mathrm{N}}}\mathrm{Q}$. The executable obtains

$$\mathcal{D}_9 = 0.394345, \quad \mathcal{Z}_9 = 0.553618, \quad \delta_{\text{frame}} = 5.551\mathrm{e}-17. \tag{110}$$

Its fixed-frequency benchmark recovers a primary 4 amplitude 7.932665−03 and a secondary 9 amplitude 3.420090e−03. This is structurally aligned with the separate DNS paper, which reports $\beta = 4$ as primary and $\beta = 9$ more selectively, but no DNS likelihood is recomputed here [13] [14].

15.2. $N = 16$: Ordered-Regional Transfer

Let an ordered transport tetrad split one ordering coordinate from three regional coordinates. A complete transfer source is an endomorphism

$$\mathrm{J}_{16} = \begin{pmatrix} J_{00} & J_{0j} \\ J_{i0} & J_{ij} \end{pmatrix} \in \mathrm{End}(V_4), \tag{111}$$

with mixed order-region blocks retained. For a weak-lensing-style screen projection, define

$$\kappa_{16} = \frac{1}{2}\left(Q^1{}_1 + Q^2{}_2\right) + \frac{1}{4}\left(Q^0{}_1 + Q^1{}_0 + Q^0{}_2 + Q^2{}_0\right). \tag{112}$$

The benchmark gives $\kappa_{16} = 3.940073\mathrm{e}-02$. Removing the mixed blocks changes it by 2.317690e−03, while the covariant-frame residual is 1.528e−16. A fixed-16 observable is recovered with amplitude 6.304116e−03; the wrong-9 control has amplitude 1.457178e−04. The branch, therefore, commissions the complete ordered-regional module rather than merely relabelling a 3 × 3 response.

15.3. $N = 27$: Typed Source-Probe Interaction

For unit output, source, and probe vectors (a, b, c), the operational interaction channel is

$$\mathcal{I}_{27} = a_i b^j c^k \left(Q_{27}\right)^i{}_{jk}. \tag{113}$$

The frozen benchmark gives $\mathcal{I}_{27} = 8.050476\mathrm{e}-02$ and recovers the fixed-27 amplitude 1.046562e−02; the wrong-16 amplitude is 1.358407e−04. This supplies an end-to-end typed interaction observable. It does not identify every astrophysical interaction with this projection.

The common numerical audit of all four modules is collected in Table 6.

Table 6. Cross-registry numerical audit under the common constitutive law.

N	$\lVert J \rVert_N$	$\lVert C_N(J) \rVert_N$	Covariance	Gradient Error	RK Order
4	1.005112	0.592071	1.028e−16	1.075e−09	4.060
9	0.223632	0.198202	1.179e−16	1.198e−09	4.060
16	0.223467	0.199021	2.010e−16	1.328e−09	4.060
27	0.229546	0.204334	1.947e−16	5.063e−09	4.060

16. Finite Memory and the Merger Demonstrator

The stress relaxation and internal module equations give ordinary constitutive memory. For a process of duration t_{proc} with slowly varying target,

$$m_{\mathrm{mem}} = \mathrm{e}^{-t_{\mathrm{proc}}/\tau_{27}}. \tag{114}$$

The frozen Bullet-like reduction uses a positive curvature proxy, response masses inferred from the $N = 4$ spherical law at declared apertures, a prescribed initial response flow aligned with the collisionless galaxies, and Gaussian thin-lens components. It obtains

$$\mathcal{C}_v^{\mathrm{proxy}} = 2.247019\mathrm{e}-98\ \mathrm{m}^{-4}, \tag{115}$$

$$\ell_{27}^4 \mathcal{C}_v^{\mathrm{proxy}} = 1.117825\mathrm{e}-234, \tag{116}$$

$$\tau_{27} = 3.910404\mathrm{e}+17\ \mathrm{s}, \tag{117}$$

$$t_{\mathrm{coll}}/\tau_{27} = 0.018938, \tag{118}$$

$$m_{\mathrm{mem}} = 0.981240. \tag{119}$$

The two one-dimensional convergence peaks lie at −354.60 kpc and 357.12 kpc, close to the prescribed galaxy centroids and offset from the prescribed gas centroids. This establishes a kinematic mechanism: long response memory can preserve collisionless alignment without setting curvature to zero. It does not derive the hydrodynamics, shock, galaxy-flow alignment, centroids, widths, three-dimensional mass reconstruction, or observational likelihood of 1E 0657-558 [15] [16].

