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Classification of Light Unflavored, Strange Mesons and Δ, N, Λ, Σ, Ξ Baryons Based on Heron's Triangles of Their Decays in Hyperbolic Velocity Space, and the Proposal to Search for New Resonances

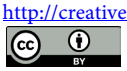
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Abstract

Two-particle decays of light unflavored, strange mesons and Δ, N, Λ, Σ, Ξ baryons are mapped to isosceles triangles in hyperbolic velocity space. A Lorentz-invariant function is defined on the isosceles triangles—the hyperbolic sine of the triangle, equal to the product of the hyperbolic sine of the altitude dropped from any vertex and the hyperbolic sine of the side opposite that vertex. Integer values of the hyperbolic sine correspond to triangles named Heron's base triangles. Based on Heron's triangles, the decays of scalar, strange mesons and Δ, N, Λ, Σ, Ξ baryons, are classified. This classification provides suggestions to search for new resonances.

Keywords

Lobachevsky-Bolyai Velocity Space, Decay Triangles, Hyperbolic Sine of the Triangle, Base Heron Triangle

1. Introduction

The endpoints of the velocity vectors of the two decay particles of the resonance represent material points in the Lobachevsky-Boyai hyperbolic velocity space with curvature $k=-1/C^2$ (where $C=1$ is the speed of light, and the points are assigned the masses of the decay particles) [1]-[5]. Two particles velocity points can be connected by line segment and an arc line, called the oricycle [6]. An oricycle is a line of rotation of curvature 0, whose center "C_0" lies among infinitely distant points of velocity space. Archimedes' laws of levers define a third point "m" on

the arc of the oricycle, to which an additive mass (the sum of the decay particle masses) is assigned. The roles of the forces in the levers are played by the rest masses of the decay particles, and the lever arms are equal to the lengths of the arcs of the oricycle. By connecting these 3 points with straight line, we get triangle of resonance decay inscribed in the oricycle. The effective resonance mass is determined by the hyperbolic cosine of the length of the side opposite the point "m", and by the masses of the decay particles [5]. When the masses of the decay particles are equal, we have an isosceles triangle $\Delta\pi_1 m\pi_2$ of the decay of the meson $f_o(980)\to\pi_1+\pi_2$ (**Figure 1**). If the masses of the decay particles are unequal, we have a different-sided decay triangle $\Delta\eta m\pi_1$ of the decay of the mezon $a_o(1450)\to\eta+\pi_1$ (**Figure 2**). The triangle $\Delta\eta m\pi_1$ can be mapped to the isosceles triangle $\Delta\pi_1 m\pi_2$ by rotating the arc $l_{m\pi_1}$ of an oricycle around the straight line ($m-G_{\eta\pi_1}$) (the point "$G_{\eta\pi_1}$" represents the center of inertia of the pair particles π_1 and η) (**Figure 2**). Thus, the two-body decays of scalar, strange mesons and Δ, N, Λ, Σ, Ξ baryons are associated with the different-sided decay triangles $\Delta bm\pi_1$ and the isosceles triangles $\Delta\pi_1 m\pi_2$ inscribed in the oricycle of hyperbolic velocity space (HVS) (**Figure 3**). Further, we will refer to the isosceles triangle $\Delta\pi_1 m\pi_2$ as the base triangle.

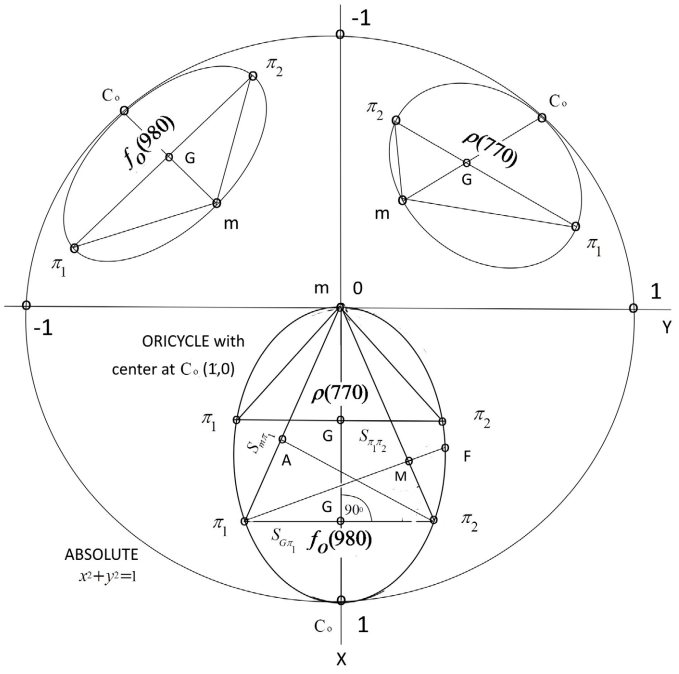


Figure 1. Decays of $\rho(770)\to\pi_1+\pi_2$ and $f_o(980)\to\pi_1+\pi_2$ in the Beltrami model of the hyperbolic velocity space. The separate ellipses of decay oricycles of $\rho(770)$, $f_o(980)$ with centers in the "C_0" points of the circle $X^2+Y^2=1$, called the ***Absolute***. Due to the congruence of the oricicles in the HVS, the ellipse with semi-axis (C_0 – 0) represents an oricicle in which the individual oricicles of the $\rho(770)$ and $f_o(980)$ meson decays are combined. The point-velocity "G" represents the centers of inertia of pairs of particles (π_1,π_2).

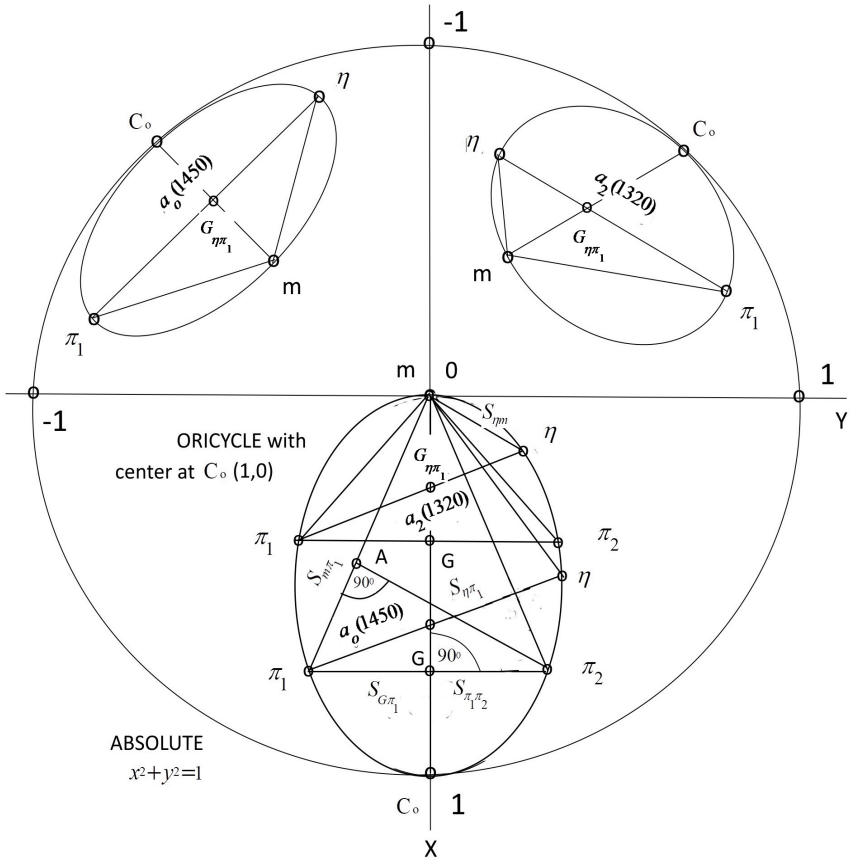


Figure 2. The different sided triangles $\Delta\eta m\pi_1$ of the decays of the $a_2(1320)\to\eta+\pi_1$ and $a_o(1450)\to\eta+\pi_1$, isosceles base triangles $\Delta\pi_1 m\pi_2$, corresponding to the triangles $\Delta\eta m\pi_1$. The point-velocity "G" represents the centers of inertia of pairs of particles (π_1, π_2). The point "$G_{\eta\pi_1}$" represents the center of inertia of the pair particles π_1 and η. The point "C_0" is the center of rotation of the oricycle.

A Lorentz-invariant function is defined on the base triangles $\Delta\pi_1 m\pi_2$—the hyperbolic sine of the triangle, equal to the product of the hyperbolic sine of the altitude dropped from any vertex and the hyperbolic sine of the side opposite that vertex (this function introduced by Y.P. Panarin [7]). Integer values of the hyperbolic sine (*HST*) correspond to triangles named Heron's base triangles.

We attempted to determine how closely the decay triangles of light unflavored, strange mesons and Δ, N, Λ, Σ, Ξ baryons, were related to Heron's base triangles. For this purpose, data on two-particle decays of all the listed resonances is selected from [8] (**Tables A1-A7** in **Appendix**). Based on the data from **Tables A1-A7**, we calculated the hyperbolic sines of the HST_{res} triangles of their decays, the integer values of HST_{her}, the masses m_{her}^{b,π_1} and $m_{her}^{\pi_1,\pi_2}$, and the absolute and relative values of the deviations of the mass m_{her}^{b,π_1} from the effective mass m_{res} (**Tables 1-17**, Equations (3)-(13), $b=\pi_2$, P, n, η, $\rho(770)$, $\omega(782)$, Λ, Σ, $\Sigma_c^0(2455)$, Ξ^0, $\Delta(1232)$, $K(493)$, $K^*(892)$). **Tables 1-17** show that the calculated effective base mass m_{her}^{b,π_1} for Heron's triangles agree well with the

published values of the effective mass m_{res} (the average deviation over all tables equals 2.2%). In Chapter 2, the correspondence between the decay triangles of the listed resonances and Heron's base triangles is studied in more details.

Table 18 presents a summary classification of light unflavored, strange mesons and Δ, N, Λ, Σ, Ξ baryons, based on the values of the hyperbolic sines HST_{her} of their Heron base triangles. Table 18 can be used to propose a method for searching for new resonances. Chapter 3 looks more closely at proposals for finding new resonances.

2. Classification of Light Unflavored, Strange Mesons and Δ, N, Λ, Σ, Ξ Baryons Based on Heron's Triangles of Their Decays in Hyperbolic Velocity Space

Consider a reaction in which the collision of the initial particles B and T produces a pair of particles b and π_1:

$$B+T \to b+\pi_1+all \tag{1}$$

Let us assume that the velocities of the particles B, b and π_1 have been measured in reference frame "0". The ends of the particle velocity vectors represent material velocity points "B," "b", "π_1" in Lobachevsky-Bolyaï hyperbolic velocity space (HVS), located inside a sphere of radius C (hereinafter, the speed of light C = 1; the points are assigned the masses m_B, m_b and m_{π_1} of the reaction particles) [1]-[5]. Let's draw a plane in the HVS passing through the points "B", "π_1" and "b". In this plane, we introduce a rectangular coordinate system $X0Y$ with the origin at the point "0" (Figure 3). The orthogonal projections (X_{π_1}, Y_{π_1}) of the velocity vector of the particle π_1 on the axes $0X$, $0Y$ are called the Beltrami coordinates of the point "π_1" in HVS [6] [9] [10]. The length $S_{b\pi_1}$ of the straight line segment ($b-\pi_1$) with the Beltrami coordinates of its ends "b" (X_b, Y_b), "π_1" (X_{π_1}, Y_{π_1}) is represented by the formula [6]:

$$\operatorname{ch}\left(S_{b\pi_1}\right)=\left(1-X_{\pi_1}*X_b-Y_{\pi_1}*Y_b\right)\Big/\left(R_{\pi_1}*R_b\right) \tag{2}$$

$$R_b=\sqrt{1-X_b^2-Y_b^2},\ R_{\pi_1}=\sqrt{1-X_{\pi_1}^2-Y_{\pi_1}^2}$$

The length $S_{b\pi_1}$ of the straight line ($b-\pi_1$) is called the rapidity [11].