17. Tensor Propagation and Radiation Jurisdiction

On an equilibrium FLRW background with $u^a = v^a$, $a^a = 0$, and vanishing equilibrium and initial TT response transient, the phase and registry terms add no TT kinetic or spatial-gradient term to the metric principal action. The two metric tensor modes obey

$$\ddot{h}_A + 3H\dot{h}_A + \frac{c^2k^2}{A^2}h_A = \frac{16\pi G}{c^4}\Pi_A^{\mathrm{TT}}, \quad A = +,\times. \tag{120}$$

Thus, on this branch,

$$c_T = c, \quad N_{\mathrm{metric\ TT}} = 2, \quad d_L^{\mathrm{GW}} = d_L^{\mathrm{EM}}. \tag{121}$$

For the frozen $30M_\odot$ chirp-mass, $z = 0.1$, 100 Hz example, the leading optimal strain is 1.820808e−21. A nonzero response TT transient can source Equation (20) but does not change the metric principal cone in this linearized branch. No statement is made about scalar/vector response spectra, binary generation, nonlinear compact-source dynamics, or unconditional global hyperbolicity. The luminal result is compatible with the GW170817 multimessenger constraint [17].

18. Exact Regular Spherical $N = 27$ Quotient

The strong-field result is a declared static spherical quotient, not the elimination of every black-hole problem. Let

$$\ell_{27} = \sqrt{27}\,\ell_{\mathrm{P}}, \quad r_s = \frac{2GM}{c^2}, \quad r_c^3 = r_s\ell_{27}^2, \tag{122}$$

with

$$m_{27}(r) = M\mu_{27}\left(r^3/r_c^3\right), \quad f(r) = 1 - \frac{2Gm_{27}(r)}{c^2r}. \tag{123}$$

Einstein's equation defines a conserved anisotropic source. Near the origin,

$$m_{27}(r) = M\frac{r^3}{r_c^3} + O\left(r^6\right), \quad f(r) = 1 - \frac{r^2}{\ell_{27}^2} + O\left(r^5\right), \quad K(0) = \frac{24}{\ell_{27}^4} < \infty. \tag{124}$$

At infinity the metric is Schwarzschild. The solver gives

$$M_{\mathrm{crit}} = 1.011363\mathrm{e}-07\ \mathrm{kg}, \quad r_{\mathrm{crit}} = 1.266207\mathrm{e}-34\ \mathrm{m}. \tag{125}$$

For $M = 10M_{\odot}$,

$$r_{-} = 8.398307\text{e}-35\ \text{m},\quad r_{+} = 2.953339\text{e}+04\ \text{m},\quad \tau_{27}(0) = 2.801374\text{e}-43\ \text{s}. \tag{126}$$

The central curvature singularity is absent within this family. Generic collapse formation, rotation, inner-horizon mass inflation, nonlinear stability, evaporation, and uniqueness among regular geometries are not established [18] [19].

19. External β Tribunals, BAO Silence, and Their Exact Role

The programme's empirical page lists selected tests in earthquakes, planetary architecture, solar flares, DNS turbulence, atrial fibrillation, tropical cyclones, and volcanic recurrence [14]. The broader empirical corpus and the *Harmonic Law* additionally contain SPARC, weak-field acceleration, binary-black-hole, ACT-lensing, supernova/expansion, microphysical, and BAO analyses [20]. The tests are not homogeneous and must not be collapsed into one significance number. Their proper role is a typed ledger: each route states its comparison coordinate, fixed parent support, null, transfer burden, and admissible conclusion.

The positive, selective, shifted, and silent routes are listed in **Table 7**.

Table 7. Restored positive, selective, shifted, and silent empirical ledger.

Route	Reported Fixed Support	Causal-Spectral Reading	Exact Status and Principal Limitation
SPARC rotation residuals and weak-field scale	4-line contact; separate acceleration centering	Minimal phase response may survive radial projection; the acceleration descendant tests an equation-number-observable relation rather than merely harmonic power	Retrospective external test plus a small embedded commissioning subset; halo/systematic and population-likelihood questions remain [12] [20]
Earthquake critical dynamics	4 in logarithmic time and radius	Minimal complete phase response in two comparison coordinates; the decisive future test is common-phase locking, which would imply $r \propto \tau$ for equal parents	External fixed-frequency, multi-channel tribunal; very large formal effects do not by themselves establish prospective forecasting [21]
DNS turbulence	4 primary, 9 selective	Phase closure is robust while a complete regional endomorphism is route-selective	External test connected to the explicit J_9 benchmark; local turnover identity $\omega L/v \sim 1$ forbids a naive cavity reading [13]
Atrial-fibrillation onset	4	Retrospective log-time approach to a boundary with minimal response burden	External tribunal; no clinical prediction or causal intervention claim follows [22]

Continued

Binary-black-hole chirp masses	Canonical $\{4,9\}$; full current subset $\{4,9,16,27\}$	Incoherent multi-module population: lighter parents dominate, higher parents are more fragile, and ensemble power need not imply event-level alignment	External $N=98$ study; modest population-level evidence, selection, and hierarchical population inference incomplete [20] [23]
ACT/Planck lensing routes	4 dominant in one residual analysis; a separate map mixture used 4, 9, 16, 27.57	Evolving projection can retain phase, regional, ordered-regional, and shifted interaction contacts in different angular bands	External survey analyses; exact 27 is not established by a 27.57 template and full end-to-end survey simulations remain required [20] [24]
Supernova and expansion channels	9 in differential expansion analyses; high contact near 27 - 27.57 in low-redshift residual routes	Differential observables may retain regional response while integrated distances smooth it; the high line is an interaction-parent neighbourhood unless transfer-phase recovery returns it to 27	External and model-dependent; calibration, covariance, survey offsets, and look-elsewhere discipline remain load-bearing [20] [25]
H-α solar-flare durations	4 and 27.57	Active source-driven durations retain a light phase line and a strong shifted high contact	External smooth-null/ Rayleigh analysis; analytic tails are stronger than finite Monte Carlo resolution, and 27.57 is not silently promoted to exact 27 [26]
Volcanic recurrence	16 and 27.57 in the archived route	Ordered-regional burden plus a shifted interaction contact in a source-driven recurrence process	External volcano-aware recurrence tribunal; catalog construction and rate heterogeneity remain decisive [20] [27]
Tropical-cyclone intensification	Fixed multi-mode templates	An active, non-equilibrium atmospheric trajectory may project several response burdens during intensification	External cross-validated route; basin dependence, storm dependence, serial correlation, and forecast separation must be retained [28]
Planetary architecture	Geometric and quantised ladder; historical β contacts	Architecture may encode multiplicative closure, but the elementary orbital identity $\omega a/v=1$ prevents direct identification of β with an ordinary orbital wavelength count	External architectural tribunal; useful for discrete-scale structure, not yet a clean derivation of carrier rank [29]

Continued

Microphysical excitation ladders	Historical fixed harmonic contacts	Finite bound-state spectra may preserve scale recurrence, but mapping a spectroscopic ladder to one current response module requires a typed carrier and transfer proof	External exploratory tribunal; not load-bearing for the exact registry without modern covariance and selection controls [20] [30]
BOSS LOWZ BAO monopole	No positive support expected or reported for historical $\{4, 9, 16, 27.57\}$	Fossil standard-ruler route: generation has ceased, and projection, propagation, reconstruction, and phase mixing may drive D_{jN} toward zero	Strategically necessary negative control; diagonal covariance, one region, simple templates, and finite mocks prevent a definitive cosmological null [31]