Besides the straight line ($b-\pi_1$), one pair of symmetrical arcs of zero curvature, called oricycles, passes through the velocity points "b" and "π_1" [6]. An oricycle is a line of rotation with curvature 0, the center "C_0" of which lies among infinitely distant points of the HVS (infinitely distant points are referred to as the ***ABSOLUTE*** of the HVS). All oricycles in HVS are congruent as straight lines of zero curvature in Euclidean space are congruent [6]. Therefore, the ellipse with semi-axis (C_0-0) represents an oricycle in which the individual oricycles of the $\rho(770)$ and $f_o(980)$ meson decays are combined (Figure 1).

Archimedes' laws of levers define a third point "m" on the arc of the oricycle, to which the additive mass of the decay particles is attributed. Point "m" with additive mass $m_{b\pi_1}=m_b+m_{\pi_1}$ ($m_b \geq m_{\pi_1}$) is defined by Equations (3). The role of

forces in Archimedes' levers is played by the masses m_b and m_{π_1}, and the lever arms are equal to the Euclidean lengths $l_{b\pi_1}, l_{mb}, l_{m\pi_1}$ of the arcs of the oricycle [12]-[14]:

$$\begin{aligned} & l_{b\pi_1} = l_{m\pi_1} + l_{mb}, \quad m_{b\pi_1} = m_b + m_{\pi_1} \\ & m_b l_{mb} = m_{\pi_1} l_{m\pi_1} = m_{\pi_1} \left(l_{b\pi_1} - l_{mb} \right) \\ & l_{m\pi_1} = m_b l_{b\pi_1} \Big/ \left(m_b + m_{\pi_1} \right) \\ & l_{mb} = m_{\pi_1} l_{b\pi_1} \Big/ \left(m_b + m_{\pi_1} \right) \end{aligned} \tag{3}$$

Equations (3) follow from the fact that Euclidean geometry of zero curvature is valid on the oricycle.

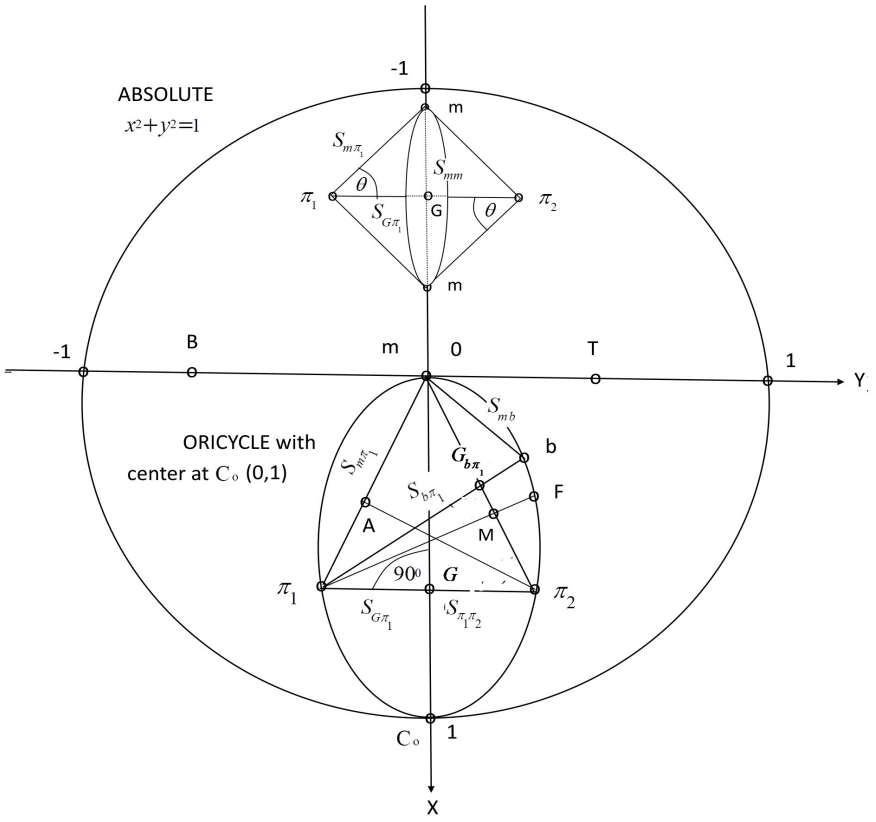


Figure 3. The different sided triangles $\Delta bm\pi_1$ of reaction $B + T \to b + \pi_1 + all$, the base isosceles triangle $\Delta\pi_1 m\pi_2$. The point "C_0" is the center of rotation of the oricycle. The point "$G_{b\pi_1}$" represents the center of inertia of the pair particles π_1 and b. The point-velocity "G" represents the centers of inertia of pairs of particles (π_1, π_2). A straight line drawn from point "G" through the midpoint "M" of side ($m - \pi_2$) intersects the oricycle at point "F". The ratio of the arc lengths $l_{m\pi_2}$ to l_{mF} of the oricycle is equal to the golden ratio 1.61803…

Connecting the points "π_1", "m", "b" with each other by straight lines, we obtain a different sided triangle $\Delta bm\pi_1$ inscribed in the oricycle (**Figure 3**). The

straight line ($C_0 - m$) is called the axis of the oricycle [10]. The lengths of the sides of the triangle $\Delta bm\pi_1$ are related to the lengths of the arcs of the oricycle by the following relationships [6]:

$$l_{b\pi_1} = 2\mathrm{sh}\left(S_{b\pi_1}/2\right),\ \ l_{m\pi_1} = 2\mathrm{sh}\left(S_{m\pi_1}/2\right),\ \ l_{bm} = 2\mathrm{sh}\left(S_{bm}/2\right) \tag{4}$$

In **Figure 3**, in the Beltrami model of HVS, the ellipse tangent to the circle $X^2 + Y^2 = 1$ is a oricycle with center of rotation at the points of tangency " C_0 ". The circle $X^2 + Y^2 = 1$, called the ***Absolute***, represents, according to formula (2), the infinitely distant points of the HVS.

The straight line ($m - G_{b\pi_1}$) passes through point " C_0 " - the center of rotation of the oricycle (the point " $G_{b\pi_1}$ " represents the center of inertia of the pair particles π_1 and b) (**Figure 3**). The infinitely large radius ($m - C_0$) is called the axis of the oricycle. Rotating the arc $l_{m\pi_1}$ of the oricycle around the axis ($m - C_0$), the point " π_1 " will move to the point " π_2 " on the oricycle. By connecting the points " π_1 " "m," and " π_2 " with line segments, we obtain an isosceles triangle $\Delta\pi_1 m\pi_2$. Here, the line segment ($\pi_1 - \pi_2$) of the side of triangle $\Delta\pi_1 m\pi_2$ is perpendicular to the axis ($m - C_0$) of the oricycle. Accordingly, the line segment ($G - m$) is the altitude of $\Delta\pi_1 m\pi_2$ (point "G" is the midpoint of the side ($\pi_1 - \pi_2$)). Thus, the pair of particles b and π_1 in reaction (1) corresponds to a different-sided triangle $\Delta bm\pi_1$ and the isosceles triangle $\Delta\pi_1 m\pi_2$. And vice versa: every isosceles triangle $\Delta\pi_1 m\pi_2$ corresponds to a different-sided triangle $\Delta bm\pi_1$. Further, we will refer to the isosceles triangle $\Delta\pi_1 m\pi_2$ as the base triangle. The base triangle $\Delta\pi_1 m\pi_2$, inscribed in an oricycle, has a very interesting property. A straight line drawn from point "G" through the midpoint "M" of side ($m - \pi_2$) intersects the oricycle at point "F" (**Figure 3**). In this case, the ratio of the arc lengths $l_{m\pi_2}$ to l_{mF} of the oricycle is equal to the golden ratio: 1.61803… [12].

The effective mass m_{res} of the pair of particles b and π_1 is calculated using their rest masses m_b and m_{π_1} and the length $S_{b\pi_1}$ of the side ($b - \pi_1$) of the triangle $\Delta bm\pi_1$ [5]:

$$m_{res}^2 = m_b^2 + m_{\pi_1}^2 + 2m_b m_{\pi_1} \mathrm{ch}\, S_{b\pi_1} \tag{5}$$

The point " π_2 " is assigned the rest mass m_{π_1} of the π_1 meson ($m_{\pi_1} = m_{\pi_2}$). The base effective mass $m_{res}^{\pi_1,\pi_2}$ of the particles π_1 and π_2 is calculated using their rest masses and the length $S_{\pi_1\pi_2}$ of the side ($\pi_1 - \pi_2$) of the base triangle $\Delta\pi_1 m\pi_2$ [5]:

$$m_{base}^{\pi_1,\pi_2} = \sqrt{m_{\pi_1}^2 + m_{\pi_2}^2 + 2m_{\pi_1} m_{\pi_2} \mathrm{ch} S_{\pi_1\pi_2}} \tag{6}$$

From the correspondence between the triangles $\Delta bm\pi_1$ and $\Delta\pi_1 m\pi_2$, it follows that the masses m_{res} and $m_{base}^{\pi_1,\pi_2}$ are also in correspondence. Each value of the effective mass m_{res} corresponds to a base mass $m_{base}^{\pi_1,\pi_2}$. The reverse is also true: each value of the base mass $m_{base}^{\pi_1,\pi_2}$ corresponds to an effective mass m_{res}.

To classify light unflavored, strange mesons and Δ, N, Λ, Σ, Ξ baryons of the reaction (1) used base triangles $\Delta\pi_1 m\pi_2$, obtained by rotating the arc $l_{m\pi_1}$ of the oricycle around the axis ($m - C_0$). For the base triangle $\Delta\pi_1 m\pi_2$, the hyperbolic

sine HST_{res} (introduced by Y.P. Panarin [7]), defined as the product of the hyperbolic sine of the altitude dropped from any vertex and the hyperbolic sine of the side opposite that vertex (**Figures 1-3**):

$$HST_{res} = \mathrm{sh}\, S_{Gm}\, \mathrm{sh}\, S_{\pi_1\pi_2} = \mathrm{sh}\, S_{A\pi_1}\, \mathrm{sh}\, S_{m\pi_2} \tag{7}$$

The resonance decay triangle in the HVS provides more information than the line segment whose length is used to calculate its effective mass. Therefore, the hyperbolic sine HST_{res} of the resonance decay triangle provides more information than its effective mass m_{res}.

Using Equations (3)-(7) and standard transformations of hyperbolic functions, we obtain:

$$\mathrm{ch}\left(S_{b\pi_1}\right) = \left(m_{res}^2 - m_{\pi_1}^2 - m_b^2\right)\Big/ 2m_{\pi_1}m_b, \quad \mathrm{sh}\left(S_{b\pi_1}/2\right) = \sqrt{\left(\mathrm{ch}\left(S_{b\pi_1}/2\right)-1\right)\Big/2} \tag{8}$$

$$l_{b\pi_1} = 2\mathrm{sh}\left(S_{b\pi_1}/2\right), \quad l_{m\pi_1} = m_b l_{b\pi_1}\Big/\left(m_{\pi_1}+m_b\right) = \mathrm{sh}\left(S_{\pi_1\pi_2}/2\right) \tag{9}$$

$$\mathrm{ch}\left(S_{\pi_1\pi_2}/2\right) = \sqrt{\mathrm{sh}^2\left(S_{\pi_1\pi_2}/2\right)+1}, \quad \mathrm{sh}\left(S_{\pi_1\pi_2}\right) = 2\mathrm{sh}\left(S_{\pi_1\pi_2}/2\right)\mathrm{ch}\left(S_{\pi_1\pi_2}/2\right) \tag{10}$$

$$\mathrm{sh}\left(S_{m\pi_1}/2\right) = l_{m\pi_1}/2, \quad \mathrm{ch}\left(S_{m\pi_1}/2\right) = \sqrt{\mathrm{sh}^2\left(S_{m\pi_1}/2\right)+1}, \tag{11}$$

$$\mathrm{sh}\left(S_{m\pi_1}\right) = 2\mathrm{sh}\left(S_{m\pi_1}/2\right)\mathrm{ch}\left(S_{m\pi_1}/2\right), \quad \mathrm{ch}\left(S_{m\pi_1}\right) = \sqrt{\mathrm{sh}^2\left(S_{m\pi_1}\right)+1} \tag{12}$$

From the right-angled triangle $\Delta Gm\pi_1$ we obtain:

$$\mathrm{ch}\left(S_{Gm}\right) = \mathrm{ch}\left(S_{m\pi_1}\right)\Big/\mathrm{ch}\left(S_{\pi_1\pi_2}/2\right), \quad \mathrm{sh}\left(S_{Gm}\right) = \sqrt{\mathrm{ch}^2\left(S_{Gm}\right)-1} \tag{13}$$

The integer values of the hyperbolic sine of a triangle (HST_{her}) correspond to base triangles $\Delta\pi_1 m\pi_2$ that we have named Heron's base triangles. The integer values of HST_{her} for the Heron's base triangles $\Delta\pi_1 m\pi_2$ was used to classify scalar, strange mesons and Δ, N, Λ, Σ, Ξ baryons, (**Tables 1-17**).

In the case of the decay of a Light Unflavored Meson into π_1 and π_2 mesons with equal masses, Equations (3)-(13) take the form ($m_b = m_{\pi_2} = m_{\pi_1}$) (**Figure 1**) [12]-[14]:

$$\begin{gathered} l_{\pi_1\pi_2} = l_{m\pi_1} + l_{m\pi_2}, \quad m_{\pi_1\pi_2} = m_{\pi_1} + m_{\pi_2} \\ m_{\pi_1} l_{m\pi_1} = m_{\pi_2} l_{m\pi_2} = m_{\pi_1}\left(l_{\pi_1\pi_2} - l_{m\pi_1}\right) \\ l_{m\pi_1} = m_{\pi_2} l_{\pi_1\pi_2}\Big/\left(m_{\pi_2}+m_{\pi_1}\right) = l_{m\pi_1} = m_{\pi_1} l_{\pi_1\pi_2}\Big/\left(m_{\pi_1}+m_{\pi_2}\right) \end{gathered} \tag{14}$$

The effective mass $m_{her}^{\pi_1,\pi_2}$ is calculated using the base length $S_{\pi_1\pi_2}$ of the Heron's triangle $\Delta\pi_1 m\pi_2$:

$$m_{her}^{\pi_1,\pi_2} = \sqrt{m_{\pi_1}^2 + m_{\pi_2}^2 + 2m_{\pi_1}m_{\pi_2}\mathrm{ch}S_{\pi_1\pi_2}} \tag{15}$$

The hyperbolic sine HST_{res} of the base triangle $\Delta\pi_1 m\pi_2$ (**Figure 2**):

$$HST_{res} = \mathrm{sh}\, S_{Gm}\, \mathrm{sh}\, S_{\pi_1\pi_2} \tag{16}$$

The hyperbolic sine HST_{her} of the Heron's base triangle $\Delta\pi_1 m\pi_2$ rounded to the nearest integer of HST_{res}. Using Equations (3)-(7) and standard transfor-

mations of hyperbolic functions, we obtain:

Table 1. Hyperbolic Heron triangles in decay of Scalar Mezon $\to \pi_1 + \pi_2$.

Name Scalar Mezon	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her}	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{\pi_1,\pi_2}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)
1	2	3	4	5	6	7	8
Scalar Mezon $\to \pi_1 + \pi_2$, $m_{\pi_1} = m_{\pi_2}$ 139.57001 Mev , AP = 3.5%							
$f_o(500)$	600.0 ± 50.0	450.0	$6.89^{+2.41}_{-1.99}$	7	602.5	2.5	0.6
$\rho(770)$	775.3 ± 0.2	147.0	$17.39^{+0.02}_{-0.02}$	17	770.2	5.1	3.5
$\omega(782)$	782.7 ± 0.13	8.68	$17.97^{+0.01}_{-0.01}$	18	783.0	0.3	3.9
$f_o(980)$	980.0 ± 20.0	55.0	$39.40^{+2.65}_{-2.54}$	39	986.9	3.1	5.6
$\phi(1020)$	1019.46 ± 0.02	4.25	$43.34^{+0.02}_{-0.02}$	43	1017.0	2.5	58.0
$f_2(1275)$	1275.4 ± 0.80	186.6	$88.61^{+0.18}_{-0.18}$	89	1277.2	1.8	1.0
$f_o(1370)$	1350.0 ± 50.0	350.8	$105.94^{+12.77}_{-11.84}$	106	1350.2	0.2	0.1
$\rho(1450)$	1465.0 ± 25.0	400.0	$137.76^{+7.39}_{-7.14}$	138	1465.8	0.8	0.2
$f_o(1500)$	1522.0 ± 25.0	108.0	$153.99^{+7.98}_{-7.22}$	154	1522.0	0.03	0.04
$f_2'(1525)$	1517.0 ± 13.0	132.0	$152.52^{+7.98}_{-7.22}$	153	1517.3	1.5	2.1
$f_2(1565)$	1571.0 ± 13.0	132.0	$169.89^{+0.81}_{-0.80}$	170	1571.3	0.3	0.3
$\rho_3(1690)$	1686.0 ± 4.0	161.0	$211.35^{+1.55}_{-1.54}$	211	1685.1	0.9	0.6
$\rho(1700)$	1720.0 ± 20.0	250.0	$224.77^{+8.15}_{-7.96}$	225	1720.6	0.6	0.2
$f_o(1710)$	1733.0 ± 8.0	150.0	$230.04^{+3.29}_{-3.25}$	230	1732.9	0.1	0.1
$f_o(1770)$	1804.0 ± 16.0	138.0	$260.29^{+7.16}_{-7.03}$	260	1803.3	0.7	0.5
$f_2(1810)$	1815.0 ± 12.0	197.0	$265.20^{+5.42}_{-5.35}$	265	1814.6	0.4	0.2
$f_2(1950)$	1936.0 ± 12.0	197.0	$323.27^{+6.18}_{-6.10}$	323	1935.5	0.5	0.1
$f_0(2020)$	1982.0 ± 54.0	464.0	$347.37^{+29.77}_{-28.18}$	347	1981.3	0.7	0.2
$f_4(2050)$	2018.0 ± 11.0	237.0	$367.04^{+6.15}_{-6.09}$	367	2017.9	0.1	0.03
$\rho_4(2300)$	2312.0 ± 15.0	300.0	$555.82^{+11.05}_{-10.91}$	556	2312.2	0.2	0.1
$f_6(2510)$	2470.0 ± 50.0	260.0	$679.60^{+42.66}_{-40.96}$	680	2470.5	0.5	0.2

$$\mathrm{ch}\left(S_{\pi_1\pi_2}\right) = \left(m_{res}^2 - m_{\pi_1}^2 - m_{\pi_2}^2\right) \Big/ 2m_{\pi_1} m_{\pi_2} , \quad \mathrm{sh}\left(S_{\pi_1\pi_2}\right) = \sqrt{\mathrm{ch}^2\left(S_{\pi_1\pi_2}\right) - 1} \tag{17}$$

$$l_{\pi_1\pi_2} = 2\mathrm{sh}\left(S_{\pi_1\pi_2}/2\right), \quad l_{m\pi_1} = \mathrm{sh}\left(S_{G\pi_1}\right) = m_{\pi_1} l_{\pi_1\pi_2} \Big/ \left(m_{\pi_1} + m_{\pi_2}\right) \tag{18}$$

$$\mathrm{sh}\left(S_{G\pi_1}\right) = \mathrm{sh}\left(S_{\pi_1\pi_2}/2\right) = \sqrt{\left(\mathrm{ch}\left(S_{\pi_1\pi_2}\right) - 1\right)/2} , \tag{19}$$

$$\mathrm{sh}\left(S_{m\pi_1}/2\right) = l_{m\pi_1}/2 = \mathrm{sh}\left(S_{\pi_1\pi_2}/2\right)/2 , \quad \mathrm{ch}\left(S_{m\pi_1}/2\right) = \sqrt{\mathrm{sh}^2\left(S_{m\pi_1}/2\right) + 1}$$

$$\mathrm{sh}\left(S_{m\pi_1}\right)=2\mathrm{sh}\left(S_{m\pi_1}/2\right)\mathrm{ch}\left(S_{m\pi_1}/2\right),\quad \mathrm{ch}\left(S_{m\pi_1}\right)=\sqrt{\mathrm{sh}^2\left(S_{m\pi_1}\right)+1}$$

From the right-angled triangle $\Delta Gm\pi_1$ we obtain:

$$\mathrm{ch}\left(S_{Gm}\right)=\mathrm{ch}\left(S_{m\pi_1}\right)\Big/\mathrm{ch}\left(S_{\pi_1\pi_2}/2\right),\quad \mathrm{sh}\left(S_{Gm}\right)=\sqrt{\mathrm{ch}^2\left(S_{Gm}\right)-1} \tag{20}$$

In **Table 1**, based on data from **Table A1**, the values of HST_{res} for the decay Scalar Mezon $\to \pi_1+\pi_2$ in column 4 were calculated using Equations (14)-(20). Columns 2 and 3 of **Table 1** list the values of the effective masses m_{res} and widths Γ of scalar mesons from **Table A1**. Columns 4, 5 list the HST_{res}, HST_{her} values (HST_{her} value for Heron's triangles is rounded to the nearest integer HST_{res} value). The base mass $m_{her}^{\pi_1,\pi_2}$ (column 6) is calculated using Equation (14) (the points "π_1" and "π_2" on the side ($\pi_1-\pi_2$) of the Heron's base triangle $\Delta\pi_1 m\pi_2$ are assigned a mass of $m_{\pi_1}=m_{\pi_2}$). Column 7 lists the absolute deviation Δm_{res}^{her} of the base mass $m_{her}^{\pi_1,\pi_2}$ from the published values m_{res}. Column 8 lists the ratio of the absolute deviation Δm_{res}^{her} to the width Γ, expressed as a percentage. **Table 1** shows that the values of the base masses $m_{her}^{\pi_1,\pi_2}$ for Heron's triangles agree well with the published values of m_{res} (the average percentage (AP) based on **Table 1** equals 3.5%). Note that, overall, the agreement between the mass $m_{her}^{\pi_1,\pi_2}$ and the measured mass m_{res} lies within the measurement error Δm_{res}. The main contribution to the calculation error—58%—comes from the $\phi(1020)$ meson, whose measured width is very small (4.25 MeV).