19.1. What the Black-Hole Proportions Do and Do Not Mean

The binary black hole study is the clearest population example of Equation (46). The canonical result was carried mainly by 4 and 9. The full fixed set included the four current parents, but the higher lines were subdominant, and the per-event multi-frequency amplitude was consistent with the smooth-baseline null. This is not four detections and not evidence that each black hole is a four-frequency cavity. It is a weak population-level mixture compatible with several orthogonal response modules being sampled by heterogeneous formation channels and then differently attenuated by selection, posterior width, projection, and decoherence.

The earlier weighting $a(N) \propto N^{-3/2}$ was phenomenological. Equation (50) supplies a possible asymptotic origin through a gamma mixture with $q = 3/4$, but it becomes predictive only when the diffusion distribution is fixed from independent formation modelling, posterior propagation, or a disjoint training catalog.

19.2. BAO as the Necessary Adverse Tribunal

The BAO monopole is not an omitted positive test. It is the cleanest adverse channel in the empirical programme. The archived analysis fitted fixed templates in $x = \ln k$ and reported no positive response at

$$B_{\text{hist}} = \{4, 9, 16, 27.57\}, \tag{127}$$

with the combined statistic lying in approximately the lowest 0.3% of its simplified diagonal-covariance Monte Carlo null. The admissible conclusion is limited: under that frozen protocol, one LOWZ South monopole was unusually β-silent relative to generic residual noise. It is not a full-covariance proof that BAO must be silent in every sample.

The modern registry requires a stricter two-template comparison:

$$B_{\text{exact}} = \{4, 9, 16, 27\}, \tag{128}$$

$$B_{\text{hist}} = \{4, 9, 16, 27.57\}. \tag{129}$$

A decisive rerun should use the published covariance or survey mocks, LOWZ North and South, CMASS, pre- and post-reconstruction spectra, and higher multipoles. It should test 27 and 27.57 separately. If the fossil route is genuinely silent, neither fixed set should acquire persistent positive power. If only the shifted line survives, the transfer-phase account must be independently demonstrated rather than inferred from the same residual.

19.3. The Empirical Logic

The forward direction is

$$\begin{aligned}&\mathfrak{M}[\mathbf{U}] = 0 \to \mathrm{Q}_N \to G_{jN} \to P_{jN} T_{jN} D_{jN} \to \mathcal{O}_j(x) \to \hat{\beta}_j \\ &\to \text{declared null comparison.}\end{aligned} \tag{130}$$

The external papers test the final links. They do not prove the carrier-rank realization, determinant theorem, finite-composition postulate, or decoherence distribution. Conversely, the master equation now supplies a sharper prospective grammar: exact parent support is fixed; shifted contacts require transfer phase; amplitudes are route-dependent; cross-channel identity requires phase locking; and fossil silence is a positive negative-control prediction rather than an embarrassment.

20. Frozen Numerical Atlas, Controls, and Fate

The embedded Python 3.10+ solver uses only the standard library. Its frozen atlas contains:

1) finite-composition, autonomous-ODE, and spectral-generator audits for all four parents;

2) a causal-spectral audit of the minimal rank-two phase carrier, $N = d^r$, scale-space recurrence ratios, the physical-time bridge, transfer-phase displacement, cross-channel scaling, and the candidate gamma-mixture decoherence hierarchy;

3) convex-gradient, frame-covariance, monotonicity, invariant-ball, and fourth-order relaxation audits;

4) a five-regime shell, overlap, and Abelian gauge-quotient audit;

5) physically typed $N = 9, 16$ and 27 source-to-observable benchmarks;

6) a fixed-frequency active/fossil protocol unit test and a machine-readable complete external ledger including BAO silence and exact-versus-shifted high templates;

7) a frozen conditional-principal-symbol audit;

8) the selected galaxy, lensing, tensor, merger-memory, and regular-spherical descendants;

9) defect-sensitive controls for the wrong parent, wrong frame, uncompensated gauge, unstable sign, quotient loss, and negative curvature.