In the case of the decay of a Light Unflavored Mezon into daughter mesons η and π_1 with different masses ($m_b=m_\eta$, $m_\eta>m_{\pi_1}$), Equation (3) takes the following form (point "m" with mass $m_{\eta\pi_1}=m_\eta+m_{\pi_1}$ is shifted toward point "η" of the particle with the greater mass) (**Figure 2**) [12]-[14]:

$$\begin{gathered} l_{\eta\pi_1}=l_{m\pi_1}+l_{m\eta},\quad m_{\eta\pi_1}=m_\eta+m_{\pi_1} \\ m_\eta l_{m\eta}=m_{\pi_1}l_{m\pi_1}=m_{\pi_1}\left(l_{\eta\pi_1}-l_{m\eta}\right) \\ l_{m\pi_1}=m_\eta l_{\eta\pi_1}\Big/\left(m_\eta+m_{\pi_1}\right) \\ l_{m\eta}=m_{\pi_1}l_{\eta\pi_1}\Big/\left(m_\eta+m_{\pi_1}\right) \end{gathered} \tag{21}$$

By connecting the points "π_1", "m" and "η" with straight lines, we obtain a different-sided triangle $\Delta\eta m\pi_1$. The lengths of the sides of the triangle $\Delta\eta m\pi_1$ are related to the lengths of the arcs of the oricycle by the following relationships [10]:

$$l_{\eta\pi_1}=2\mathrm{sh}\left(S_{\eta\pi_1}/2\right),\quad l_{m\pi_1}=2\mathrm{sh}\left(S_{m\pi_1}/2\right),\quad l_{m\eta}=2\mathrm{sh}\left(S_{m\eta}/2\right) \tag{22}$$

By rotating the arc $l_{m\pi_1}$ of the oricycle around the axis ($m-C_0$), the point "π_1" will move to the point "π_2" on the oricycle, we obtain an isosceles base triangle $\Delta\pi_1 m\pi_2$, corresponding to the different-sided triangle $\Delta\eta m\pi_1$ (**Figure 2**).

The effective mass m_{res} of the particle pair η and π_1 is calculated using their rest masses m_η and m_{π_1} and the length $S_{\eta\pi_1}$ of the side ($\eta-\pi_1$) of the triangle $\Delta\eta m\pi_1$:

$$m_{res}^2 = m_{\pi_1}^2 + m_{\eta}^2 + 2m_{\pi_1}m_{\eta}\mathrm{ch}\left(S_{\eta\pi_1}\right) \quad (23)$$

The HST_{res} for the base triangle $\Delta\pi_1 m\pi_2$ of the Scalar Mezon $\to \eta + \pi_1$ decay (Figure 2):

$$HST_{res} = \mathrm{sh}\left(S_{Gm}\right)\mathrm{sh}\left(S_{\pi_1\pi_2}\right) \quad (24)$$

Using Equations (3)-(13) and standard transformations of hyperbolic functions, we obtain:

$$\mathrm{ch}\left(S_{\eta\pi_1}\right) = \left(m_{res}^2 - m_{\pi_1}^2 - m_{\eta}^2\right)\Big/2m_{\pi_1}m_{\eta}, \quad \mathrm{sh}\left(S_{\eta\pi_1}/2\right) = \sqrt{\left(\mathrm{ch}\left(S_{\eta\pi_1}/2\right)-1\right)/2} \quad (25)$$

$$l_{\eta\pi_1} = 2\mathrm{sh}\left(S_{\eta\pi_1}/2\right), \quad l_{m\pi_1} = m_{\eta}l_{\eta\pi_1}\Big/\left(m_{\pi_1}+m_{\eta}\right) \quad (26)$$

$$\mathrm{sh}\left(S_{m\pi_1}/2\right) = l_{m\pi_1}/2, \quad \mathrm{ch}\left(S_{m\pi_1}/2\right) = \sqrt{\mathrm{sh}^2\left(S_{m\pi_1}/2\right)+1},$$

$$\mathrm{sh}\left(S_{m\pi_1}\right) = 2\mathrm{sh}\left(S_{m\pi_1}/2\right)\mathrm{ch}\left(S_{m\pi_1}/2\right), \quad \mathrm{ch}\left(S_{m\pi_1}\right) = \sqrt{\mathrm{sh}^2\left(S_{m\pi_1}\right)+1} \quad (27)$$

$$\mathrm{sh}\left(S_{\pi_1\pi_2}\right) = 2\mathrm{sh}\left(S_{m\pi_1}\right)\mathrm{ch}\left(S_{m\pi_1}\right), \quad \mathrm{ch}\left(S_{\pi_1\pi_2}\right) = \sqrt{\mathrm{sh}^2\left(S_{\pi_1\pi_2}\right)+1} \quad (28)$$

From the right-angled triangle $\Delta Gm\pi_1$ we obtain:

$$\mathrm{ch}\left(S_{Gm}\right) = \mathrm{ch}\left(S_{m\pi_1}\right)\Big/\mathrm{ch}\left(S_{\pi_1\pi_2}/2\right), \quad \mathrm{sh}\left(S_{Gm}\right) = \sqrt{\mathrm{ch}^2\left(S_{Gm}\right)-1} \quad (29)$$

Table 2. Hyperbolic Heron triangles in decays of Scalar Mezon $\to \eta + \pi_1$.

Name Scalar Mezon	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her} $HST_{her}/\mathrm{ch}\,S_{\pi_1\pi_2}$	Effective Mass m_{her}^{η,π_1} in $\Delta\eta m\pi_1$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{\eta,\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Scalar Mezon $\to \eta + \pi_1$, $m_{\eta} = 547.862$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 1.2%								
$a_o(980)$	980.0 ± 20.0	75.0	$8.16^{+1.01}_{-0.94}$	8/9	976.8	3.2	4.3	624.2
$a_2(1320)$	1317.7 ± 1.4	107.0	$34.01^{+0.17}_{-0.16}$	34	1317.6	0.1	0.1	946.4
$\pi_1(1600)$	1354.0 ± 25.0	330.0	$38.00^{+3.25}_{-3.08}$	38	1354.0	0.1	0.0	979.1
$a_o(1450)$	1439.0 ± 34.0	258.0	$48.37^{+5.20}_{-4.86}$	48	1436.2	2.8	1.1	1052.2
$a_2(1700)$	1706.0 ± 14.0	380.0	$91.13^{+3.16}_{-3.08}$	91	1705.3	0.7	0.2	1286.2
$a_0(1710)$	1713.0 ± 19.0	107.0	$92.47^{+4.34}_{-4.21}$	92	1710.6	2.5	2.3	1290.7
$a_4(1970)$	1967.0 ± 16.0	324.0	$149.86^{+4.99}_{-4.88}$	150	1967.5	0.5	0.2	1509.2

Table 2 shows the results of the calculations for the Scalar Mezon $\to \eta + \pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (21)-(29) based on data from Table A1. Columns 2 and 3 of Table 2 list the values of the effective masses m_{res} and widths Γ from Table A1. Columns 4, 5 list the HST_{res} and HST_{her} values (HST_{her} value for Heron's triangles is rounded to the nearest integer HST_{res} value). Column 6 lists the values of the effective

masses m_{her}^{η,π_1} in triangles $\Delta\eta m\pi_1$. Column 7 lists the absolute deviation Δm_{res}^{her} of the effective mass m_{her}^{η,π_1} from the published values m_{res}. Column 8 lists the ratio of the absolute deviation Δm_{res}^{her} to the width Γ, expressed as a percentage. Column 9 lists the values of the effective masses $m_{her}^{\pi_1,\pi_2}$ in Heron's triangles $\Delta\pi_1 m\pi_2\ \pi_2$. **Table 2** shows that the values of the effective masses m_{her}^{η,π_1} agree well with the published values of m_{res} (the average percentage (AP) based on **Table 2** equals 1.2%). From Equation (28), we write the following expression:

$$\mathrm{ch}\left(S_{\pi_1\pi_2}\right)=\sqrt{\mathrm{sh}^2\left(S_{\pi_1\pi_2}\right)+1}$$

Column 5 lists, separated by a slash, the value of the hyperbolic cosine $\mathrm{ch}\left(S_{\pi_1\pi_2}\right)$ of the side $(\pi_1-\pi_2)$ of Heron's base triangle $\Delta\pi_1 m\pi_2$. For certain values of HST_{her}, the hyperbolic cosines $\mathrm{ch}\left(S_{\pi_1\pi_2}\right)$ of the side $(\pi_1-\pi_2)$ of the triangles $\Delta\pi_1 m\pi_2$ are also equal to integer values. The integer value of $\mathrm{ch}\left(S_{\pi_1\pi_2}\right)$ for a Heron triangle corresponds to the integer value of the oricyclic cotangent of the triangle, as introduced in [15].

Table 3. Hyperbolic Heron triangles in decays of Scalar Mezon $\to\rho(770)+\pi_1$.

Name Scalar Mezon	Effective Mass Resonance $m_{res}\pm\Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}^{i} / HST_{res}	HST_{her}^{i} / HST_{her}	Effective Mass $m_{her}^{\rho(770),\pi_1}$ in $\Delta\rho(770)m\pi_1$ (Mev)	$\Delta m_{res}^{her}=m_{res}-m_{her}^{\rho(770),\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Scalar Mezon $\to\rho(770)+\pi_1$, $m_\rho=775.29999$ Mev, $m_{\pi_1}=139.57$ Mev, AP = 1.2%								
$\eta'(958)$	957.8 ± 0.06	0.23	$-19.98^{+0.02}_{-0.02}$	-20	957.8	0.0	0.03	345.6
$h_1(1170)$	1166.2 ± 19.0	107.0	$6.47^{+0.47}_{-0.47}$	6	1152.2	11.0	2.9	579.0
$a_1(1260)$	1240.0 ± 40.0	422.0	$10.03^{+0.03}_{-0.03}$	10	1239.5	0.5	0.1	663.0
$\pi(1300)$	1300.0 ± 100.0	400.0	$13.47^{+6.88}_{-5.46}$	13	1292.3	7.7	1.9	713.2
$a_2(1320)$	1317.7 ± 1.4	107.0	$14.58^{+0.10}_{-0.10}$	15	1324.2	6.5	6.0	742.9
$\omega(1420)$	1410.0 ± 60.0	290.0	$21.12^{+5.66}_{-4.84}$	21	1408.5	1.5	0.5	819.2
$\pi_1(1600)$	1645.0 ± 16.0	370.0	$43.69^{+2.25}_{-2.18}$	44	1647.7	2.7	0.7	1024.2
$\omega(1650)$	1670.0 ± 30.0	315.0	$46.63^{+4.47}_{-4.20}$	47	1673.1	3.1	1.0	1045.3
$\omega_3(1670)$	1667.0 ± 4.0	168.0	$46.27^{+0.58}_{-0.57}$	46	1664.8	2.2	1.3	1038.4
$\pi_2(1670)$	1670.6 ± 2.9	288.0	$46.70^{+0.42}_{-0.42}$	47	1673.1	2.5	0.9	1045.3
$a_2(1700)$	1706.0 ± 14.0	380.0	$51.05^{+1.78}_{-1.75}$	51	1705.6	0.4	0.1	1072.1
$a_0(1710)$	1713.0 ± 19.0	107.0	$52.05^{+0.07}_{-0.07}$	52	1713.5	0.5	0.5	1078.6
$a_4(1970)$	1967.0 ± 16.0	324.0	$90.27^{+3.59}_{-3.50}$	90	1965.5	1.6	0.5	1281.7
$\pi_2(2100)$	2090.0 ± 29.0	620.0	$113.45^{+7.62}_{-7.30}$	113	2087.8	2.2	0.4	1378.1

Table 3 shows the results of the calculations for the Scalar Mezon $\to\rho(770)+\pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) based on data from **Table A1** ($m_b=m_{\rho(770)}$).

Columns 2 and 3 of Table 2 list the values of the effective masses m_{res} and widths Γ from Table A1. Column 5 lists the value of HST_{her} for Heron's triangles $\Delta\pi_1 m\pi_2$, rounded to the nearest integer of HST_{res}. The nearest integer for values of HST_{res} < 1.1 was determined using inversion. Namely, an inverse value of HST_{res}^{i} was specified with the coefficient:

$$HST_{res}^{i} = \mathrm{PI}\Big/\left(\mathrm{sh}\, S_{Gm}\, \mathrm{sh}\, S_{G\pi_1}\right) \tag{30}$$

Value PI = 3.1415… Column 4 list the HST_{res} and HST_{res}^{i} values calculated using Equations (3)-(13) ($m_b = m_{\rho(770)}$) based on data from Table A1. Values of HST_{res}^{i} are shown with a minus sign. Columns 2 and 3 of Table 3 list the values of the effective masses m_{res} and widths Γ from Table A1. Column 5 list the HST_{her} and HST_{her}^{i} values for Heron's triangles $\Delta\pi_1 m\pi_2$, rounded to the nearest integer. Column 6 lists the values of the effective masses $m_{her}^{\rho(770),\pi_1}$ in triangles $\Delta\rho(770)m\pi_1$. Column 7 lists the absolute deviation Δm_{res}^{her} of the effective mass $m_{her}^{\rho(770),\pi_1}$ from the published values m_{res}. Column 8 lists the ratio of the absolute deviation Δm_{res}^{her} to the width Γ, expressed as a percentage. Column 9 lists the values of the effective masses $m_{her}^{\pi_1,\pi_2}$ in Heron's triangles $\Delta\pi_1 m\pi_2$. Table 3 shows that the values of the effective masses $m_{her}^{\rho(770),\pi_1}$ agree well with the published values of m_{res} (the average percentage (AP) based on Table 3 equals 1.2%).

Table 4 shows the results of the calculations for the Scalar Mezon $\to \omega(782)+\pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_{\omega(782)}$) based on data from Table A1. The calculations in the remaining columns of Table 4 are similar to those in the columns of Table 2. Table 4 shows that the values of the effective masses $m_{her}^{\omega(782),\pi_1}$ agree well with the published values of m_{res} (average percentage based on Table 4 equals 0.5%).

Table 4. Hyperbolic Heron triangles in decays of Scalar Mezon $\to \omega(782)+\pi_1$.

Name Scalar Mezon	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her}	Effective Mass $m_{her}^{\omega(782),\pi_1}$ in $\Delta\omega(782)m\pi_1$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{\omega(782),\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Scalar Mezon $\to \omega(782)+\pi_1$, $m_\omega = 782.65002$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 0.5%								
$b_1(1235)$	1229.5 ± 3.30	142.9	$9.10^{+0.17}_{-0.16}$	9	1227.5	2.0	1.4	644.2
$\rho(1450)$	1465.8 ± 25.0	400.0	$25.04^{+2.50}_{-2.35}$	25	1465.3	0.5	0.1	862.6
$\rho_3(1690)$	1688.8 ± 2.1	161.0	$47.94^{+0.31}_{-0.31}$	48	1689.3	0.5	0.3	1052.2
$\rho_3(2150)$	2150.8± 40.0	300.0	$124.03^{+0.04}_{-0.04}$	124	2149.9	0.1	0.04	1419.7

Table 5 shows the results of the calculations for the StrangeMezon $\to K(493)+\pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_{k(493)}$) based on data from Table A2. The calculations in the remaining columns of Table 5 are similar to those in the

columns of Table 2. Table 5 shows that the values of the effective masses $m_{her}^{k(493),\pi_1}$ agree well with the published values of m_{res} (the average percentage (AP) based on Table 5 equals 3.0%).

Table 5. Hyperbolic Heron triangles in decays of Strange Mezon $\to K(493)+\pi_1$.

Name Strange Mezon	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her}	Effective Mass m_{her}^{k493,π_1} in $\Delta k493m\pi_1$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{k493,\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Strange Mezon $\to K(493)+\pi_1$, $m_{k493}=493.677$ Mev, $m_{\pi_1}=139.57$ Mev, AP = 3.0%								
$K^*(700)$	838.0 ± 11.0	463.0	$4.33^{+0.38}_{-0.36}$	4	828.7	9.3	2.0	523.7
$K^*(892)$	891.8 ± 0.25	50.3	$6.48^{+0.01}_{-0.01}$	6	880.5	11.2	22.3	579.0
$K^*(1410)$	1421.8 ± 9.0	236.0	$53.93^{+1.44}_{-1.42}$	54	1421.5	0.5	0.2	1091.4
$K_0^*(1430)$	1425.6 ± 18.0	270.0	$54.58^{+2.93}_{-2.83}$	55	1428.5	2.9	1.1	1097.6
$K_2^*(1430)$	1427.3 ± 1.5	100.0	$54.83^{+0.24}_{-0.24}$	55	1428.5	1.2	1.2	1097.6
$K_3^*(1780)$	1779.0 ± 8.0	161.0	$120.37^{+2.15}_{-2.13}$	120	1777.4	1.6	1.0	1404.9
$K_0^*(1950)$	1957.0 ± 14.0	170.5	$166.33^{+4.67}_{-4.58}$	166	1955.8	1.2	0.7	1559.3
$K_4^*(2045)$	2045.0 ± 9.0	199.0	$192.61^{+3.29}_{-3.26}$	193	2046.3	1.3	0.6	1637.1
$K_5^*(2380)$	2382.8 ± 14.0	178.0	$317.54^{+7.13}_{-7.02}$	318	2383.9	1.1	0.6	1925.6

Table 6. Hyperbolic Heron triangles in decays of Strange Mezon $\to K^*(892)+\pi_1$.

Name Strange Mezon	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her}	Effective Mass m_{her}^{k892,π_1} in $\Delta k892m\pi_1$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{k892,\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Strange Mezon $\to K^*(892)+\pi_1$, $m_{k892}=891.76$ Mev, $m_{\pi_1}=139.57$ Mev, AP = 2.7%								
$K_1(1270)$	1253.0 ± 7.0	10.0	$5.42^{+0.23}_{-0.23}$	5	1245.2	10.9	12.1	552.9
$K_1(1400)$	1403.0 ± 7.0	174.0	$12.67^{+0.45}_{-0.44}$	13	1408.6	5.6	3.2	713.2
$K_1^*(1410)$	1414.0 ± 15.0	232.0	$13.33^{+1.00}_{-0.96}$	13	1408.6	5.4	2.4	713.2
$K_2^*(1430)$	1427.3 ± 1.5	100.0	$14.14^{+0.10}_{-0.10}$	14	1425.0	2.3	2.3	728.4
$K(1460)$	1482.4 ± 15.0	335.6	$17.78^{+1.20}_{-1.16}$	18	1485.6	3.2	0.9	783.0
$K_2(1580)$	1580.8 ± 5.0	110.0	$25.32^{+0.49}_{-0.49}$	25	1577.0	3.8	3.5	862.6
$K^*(1680)$	1718.8 ± 18.0	320.0	$38.19^{+2.34}_{-2.25}$	38	1716.2	1.8	0.6	979.1
$K_3^*(1780)$	1779.0 ± 8.0	161.0	$44.84^{+1.13}_{-1.12}$	45	1780.4	1.4	0.9	1031.4
$K_2(1770)$	1773.8 ± 8.0	186.0	$44.25^{+1.13}_{-1.10}$	44	1771.6	2.2	1.2	1024.2
$K_2(1820)$	1819.0 ± 12.0	264.0	$49.53^{+1.82}_{-1.78}$	50	1822.9	3.9	1.5	1065.6
$K_2^*(1980)$	1990.0 ± 50.0	348.0	$72.58^{+10.03}_{-9.21}$	73	1992.8	2.8	0.8	1199.5

Table 6 shows the results of the calculations for the Strange Mezon $\to K^{*}(892)+\pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_{k(892)}$) based on data from Table A2. The calculations in the remaining columns of Table 6 are similar to those in the columns of Table 2. Table 6 shows that the values of the effective masses $m_{her}^{k(892),\pi_1}$ agree well with the published values of m_{res} (the average percentage (AP) based on Table 6 equals 2.7%).

Table 7. Hyperbolic Heron triangles in decays of Δ Barion $\to P+\pi_1$.

Name Δ Barion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her}	Effective Mass m_{her}^{P,π_1} in $\Delta P m \pi_1$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{P,\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta \pi_1 m \pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Δ Barion $\to P+\pi_1$, $m_P = 938.27209$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 0.5%								
$\Delta(1232)$	1232.0 ± 0.40	117.0	$2.96^{+0.01}_{-0.01}$	3	1233.4	1.4	1.2	489.9
$\Delta(1600)$	1570.0 ± 0.25	250.0	$20.71^{+0.02}_{-0.02}$	21	1573.9	3.9	1.6	819.2
$\Delta(1620)$	1610.0 ± 20.0	130.0	$23.81^{+1.64}_{-1.58}$	24	1612.3	2.3	1.8	852.2
$\Delta(1750)$	1715.0 ± 10.0	350.0	$32.57^{+0.96}_{-0.94}$	33	1714.6	0.5	0.1	937.9
$\Delta(1900)$	1865.0 ± 30.0	300.0	$49.08^{+3.60}_{-3.46}$	49	1864.3	0.6	0.3	1058.9
$\Delta(1905)$	1880.0 ± 30.0	300.0	$50.87^{+3.60}_{-3.46}$	51	1881.3	1.0	0.3	1072.1
$\Delta(1910)$	1900.0 ± 30.0	300.0	$53.32^{+4.86}_{-4.59}$	53	1897.4	2.6	0.9	1085.0
$\Delta(1920)$	1920.0 ± 30.0	300.0	$55.84^{+5.00}_{-4.84}$	56	1921.3	1.3	0.4	1103.8
$\Delta(1930)$	1950.0 ± 30.0	400.0	$59.74^{+5.22}_{-4.95}$	60	1952.0	2.0	0.7	1127.9
$\Delta(1940)$	2000.0 ± 24.0	500.0	$66.56^{+4.46}_{-4.27}$	67	2003.1	3.1	0.8	1167.6
$\Delta(1950)$	1930.0 ± 10.0	285.0	$57.12^{+1.30}_{-1.28}$	57	1929.1	0.9	0.3	1109.9
$\Delta(2000)$	2015.0 ± 24.0	500.0	$68.69^{+5.71}_{-5.42}$	69	2017.2	2.2	0.4	1178.4
$\Delta(2150)$	2150.0 ± 100.0	200.0	$89.62^{+23.87}_{-20.33}$	90	2152.3	2.3	1.1	1281.7
$\Delta(2200)$	2200.0 ± 30.0	300.0	$98.20^{+25.27}_{-21.63}$	98	2198.9	1.1	0.4	1316.9
$\Delta(2300)$	2300.0 ± 100.0	350.0	$116.76^{+28.17}_{-24.33}$	117	2301.2	1.2	0.3	1393.5
$\Delta(2350)$	2350.0 ± 100.0	400.0	$126.76^{+31.20}_{-27.15}$	127	2351.2	1.2	0.3	1430.6
$\Delta(2390)$	2375.0 ± 100.0	300.0	$131.95^{+31.20}_{-27.15}$	132	2375.3	0.3	0.1	1448.4
$\Delta(2400)$	2450.0 ± 100.0	500.0	$148.26^{+32.76}_{-28.61}$	148	2470.9	0.9	0.2	1518.5
$\Delta(2420)$	2483.0 ± 100.0	500.0	$155.80^{+32.76}_{-28.61}$	156	2483.9	0.9	0.2	1528.4
$\Delta(2750)$	2794.0 ± 80.0	350.0	$238.43^{+35.01}_{-31.97}$	238	2792.6	1.4	0.4	1752.3
$\Delta(2950)$	2990.0 ± 100.0	330.0	$301.98^{+51.57}_{-46.35}$	302	2990.1	0.1	0.02	1893.5

Table 7 shows the results of the calculations for the Δ Barion $\to P+\pi_1$ decay

triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_P$) based on data from Table A3. The calculations in the remaining columns of Table 7 are similar to those in the columns of Table 2. Table 6 shows that the values of the effective masses m_{her}^{P,π_1} agree well with the published values of m_{res} (the average percentage (AP) based on Table 7 equals 0.5%).

Table 8 shows the results of the calculations for the Δ Barion $\to \Delta(1232) + \pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_{\Delta(1232)}$) based on data from Table A3. The calculations in the remaining columns of Table 8 are similar to those in the columns of Table 2. Table 8 shows that the values of the effective masses $m_{\Delta,her}^{\Delta(1232),\pi_1}$ agree well with the published values of m_{res} (the average percentage based on Table 8 equals 1.4%).