Every mandatory audit passes, and every injected defect is detected. The numerical fate is

$$\boxed{\text{PASS on the frozen regular executable jurisdiction.}} \tag{131}$$

This is not an empirical verdict and not a theorem outside the finite atlas. It means that the archived implementation matches the equations, converges under the stated refinements, respects its declared quotients, and rejects its own known defects.

21. Parameter, Assumption, and Provenance Ledger

The status and prohibited promotion of each load-bearing input are recorded in Table 8.

Table 8. Load-Bearing provenance.

Entry	Status	Role	Prohibited Promotion
$c, G, \hbar$	Metrological/empirical	Propagation, coupling, conversion	Not predicted here
$H_\star$	Boundary/background datum	Infrared rate and a_N	Not a derivation of cosmology
$N = 4, 9, 16, 27$	Corpus-relative uniqueness theorem	Primitive response-number spectrum	Not a classification of theories or representation sizes outside the frozen jurisdiction
$x \oplus_N y$	Realization postulate	Uniquely selects μ_N	Not forced by the registry theorem alone
L_j, T_j, B_j	Channel-derived or commissioned	Observable map and visibility	May not be chosen after residual inspection
$\Lambda_\star$	Total vacuum datum	Counted once in the equilibrium action	No duplicate metric term
ω_0, m, q	Sector data	Localization and gauge branch	Commissioned values are not spectrum predictions
Initial $Q_N, \mathcal{R}_{ab}$	Initial data	Transient history and memory	Not universal constants
SPARC inputs	Retrospective commissioning	Seven-galaxy internal audit	Not a population likelihood
Cluster geometry/flow	Controlled inputs	Memory mechanism demonstrator	Not derived hydrodynamics
Black-hole mass	Source datum	Static spherical quotient	Not a collapse prediction
External β papers	Separate empirical records	Positive, selective, shifted, and silent tribunals	Their significance is not regenerated here; exact 27 and historical 27.57 remain distinct
Solver and tolerances	Numerical protocol	Reproducibility	Not physical parameters

22. Failure Conditions and Unresolved Obligations

The representative fails, branches, or becomes undefined if any of the following occur:

1) The terminal complex operator envelope, primitive-factor quotient, diagnostic minimality, typed-role discipline, or complete coefficient-retention premise fails; in that case, the primitive carrier-signature theorem loses jurisdiction, and the alternative branch must be restored rather than suppressed; if instead the co-

efficient-transport or least-character premise fails, the carrier dimensions survive but the observable β-winding corollary does not;

2) A larger representation dimension is promoted to a universal parent without an independently irreducible response role, or a process, effect, record, normalization line, nonlinear jet, ancillary extension, or determinant harmonic is relabelled as a new primitive signature;

3) $\nabla_a T$ ceases to be timelike or the positive energy sheet crosses a branch point;

4) $\mathcal{R}_{ab}$ leaves the simple type-I domain, and no independently defined continuation exists;

5) A source or observable map fails to descend through its frame, gauge, or coordinate quotient;

6) Finite-load composition, convexity, covariance, spectral support, or a declared negative control fails;

7) Matter and response stresses violate the common conservation identity;

8) A commissioned matter/ordering branch fails the hypotheses of the conditional Cauchy theorem;

9) A boundary, singular extension, or regulator changes an observable without an explicit branch split;

10) A physical sector requires a new coefficient, frequency, projection, or transfer repair after data inspection;

11) A prospectively frozen active channel lacks a predicted detectable parent, or a fossil control shows persistent registry power incompatible with its transfer model;

12) A claimed causal-cavity realization identifies d using a local frequency and length that $\omega L/v$ is kinematically fixed or unrelated to the complete response carrier;

13) A decoherence exponent or variance distribution is inferred from the tested parent amplitudes and then presented as an independent prediction;

14) A shifted contact such as 27.57 is promoted to an exact parent without an independently derived transfer-phase gradient, or the transfer-corrected line fails to return to 27;

15) A fossil negative control, such as BAO, develops persistent registry power under full covariance and predeclared templates without a licensed active source or transfer branch;

16) The gravitational descendants fail comparison with GR plus the same matter and nuisance assumptions.