Table 8. Hyperbolic Heron triangles in decays of Δ Barion $\to \Delta(1232) + \pi_1$.

Name Δ Barion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her} $HST_{her}/\mathrm{ch}\, S_{\pi_1\pi_2}$	Effective Mass $m_{\Delta,her}^{\Delta(1232),\pi_1}$ in $\Delta\pi_1 m\Delta(1232)$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{\Delta,her}^{\Delta(1232),\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Δ Barion $\to \Delta(1232) + \pi_1$, $m_{\Delta(1232)} = 1232.0$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 1.4%								
$\Delta(1600)$	1570.0 ± 20.0	250.0	$4.53^{+0.76}_{-0.70}$	5	1582.5	12.5	5.0	553.0
$\Delta(1620)$	1610.0 ± 10.0	130.0	$6.09^{+0.42}_{-0.41}$	6	1607.8	2.8	1.8	579.0
$\Delta(1700)$	1710.0 ± 2.0	300.0	$10.83^{+0.11}_{-0.11}$	11	1713.3	3.3	1.1	680.7
$\Delta(1900)$	1860.0 ± 20.0	250.0	$20.16^{+0.45}_{-0.40}$	20	1857.8	2.2	0.9	807.5
$\Delta(1905)$	1880.0 ± 20.0	330.0	$21.61^{+1.50}_{-1.45}$	22	1885.3	5.3	1.6	830.5
$\Delta(1910)$	1900.0 ± 20.0	300.0	$23.10^{+1.55}_{-1.50}$	23	1898.5	1.4	0.5	841.5
$\Delta(1920)$	1920.0 ± 30.0	300.0	$24.65^{+3.00}_{-2.78}$	25	1924.4	4.4	1.5	862.6
$\Delta(1930)$	1950.0 ± 30.0	300.0	$27.07^{+3.17}_{-2.95}$	27/19	1949.2	0.8	0.3	882.7
$\Delta(1950)$	1930.0 ± 10.0	300.0	$25.45^{+0.80}_{-0.79}$	25	1924.4	5.6	1.9	862.6
$\Delta(1940)$	2000.0 ± 40.0	400.0	$31.35^{+3.65}_{-3.45}$	31	1996.1	3.9	1.0	920.2
$\Delta(2000)$	2150.0 ± 24.0	500.0	$46.13^{+3.53}_{-3.36}$	46	2148.8	1.2	0.2	1038.4
$\Delta(2200)$	2200.0 ± 30.0	300.0	$51.73^{+3.52}_{-3.40}$	52	2202.3	2.3	0.8	1078.6

Table 9 shows the results of the calculations for the N Barion $\to n + \pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_n$) based on data from Table A4. The calculations in the remaining columns of Table 9 are similar to those in the columns of Table 2. Table 9 shows that the values of the effective masses $m_{N,her}^{n,\pi_1}$ agree well with the published values of m_{res} (the average percentage (AP) based on Table 9 equals 1.0%).

Table 9. Hyperbolic Heron triangles in decays of N Barion $\to n+\pi_1$.

Name NBarion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her} $HST_{her}/\mathrm{ch}\, S_{\pi_1\pi_2}$	Effective Mass m_{her}^{n,π_1} in $\Delta nm\pi_1$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{n,\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
N Barion $\to n+\pi_1$, $m_n = 939.56542$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 1.0%								
$N(1440)$	1440.0 ± 30.0	350.0	$12.05^{+1.78}_{-1.66}$	12	1439.2	0.8	0.2	697.4
$N(1520)$	1515.0 ± 10.0	110.0	$16.71^{+0.68}_{-0.66}$	17	1519.3	4.3	3.9	770.1
$N(1535)$	1530.0 ± 15.0	150.0	$17.73^{+0.35}_{-0.34}$	18	1533.8	3.8	2.6	783.0
$N(1650)$	1650.0 ± 6.0	125.0	$27.03^{+0.52}_{-0.51}$	27/19	1649.7	0.3	0.3	882.7
$N(1675)$	1675.0 ± 5.0	145.0	$29.22^{+0.45}_{-0.45}$	29	1672.5	2.5	1.7	901.9
$N(1680)$	1685.0 ± 5.0	120.0	$30.12^{+0.46}_{-0.45}$	30	1683.7	1.3	1.1	911.2
$N(1700)$	1720.0 ± 20.0	200.0	$33.39^{+1.95}_{-1.89}$	33	1715.9	4.1	2.0	937.9
$N(1875)$	1875.0 ± 20.0	200.0	$50.10^{+2.43}_{-2.37}$	50	1874.1	0.9	0.4	1065.6
$N(1880)$	1880.0 ± 20.0	300.0	$50.71^{+2.45}_{-2.38}$	51	1822.4	2.4	0.8	1072.1
$N(1900)$	1920.0 ± 10.0	200.0	$55.66^{+2.49}_{-2.43}$	56	1922.7	2.6	1.3	1103.8
$N(1895)$	1985.0 ± 20.0	120.0	$52.53^{+2.49}_{-2.43}$	53	1898.8	3.8	3.1	1085.1
$N(2000)$	2050.0 ± 20.0	400.0	$73.60^{+3.95}_{-3.81}$	74	2052.7	2.7	0.7	1204.7
$N(1990)$	2020.0 ± 40.0	300.0	$69.20^{+5.89}_{-5.62}$	69	2018.6	1.4	0.5	1178.5
$N(2060)$	2100.0 ± 15.0	400.0	$81.26^{+2.39}_{-2.35}$	81	2098.3	1.7	0.4	1239.6
$N(2100)$	2100.0 ± 20.0	260.0	$81.26^{+2.39}_{-2.35}$	81	2098.3	1.7	0.6	1239.6
$N(2120)$	2120.0 ± 20.0	400.0	$84.45^{+4.31}_{-4.17}$	84	2117.2	2.8	0.9	1254.0
$N(2190)$	2180.0 ± 20.0	400.0	$94.46^{+4.64}_{-4.49}$	94	2177.3	2.6	0.7	1299.5
$N(2220)$	2250.0 ± 15.0	400.0	$106.96^{+0.39}_{-0.39}$	107	2250.2	0.2	0.1	1354.3
$N(2250)$	2280.0 ± 15.0	500.0	$112.60^{+7.79}_{-7.49}$	113	2282.1	2.1	0.4	1378.1
$N(2600)$	2600.0 ± 100.0	650.0	$183.96^{+40.96}_{-36.06}$	184	2611.8	0.2	0.02	1612.0
$N(2700)$	2612.0 ± 45.0	650.0	$187.06^{+11.88}_{-11.44}$	187	2611.8	0.2	0.1	1620.5
$N(3000)$	3000.0 ± 200.0	1650.0	$304.84^{+75.15}_{-65.12}$	305	3000.5	0.5	0.03	1899.6

Table 10 shows the results of the calculations for the N Barion $\to \Delta(1232)+\pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_{\Delta(1232)}$) based on data from **Table A4**. The calculations in the remaining columns of **Table 10** are similar to those in the columns of **Table 2**. **Table 10** shows that the values of the effective masses $m_{N,her}^{\Delta(1232),\pi_1}$ agree well with the published values of m_{res} (the average percentage (AP) based on **Table 10** equals 1.7%). From Equation (28), we write the following expression:

$$\mathrm{ch}\left(S_{\pi_1\pi_2}\right) = \sqrt{\mathrm{sh}^2\left(S_{\pi_1\pi_2}\right)+1}$$

Column 5 lists, separated by a slash, the value of the hyperbolic cosine $\mathrm{ch}\left(S_{\pi_1\pi_2}\right)$ of the base ($\pi_1 - \pi_2$) of Heron's basic triangle $\Delta\pi_1 m\pi_2$.

Table 10. Hyperbolic Heron triangles in decays of N Barion $\to \Delta(1232)+\pi_1$.

Name N Barion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her} $HST_{her}/\mathrm{ch}\, S_{\pi_1\pi_2}$	Effective Mass $m_{N,her}^{\Delta(1232),\pi_1}$ in $\Delta\pi_1 m\Delta(1232)$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{N,her}^{\Delta(1232),\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
N Barion $\to \Delta(1232)+\pi_1$, $m_{\Delta(1232)}=1232.0$ Mev, $m_{\pi_1}=139.57$ Mev, AP = 1.7%								
$N(1440)$	1440.0 ± 30.0	350.0	$-10.01^{+0.20}_{-0.19}$	−10	1440.5	0.5	0.2	394.8
$N(1520)$	1515.0 ± 10.0	110.0	$2.71^{+0.30}_{-0.29}$	3	1524.3	9.6	8.8	489.9
$N(1535)$	1530.0 ± 5.0	150.0	$3.17^{+0.16}_{-0.16}$	3	1524.6	5.4	3.6	489.9
$N(1650)$	1650.0 ± 5.0	125.0	$7.84^{+0.23}_{-0.23}$	8/9	1653.4	3.4	2.7	624.2
$N(1675)$	1675.0 ± 5.0	150.0	$9.03^{+0.25}_{-0.24}$	9	1674.3	0.7	0.5	644.3
$N(1680)$	1685.0 ± 5.0	120.0	$9.53^{+0.25}_{-0.25}$	10	1694.2	9.2	7.7	663.0
$N(1700)$	1720.0 ± 10.0	200.0	$11.36^{+0.55}_{-0.54}$	11	1713.3	6.7	3.4	680.7
$N(1710)$	1710.0 ± 20.0	200.0	$10.83^{+1.09}_{-1.04}$	11	1713.3	3.3	1.6	680.7
$N(1720)$	1720.0 ± 10.0	250.0	$11.36^{+0.55}_{-0.54}$	11	1713.3	6.7	2.7	680.7
$N(1860)$	1860.0 ± 10.0	250.0	$20.16^{+0.72}_{-0.71}$	20	1857.8	2.2	0.9	807.5
$N(1875)$	1875.0 ± 10.0	200.0	$21.24^{+0.74}_{-0.73}$	21	1871.7	3.3	1.6	819.1
$N(1880)$	1880.0 ± 20.0	300.0	$21.61^{+1.50}_{-1.45}$	22	1885.3	5.3	1.8	830.5
$N(1895)$	1985.0 ± 10.0	200.0	$22.72^{+0.76}_{-0.75}$	23	1898.6	3.6	1.8	841.5
$N(1900)$	1920.0 ± 10.0	200.0	$24.65^{+0.79}_{-0.78}$	25	1924.4	4.4	2.2	862.6
$N(2000)$	2050.0 ± 20.0	400.0	$35.95^{+2.50}_{-2.40}$	36	2050.6	0.5	0.1	963.0
$N(2060)$	2100.0 ± 15.0	400.0	$40.87^{+2.71}_{-2.60}$	41	2101.2	1.2	0.3	1002.2
$N(2100)$	2100.0 ± 20.0	260.0	$40.87^{+1.54}_{-1.51}$	41	2101.2	1.2	0.5	1002.2
$N(2120)$	2120.0 ± 30.0	300.0	$42.94^{+4.24}_{-3.97}$	43	2120.6	0.6	0.2	1017.0
$N(2190)$	2180.0 ± 20.0	400.0	$49.45^{+3.06}_{-2.94}$	49	2176.0	4.0	1.0	1058.9

Table 11. Hyperbolic Heron triangles in decays of Σ Barion $\to \Lambda+\pi_1$.