The primitive carrier-signature theorem is now closed inside its corpus jurisdiction; the corresponding β-winding registry remains the conditional observation corollary stated in Corollary 1. Its next obligation is an independent audit of the corpus handoff and primitive-factor/no-surplus premises, not another numerical fit. The remaining physical obligations are an unconditional model-specific hyperbolicity proof for the final matter and ordering action, global existence or controlled breakdown criteria, singular-domain assembly, scalar and vector perturbation spectra, post-Newtonian and strong-coupling bounds, cosmological perturbations and structure formation, full cluster hydrodynamics, generic collapse and rotation, and prospective empirical comparisons driven directly from the source-to-observable maps.

23. Conclusions

Verdict 4. *The response registry is no longer introduced by selecting three convenient carrier ranks and four convenient response modules. Within the terminal Natural Physics architecture, the selection is now derived. Minimal central phase and the one-world complex operator envelope leave one primitive noncommutative diagnostic cell,* $M_2(\mathbb{C})$. *Its center, traceless self-adjoint contrast space, and complete self-adjoint space have real ranks* 2, 3, *and* 4. *Process typing leaves one-input/one-output self-response and source/probe/output interaction as the only primitive arities. Therefore*

$$\boxed{\{(d,r)\}_{\text{primitive}} = \{(2,2),(3,2),(4,2),(3,3)\}},$$

and complete coefficient retention fixes the real module dimensions $\{4,9,16,27\}$. *Conditional on the coefficient-basis-neutral unitary transport and least-character observation premises, these dimensions give*

$$\boxed{\mathrm{Spec}^{+}_{\text{primitive}} = \{4,9,16,27\}}, \quad \boxed{\beta = \dim E_d^{(r)} = d^r}.$$

The physical statement is correspondingly spare. Nature does not expose a bare carrier rank; it exposes a complete typed response. Within the stated observation realization, β *counts the independent phase-bearing coefficients required to carry that response once around one logarithmic scale orbit. The rank-three carrier is the traceless contrast space of the minimal operator cell, not an imported three-dimensional space. Higher system dimensions, nonlinearities, ancillas, composite sources, effects, records, and harmonics remain possible, but they are descendants or representation coordinates until an independent no-factorization certificate commissions a new primitive role.*

Logarithmic comparison turns the coefficient count into phase per e-fold, $\Omega_N = N\,\mathrm{d}(\ln X)/\mathrm{d}t$. *A cavity-like ratio may realize the primitive rank only on a branch that independently supplies the coherent domain, collective frequency, propagation speed, and boundary condition. Transfer and decoherence control visibility rather than support:* $\beta_{\text{eff}} = N + \mathrm{d}(\arg H_N)/\mathrm{d}x$ *makes shifted contacts testable, and* $\mathrm{e}^{-N^2 s/2}$ *makes unequal black-hole population powers possible without individual four-line resonators. Transfer correction must return a claimed shifted descendant to its exact parent; otherwise, the claim fails.*

The empirical record remains a tribunal, not a premise of the theorem. Positive and selective contacts are retained across galactic, seismic, turbulent, biological, compact-object, lensing, expansion, solar, volcanic, atmospheric, planetary, and microphysical routes, while BAO remains the explicit fossil silence control. None *of these observations was used to choose* $M_2(\mathbb{C})$, *the carrier decomposition, the primitive arities, or the four dimensions. Their task is now sharper: test whether nature generates and preserves the independently derived support.*

The executable master system remains internally cross-registry and operationally cross-sector on its declared regular domain. It retains the finite-composition law, memory blocks, conservation-compatible metric response, conditional local Cauchy closure, qualified galaxy and lensing branch, luminal tensor propagation on the specified FLRW equilibrium, merger memory, and regular static spherical quotient. Numerical audits certify that implementation; they do not establish nature.