Name Σ Barion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her}	Effective Mass m_{her}^{Λ,π_1} in $\Delta\Lambda m\pi_1$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{\Lambda,\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Σ Barion $\to \Lambda+\pi_1$, $m_{\Lambda}=1115.68298$ Mev, $m_{\pi_1}=139.57$ Mev, AP = 3.9%								
$\Sigma(1385)$	1383.7 ± 2.0	39.4	$2.26^{+0.06}_{-0.06}$	2	1374.3	9.5	24.0	449.0
$\Sigma(1620)$	1620.0 ± 2.0	70.0	$12.27^{+0.11}_{-0.11}$	12	1615.2	4.8	6.9	697.4
$\Sigma(1660)$	1660.0 ± 20.0	200.4	$14.65^{+1.26}_{-1.21}$	15	1665.7	5.7	2.8	742.9
$\Sigma(1670)$	1675.0 ± 2.0	70.0	$15.59^{+0.13}_{-0.13}$	16	1681.4	6.4	9.2	756.8
$\Sigma(1750)$	1750.0 ± 2.0	150.0	$20.72^{+0.15}_{-0.15}$	21	1753.8	3.8	2.6	819.1

Continued

$\Sigma(1775)$	1775.0 ± 10.0	120.0	$22.59^{+0.77}_{-0.76}$	23	1780.4	5.4	4.5	841.5
$\Sigma(1780)$	1780.0 ± 30.0	200.0	$22.97^{+2.37}_{-2.25}$	23	1780.4	0.4	0.2	841.5
$\Sigma(1900)$	1925.0 ± 20.0	165.0	$35.52^{+1.95}_{-1.90}$	36	1930.0	4.9	3.0	963.0
$\Sigma(1910)$	1910.0 ± 50.0	220.0	$34.09^{+4.88}_{-4.54}$	34	1909.0	1.0	0.4	946.4
$\Sigma(1915)$	1915.0 ± 22.0	120.0	$34.56^{+2.12}_{-2.06}$	35	1919.6	4.6	3.8	954.8
$\Sigma(1940)$	1940.0 ± 100.0	250.0	$36.98^{+10.54}_{-9.16}$	37	1940.2	0.2	0.1	971.1
$\Sigma(2010)$	2005.0 ± 14.0	178.0	$43.67^{+1.52}_{-1.49}$	44	2008.1	3.1	1.7	1024.2
$\Sigma(2030)$	2030.0 ± 10.0	180.0	$46.40^{+1.12}_{-1.10}$	46	2026.4	3.6	2.0	1038.4
$\Sigma(2100)$	2100.0 ± 50.0	310.0	$54.53^{+2.45}_{-2.39}$	55	2103.9	3.9	1.9	1097.6
$\Sigma(2110)$	2100.0 ± 50.0	310.0	$55.75^{+3.30}_{-3.19}$	56	2112.1	2.1	2.1	1103.8
$\Sigma(2230)$	2240.0 ± 27.0	347.0	$72.97^{+3.90}_{-3.79}$	73	2240.2	0.2	0.1	1199.6
$\Sigma(2250)$	2250.0 ± 20.0	140.0	$74.40^{+2.91}_{-2.85}$	74	2247.2	2.8	0.8	1204.7

Table 12. Hyperbolic Heron triangles in decays of Σ Barion $\to \Sigma + \pi_1$.

Name Σ Barion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST^i_{her} HST_{her} $HST_{her}/\mathrm{ch}\, S_{\pi_1\pi_2}$	Effective Mass m^{Λ,π_1}_{her} in $\Delta\Lambda m\pi_1$ (Mev)	$\Delta m^{her}_{res} = m_{res} - m^{\Lambda,\pi_1}_{her}$ (Mev)	$\frac{\Delta m^{her}_{res}}{\Gamma}$ (%)	Base Mass m^{π_1,π_2}_{her} in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Σ Barion $\to \Sigma + \pi_1$, $m_\Sigma = 1189.37$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 3.1%								
$\Sigma(1385)$	1383.7 ± 2.0	39.4	$-13.57^{+0.5}_{-0.5}$	−14	1382.4	1.3	3.3	363.9
$\Sigma(1620)$	1620.0 ± 2.0	70.0	$8.43^{+0.10}_{-0.10}$	8/9	1610.9	9.9	13.0	624.2
$\Sigma(1660)$	1660.0 ± 20.0	200.4	$10.43^{+1.17}_{-1.09}$	10	1651.7	8.3	4.1	663.0
$\Sigma(1670)$	1675.0 ± 2.0	70.0	$11.23^{+0.12}_{-0.12}$	11	1670.7	4.3	6.2	680.7
$\Sigma(1750)$	1750.0 ± 2.0	150.0	$15.65^{+0.14}_{-0.14}$	16	1755.6	5.6	3.7	756.7
$\Sigma(1775)$	1775.0 ± 10.0	120.0	$17.27^{+0.78}_{-0.76}$	17	1771.0	4.0	3.4	770.1
$\Sigma(1780)$	1780.0 ± 30.0	200.0	$17.60^{+2.34}_{-2.24}$	18	1785.9	5.9	3.0	783.0
$\Sigma(1900)$	1925.0 ± 20.0	165.0	$28.63^{+2.17}_{-2.07}$	29	1929.3	4.3	2.6	901.9
$\Sigma(1910)$	1910.0 ± 50.0	220.0	$27.37^{+5.45}_{-4.84}$	27/19	1905.6	4.4	2.0	882.7
$\Sigma(1915)$	1915.0 ± 22.0	120.0	$27.78^{+2.34}_{-2.25}$	28	1917.6	2.6	2.1	892.4
$\Sigma(1940)$	1940.0 ± 100.0	250.0	$29.92^{+12.19}_{-9.67}$	30	1940.9	0.9	0.3	911.2
$\Sigma(2010)$	2005.0 ± 14.0	178.0	$35.87^{+1.74}_{-1.68}$	36	2006.4	1.4	0.8	963.0
$\Sigma(2030)$	2030.0 ± 10.0	180.0	$38.30^{+1.29}_{-1.26}$	38	2027.0	3.0	1.7	979.1
$\Sigma(2070)$	2060.0 ± 10.0	200.0	$41.33^{+1.29}_{-1.26}$	41	2056.8	3.2	1.6	1002.2
$\Sigma(2100)$	2100.0 ± 50.0	310.0	$45.57^{+1.44}_{-1.41}$	46	2104.0	4.0	1.5	1038.4
$\Sigma(2230)$	2240.0 ± 27.0	347.0	$62.14^{+3.54}_{-3.42}$	62	2238.9	1.1	0.3	1139.5
$\Sigma(2250)$	2250.0 ± 20.0	140.0	$63.44^{+4.87}_{-4.64}$	63	2246.6	3.4	1.0	1145.2
$\Sigma(2110)$	2110.0 ± 20.0	100.0	$46.66^{+2.95}_{-2.83}$	47	2113.1	3.4	3.4	1045.3

Table 11 shows the results of the calculations for the Σ Barion $\to \Lambda + \pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_\Lambda$) based on data from Table A5. The calculations in the remaining columns of Table 11 are similar to those in the columns of Table 2. Table 11 shows that the values of the effective masses m_{her}^{Λ,π_1} agree well with the published values of m_{res} (the average percentage (AP) based on Table 11 equals 3.9%).

Table 12 shows the results of the calculations for the Σ Barion $\to \Sigma(1189) + \pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_\Sigma$) based on data from Table A5. The calculations in the remaining columns of Table 12 are similar to those in the columns of Table 2. Table 12 shows that the values of the effective masses $m_{her}^{\Sigma(1189),\pi_1}$ agree well with the published values of m_{res} (the average percentage (AP) based on Table 12 equals 3.1%).

Table 13 shows the results of the calculations for the Σ_c Barion $\to \Lambda_c^+(2286) + \pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_{\Lambda_c^+(2286)}$) based on data from Table A5. The calculations in the remaining columns of Table 13 are similar to those in the columns of Table 2. Table 13 shows that the values of the effective masses $m_{her}^{\Lambda_c^+(2286),\pi_1}$ agree well with the published values of m_{res} (the average percentage (AP) based on Table 13 equals 16.5%).

Table 13. Hyperbolic Heron triangles in decays of Σ_c Barion $\to \Lambda_c^+(2286) + \pi_1$.

Name Σ_c Barion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her}^i HST_{her}	Mass $m_{her}^{\Lambda_c^+(2286),\pi_1}$ in $\Delta\Lambda_c^+(2286)m\pi_1$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{\Lambda_c^+(2286),\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Σ_c Barion $\to \Lambda_c^+(2286) + \pi_1$, $m_{\Lambda_c^+} = 2286.46$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 16.5%								
$\Sigma_c(2455)$	2454.0 ± 0.1	1.9	$-31.56_{-0.1}^{+0.1}$	−32	2453.7	0.3	14.7	327.4
$\Sigma_c(2520)$	2518.4 ± 0.2	14.8	$-6.61_{-0.3}^{+0.3}$	−7	2515.0	4.2	28.4	415.2
$\Sigma_c(2800)$	2801.0 ± 5.0	75.0	$12.74_{-0.32}^{+0.32}$	13	2805.7	4.7	6.3	713.2

Table 14 shows the results of the calculations for the Λ Barion $\to \Sigma + \pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_\Sigma$) based on data from Table A6. The calculations in the remaining columns of Table 14 are similar to those in the columns of Table 2. Table 11 shows that the values of the effective masses m_{her}^{Σ,π_1} agree well with the published values of m_{res} (the average percentage (AP) based on Table 14 equals 4.6%).

Table 15 shows the results of the calculations for the Λ_c Barion $\to \Sigma_c^0(2455) + \pi_1^+$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_{\Sigma_c^0(2455)}$) based on data from Table A6. The calculations in the remaining columns of Table 12 are similar to those in the columns of Table 2. Table 15 shows that the values of the effective masses $m_{her}^{\Sigma_c^0,\pi_1}$

agree well with the published values of m_{res} (the average percentage (AP) based on Table 15 equals 10.2%).

Table 14. Hyperbolic Heron triangles in decays of Λ Barion $\to \Sigma + \pi_1$.

Name Λ Barion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her} $HST_{her}/\mathrm{ch}\, S_{\pi_1\pi_2}$	Effective Mass m_{her}^{Σ,π_1} in $\Delta\Sigma m\pi_1$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{\Sigma,\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Λ Barion $\to \Sigma + \pi_1$, $m_\Sigma = 1189.37$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 4.6%								
$\Lambda(1405)$	1405.1 ± 1.3	50.0	$-6.97^{+0.07}_{-0.07}$	−7/3	1404.8	0.3	0.7	394.8
$\Lambda(1520)$	1519.4 ± 0.2	15.7	$4.24^{+0.01}_{-0.01}$	4	1512.7	6.3	39.9	523.7
$\Lambda(1600)$	1600.0 ± 10.0	200.0	$7.50^{+0.45}_{-0.46}$	7	1588.9	11.1	5.6	602.4
$\Lambda(1670)$	1670.0 ± 5.0	30.0	$10.96^{+0.27}_{-0.27}$	11	1670.7	0.6	2.2	680.7
$\Lambda(1690)$	1690.0 ± 5.0	70.0	$12.06^{+0.28}_{-0.28}$	12	1688.9	1.1	1.6	697.4
$\Lambda(1710)$	1713.0 ± 13.0	180.0	$13.38^{+0.78}_{-0.76}$	13	1706.4	6.6	3.6	713.2
$\Lambda(1800)$	1800.0 ± 10.0	200.0	$18.97^{+0.69}_{-0.07}$	19	1800.5	0.5	0.2	795.4
$\Lambda(1810)$	1790.0 ± 10.0	110.0	$18.28^{+0.69}_{-0.68}$	18	1785.9	4.1	3.7	783.0
$\Lambda(1820)$	1820.0 ± 4.0	80.0	$20.38^{+0.29}_{-0.29}$	20	1814.7	5.3	6.6	807.5
$\Lambda(1830)$	1825.0 ± 10.0	90.0	$20.74^{+0.73}_{-0.72}$	21	1828.5	3.5	3.9	819.1
$\Lambda(1890)$	1890.0 ± 5.0	120.0	$25.72^{+0.40}_{-0.40}$	26	1893.4	3.4	2.8	872.8
$\Lambda(2050)$	2056.0 ± 22.0	493.0	$40.92^{+2.29}_{-2.22}$	41	2056.8	0.8	0.2	1002.2
$\Lambda(2070)$	2070.0 ± 24.0	370.0	$42.37^{+2.55}_{-2.47}$	42	2066.4	3.6	1.0	1009.6
$\Lambda(2080)$	2082.0 ± 13.0	181.0	$43.63^{+1.39}_{-1.37}$	44	2085.4	3.4	1.9	1024.2
$\Lambda(2100)$	2100.0 ± 22.0	200.0	$45.57^{+2.42}_{-2.35}$	46	2104.0	4.0	2.0	1038.4
$\Lambda(2110)$	2090.0 ± 22.0	250.0	$44.49^{+2.39}_{-2.32}$	44	2085.4	4.6	1.8	1024.2
$\Lambda(2350)$	2350.0 ± 22.0	150.0	$77.17^{+3.22}_{-3.15}$	77	2348.8	1.2	0.8	1219.9

Table 15. Hyperbolic Heron triangles in decays of Λ_c Barion $\to \Sigma_c^0(2455) + \pi_1^+$.