Accordingly, the present construction makes a precise foundational advance: the primitive carrier-signature registry is a corpus-relative uniqueness theorem rather than a fitted list or a declared grammar, while the β*-winding registry is its conditional observation corollary under the stated coefficient-transport and least-character premises. The limitation is exact. The carrier theorem governs the one-world complex operator jurisdiction selected by the Natural Physics corpus; it does not classify every conceivable algebraic theory, forbid higher-dimensional systems, derive every sector's coherent domain, complete nonlinear dynamics, or establish empirical superiority over general relativity,* Λ*CDM, the Standard Model, or established effective theories. Those are the remaining physical tests of the architecture, not missing definitions of* β.

24. Code Integrity and Regeneration

The standalone solver is supplied as a separate file with the submission. Running it under Python 3.10 or later, using only the standard library, regenerates the full-precision JSON audit record. The revised submission package contains the TeX source, the compiled PDF, the solver, the generated JSON record, and SHA-256 checksums for the archived content files. A valid regeneration must complete every mandatory audit and reproduce the archived record within the numerical tolerances declared by the executable.

Data and Code Availability

The complete TeX source, standard-library Python implementation, generated audit record, and checksums accompany the submission. The internal numerical audits require no proprietary software or external data files. External empirical contacts are cited to their original archival records and are not represented as reproduced analyses in this paper.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Panagis, C.N. (2026) Before Space, Time, and Force: The Necessity of Structure.

[2] Panagis, C.N. (2026) The Necessity of Quantum Structure.

[3] Panagis, C.N. (2026) The Necessity of Natural Physics, Vol. I: Composition and the Complete Space of Composite Worlds. https://philarchive.org/archive/PANTNO-5

[4] Panagis, C.N. (2026) The Necessity of Natural Physics, Vol. II: Processes, Interaction, Open Dynamics, and Exchange Structure. https://philarchive.org/archive/PANTNO-6

[5] Panagis, C.N. (2026) The Necessity of Natural Physics, Vol. III: Regionality, Locality, Propagation, and Field Structure. https://philarchive.org/archive/PANTNO-7

[6] Panagis, C.N. (2026) The Necessity of Natural Physics, Vol. IV: The Physical Completion of the Unique Operator World—Symmetry, Matter, Parameters, and Empirical Closure. https://philarchive.org/archive/PANTNO-8

[7] Murphy, G.J. (1990) C^*-Algebras and Operator Theory. Academic Press.

[8] Hall, B.C. (2015) Lie Groups, Lie Algebras, and Representations: An Elementary Introduction. 2nd Edition, Springer.

[9] Panagis, C.N. (2026) Structural Origin of the Logarithmic Response Registry: A Conditional Derivation from Complete Response Modules. Zenodo. https://zenodo.org/records/21129307

[10] Fourès-Bruhat, Y. (1952) Théorème d'existence pour certains systèmes d'équations aux dérivées partielles non linéaires. *Acta Mathematica*, **88**, 141-225. https://doi.org/10.1007/bf02392131

[11] Kato, T. (1975) The Cauchy Problem for Quasi-Linear Symmetric Hyperbolic Systems. *Archive for Rational Mechanics and Analysis*, **58**, 181-205. https://doi.org/10.1007/bf00280740

[12] Lelli, F., McGaugh, S.S. and Schombert, J.M. (2016) SPARC: Mass Models for 175 Disk Galaxies with Spitzer Photometry and Accurate Rotation Curves. *The Astronomical Journal*, **152**, 157. https://doi.org/10.3847/0004-6256/152/6/157

[13] Panagis, C.N. (2025) Evidence for a Universal $\beta = 4$ Log-Periodic Mode in DNS Turbulence Spectra Consistent with Unified Substrate Theory. Zenodo. https://doi.org/10.5281/zenodo.17886579

[14] Panagis, C.N. (2026) Empirical Tests. https://nosresearch.org/empirical-tests/

[15] Clowe, D., Bradač, M., Gonzalez, A.H., Markevitch, M., Randall, S.W., Jones, C., *et al.* (2006) A Direct Empirical Proof of the Existence of Dark Matter. *The Astrophysical Journal*, **648**, L109-L113. https://doi.org/10.1086/508162

[16] Bradač, M., Clowe, D., Gonzalez, A.H., Marshall, P., Forman, W., Jones, C., *et al.* (2006) Strong and Weak Lensing United. III. Measuring the Mass Distribution of the Merging Galaxy Cluster 1ES 0657-558. *The Astrophysical Journal*, **652**, 937-947. https://doi.org/10.1086/508601

[17] Abbott, B.P., Abbott, R., Abbott, T.D., *et al.* (2017) Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger: GW170817 and GRB 170817A. *The Astrophysical Journal Letters*, **848**, L13.

[18] Poisson, E. and Israel, W. (1990) Internal Structure of Black Holes. *Physical Review D*, **41**, 1796-1809. https://doi.org/10.1103/physrevd.41.1796

[19] Hayward, S.A. (2006) Formation and Evaporation of Nonsingular Black Holes. *Physical Review Letters*, **96**, Article 031103. https://doi.org/10.1103/physrevlett.96.031103

[20] Panagis, C.N. (2025) Unified Substrate Theory: The Harmonic Law.

[21] Panagis, C.N. (2025) Testing Unified Substrate Theory: A Dual-Channel > 50σ Detection of Log-Periodic $\beta = 4$ Dynamics Before and after Earthquake Mainshocks. Zenodo. https://zenodo.org/records/17846656

[22] Panagis, C.N. (2026) A Universal $\beta = 4$ Log-Periodic Precursor to Atrial Fibrillation. Zenodo. https://doi.org/10.5281/zenodo.20111949

[23] Panagis, C.N. (2025) Testing Unified Substrate Theory with Binary Black Holes: A ~3σ Population-Level Signature of the UST β-Spectrum. Zenodo. https://doi.org/10.5281/zenodo.21907478

[24] Panagis, C.N. (2025) Testing Unified Substrate Theory: A 5σ Detection of UST β-Modes in the ACT DR6 Lensing Convergence Map. Zenodo. https://doi.org/10.5281/zenodo.21907351

[25] Panagis, C.N. (2025) Testing Unified Substrate Theory: Harmonic Structure and Predictions for Redshift Drift. Zenodo. https://zenodo.org/records/21907426

[26] Panagis, C.N. (2025) Testing Unified Substrate Theory through Solar Flare Statistics: A 30σ Harmonic Signature in H-Alpha Flare Durations. Zenodo. https://zenodo.org/records/17853006

[27] Panagis, C.N. (2025) Harmonic Signature in Volcanic Recurrence: Observational Evidence for Unified Substrate Theory. Zenodo. https://zenodo.org/records/17872649

[28] Panagis, C.N. (2025) Discrete-Scale-Invariant Harmonic Structure in Tropical Cyclone Intensification: A Cross-Validated Multi-Mode Log-Periodic Analysis of the 1851-2023 Atlantic Basin. Zenodo. https://zenodo.org/records/17846619

[29] Panagis, C.N. (2025) Quantised Geometric Structure in Planetary Architectures: A 13σ Test of the UST φ-Harmonic Scaling Law. Zenodo. https://zenodo.org/records/17848658

[30] Panagis, C.N. (2025) Testing Unified Substrate Theory: A 4.1σ Detection of Substrate Harmonics in the Microcosm. Zenodo. https://doi.org/10.5281/zenodo.21907394

[31] Panagis, C.N. (2025) Testing Unified Substrate Theory: β-Silence in the BOSS LOWZ BAO Monopole. Zenodo. https://zenodo.org/records/21907540