Name Λ_c Barion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her}^i HST_{her} $HST_{her}/\mathrm{ch}\, S_{\pi_1\pi_2}$	Effective Mass $m_{her}^{\Sigma_c^0(2455),\pi_1}$ in $\Delta\Sigma_c^0(2455)m\pi_1$ (Mev)	$\Delta m_{res}^{her} = m_{res} - m_{her}^{\Sigma_c^0(2455),\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Λ_c Barion $\to \Sigma_c^0(2455) + \pi_1^+$, $m_\Sigma = 2453.75$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 10.2%								
$\Lambda_c(2625)$	2628.0 ± 0.2	52.0	$-13.41^{+0.03}_{-0.03}$	−23	2628.5	0.5	0.9	339.6
$\Lambda_c(2880)$	2881.6 ± 0.2	51.8	$8.38^{+0.01}_{-0.01}$	8/9	2873.2	8.4	16.2	624.2
$\Lambda_c(2910)$	2913.8 ± 3.8	51.8	$9.91^{+0.21}_{-0.21}$	10	2915.6	1.8	3.5	663.0
$\Lambda_c(2940)$	2939.6 ± 1.5	20.0	$11.21^{+0.09}_{-0.09}$	11	2935.5	4.0	20.3	680.7

Table 16 shows the results of the calculations for the Ξ Barion $\to \Xi^0 + \pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_{\Xi^0}$) based on data from Table A7. The calculations in the remaining columns of Table 16 are similar to those in the columns of Table 2. Table 16 shows that the values of the effective masses $m_{her}^{\Xi^0,\pi_1}$ agree well with the published values of m_{res} (the average percentage (AP) based on Table 16 equals 15.5%).

Table 17 shows the results of the calculations for the Ξ Barion $\to \Xi(1530) + \pi_1$ decay triangles. Column 4 lists the HST_{res} values calculated using Equations (3)-(13) ($m_b = m_{\Xi(1530)}$) based on data from Table A7. The calculations in the remaining columns of Table 17 are similar to those in the columns of Table 2. Table 17 shows that the values of the effective masses $m_{her}^{\Xi(1530),\pi_1}$ agree well with the published values of m_{res} (the average percentage (AP) based on Table 17 equals 5.4%).

Table 16. Hyperbolic Heron triangles in decays of Ξ Barion $\to \Xi^0 + \pi_1$.

Name Ξ Barion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}	HST_{her}^{i} HST_{her} $HST_{her}/\mathrm{ch}\,S_{\pi_1\pi}$	Effective Mass $m_{her}^{\Xi^0,\pi_1}$ in $\Delta\Xi^0 m\pi_1$ (Mev)	$\Delta m_r^{her} = m_{res} - m_{her}^{\Xi^0,\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Ξ Barion $\to \Xi^0 + \pi_1$, $m_{\Xi^0} = 1314.85999$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 15.5%								
$\Xi(1530)$	1531.8 ± 0.3	9.1	$-9.02^{+0.04}_{-0.04}$	−9	1532.3	0.1	1.4	393.8
$\Xi(1620)$	1620.8 ± 6.0	32.0	$3.33^{+0.20}_{-0.19}$	3	1610.4	10.4	32.5	489.9
$\Xi(1690)$	1690.0 ± 10.0	20.0	$5.85^{+0.41}_{-0.40}$	6	1693.6	3.6	18.0	579.0
$\Xi(1820)$	1823.0 ± 5.0	20.0	$12.29^{+0.28}_{-0.28}$	12	1817.9	5.1	25.6	697.4
$\Xi(1950)$	1950.0 ± 15.0	60.0	$20.35^{+1.08}_{-1.05}$	20	1945.1	5.0	24.7	807.5
$\Xi(2030)$	2025.0 ± 20.0	60.0	$26.01^{+1.62}_{-1.58}$	26	2024.9	0.1	0.2	872.8
$\Xi(2500)$	2500.0 ± 150.0	60.0	$78.48^{+23.05}_{-19.82}$	78	2496.6	3.6	5.6	1224.9

Table 17. Hyperbolic Heron triangles in decays of Ξ Barion $\to \Xi(1530) + \pi_1$.

Name Ξ Barion	Effective Mass Resonance $m_{res} \pm \Delta m_{res}$ (Mev)	Width Γ (Mev)	HST_{res}^{i} HST_{res}	HST_{her}^{i} HST_{her} $HST_{her}/\mathrm{ch}\,S_{\pi_1\pi_2}$	Effective Mass $m_{her}^{\Xi(1530),\pi_1}$ in $\Delta\Xi(1530) m\pi_1$ (Mev)	$\Delta m_r^{her} = m_{res} - m_{her}^{\Xi(1530),\pi_1}$ (Mev)	$\frac{\Delta m_{res}^{her}}{\Gamma}$ (%)	Base Mass $m_{her}^{\pi_1,\pi_2}$ in $\Delta\pi_1 m\pi_2$ (Mev)
1	2	3	4	5	6	7	8	9
Ξ Barion $\to \Xi(1530) + \pi_1$, $m_{\Xi(1530)} = 1531.8$ Mev, $m_{\pi_1} = 139.57$ Mev, AP = 5.4%								
$\Xi(1690)$	1690.0 ± 10.0	20.0	$-59.26^{+0.26}_{-0.26}$	−59	1690.1	0.1	0.2	311.1
$\Xi(1820)$	1823.0 ± 5.0	20.0	$2.99^{+0.28}_{-0.26}$	3	1823.3	0.3	1.4	697.4
$\Xi(1950)$	1950.0 ± 15.0	60.0	$7.87^{+0.24}_{-0.24}$	8/9	1952.7	2.7	13.6	624.2
$\Xi(2030)$	2025.0 ± 20.0	60.0	$11.62^{+0.93}_{-0.88}$	12	2032.0	7.0	11.7	697.4
$\Xi(2500)$	2500.0 ± 150.0	60.0	$50.00^{+3.08}_{-2.96}$	50	2500.0	0.01	0.01	1065.6

3. The Proposal to Search for New Resonances

Table 18 presents a summary classification of two-body decays of light unflavored, strange mesons and Δ, N, Λ, Σ, Ξ baryons, based on the values of the hyperbolic sines HST_{her} of their Heron triangles, using data from Tables 1-17. Column 1 of Table 18 list the HST_{her}^{i}, HST_{her} and $HST_{her}/\mathrm{ch}\left(S_{\pi_1\pi_2}\right)$ values. Values of HST_{her}^{i} are shown with a minus sign. The values of $HST_{her}/\mathrm{ch}\left(S_{\pi_1\pi_2}\right)$ are shown in bold. Column 2 lists the base mass $m_{her}^{\pi_1,\pi_2}$. Column 3 list the decays Scalar Mezon $\to \pi_1+\pi_2$ and Σ_c Barion $\to \Lambda_c^+(2286)+\pi_1$. Column 4 list the decays Δ Barion $\to P+\pi_1$ and N Barion $\to n+\pi_1$, and Scalar Mezon $\to \eta+\pi_1$. Column 5 list the decays Σ Barion $\to \Lambda+\pi_1$ and Λ Barion $\to \Sigma+\pi_1$. Column 6 list the decays Ξ Barion $\to \Xi^0+\pi_1$ and Scalar Mezon $\to \omega(782)+\pi_1$. Column 7 list the decays Scalar Mezon $\to \rho(770)+\pi_1$ and Λ_c Barion $\to \Sigma_c^0(2455)+\pi_1$, and Σ Barion $\to \Sigma+\pi_1$. Column 8 list the decays Strange Mezon $\to K(493)+\pi_1$ and N Barion $\to \Delta(1232)+\pi_1$. Column 9 list the decays Δ Barion $\to \Delta(1232)+\pi_1$ and Strange Mezon $\to K^*(892)+\pi_1$, and Ξ Barion $\to \Xi(1530)+\pi_1$.

Table 18. Hyperbolic Sine $HST_{her}, HST_{her}/\mathrm{ch}\,S_{\pi_1\pi_2}$, mass $m_{her}^{\pi_1,\pi_2}$ of Heron triangles of light unflavored, strange meson and Δ, N, Λ, Σ, Ξ baryons decays.

HST_{her}^{i} HST_{her} $HST_{her}/\mathrm{ch}\,S_{\pi_1\pi_2}$	Base Mass $m_{her}^{\pi_1,\pi_2}$ (Mev)	Scalar Mez $\to \pi_1+\pi_2$ Σ_c Barion $\to \Lambda_c^+(2286)+\pi_1$	Δ Barion $\to P+\pi_1$ N Barion $\to n+\pi_1$ Scalar Mez $\to \eta+\pi_1$	Σ Barion $\to \Lambda+\pi_1$ Λ Barion $\to \Sigma+\pi_1$	Ξ Barion $\to \Xi^0+\pi_1$ Scalar Mez $\to \omega(782)+\pi_1$	Scalar Mez $\to \rho(770)+\pi_1$ Λ_c Barion $\to \Sigma_c^0(2455)+\pi_1^+$ Σ Barion $\to \Sigma+\pi_1$	Strange Mez $\to K(493)+\pi_1$ N Barion $\to \Delta(1232)+\pi_1$	Δ Barion $\to \Delta(1232)+\pi_1$ Strange Mez $\to K^*(892)+\pi_1$ Ξ Barion $\to \Xi(1530)+\pi_1$
1	2	3	4	5	6	7	8	9
−59	311.1							$\Xi(1690)$
−32	327.4	$\Sigma_c(2455)$						
−23	339.6					$\Lambda_c(2625)$		
−20	345.6					$\eta'(958)$		
−14	363.9					$\Sigma(1385)$		
−10	394.8						$N(1440)$	
−9	393.8				$\Xi(1530)$	empty		
−7	415.2	$\Sigma_c(2520)$						
1/3	394.8			$\Lambda(1405)$	$\Xi(1530)$		empty	
2	449.0			$\Sigma(1385)$				
3	489.9		$\Delta(1232)$		$\Xi(1620)$		$N(1520)$ $N(1535)$	$\Xi(1820)$
4	523.9			$\Lambda(1520)$			$K^*(700)$	
5	552.9							$\Delta(1600)$ $K_1(1270)$

Continued

6	579.0				$\Xi(1690)$	$h_1(1170)$	$K^*(892)$	$\Delta(1620)$
7	602.5	$f_o(500)$		$\Lambda(1600)$				
8/9	624.2		$a_o(980)$			$\Sigma(1620)$ $\Lambda_c(2880)$	$N(1650)$	$\Xi(1950)$
9	644.2				$b_1(1235)$		$N(1675)$	
10	663.0					$\Sigma(1660)$ $\Lambda_c(2910)$ $a_1(1260)$	$N(1680)$	
11	680.7			$\Lambda(1670)$		$\Sigma(1670)$ $\Lambda_c(2910)$	$N(1700)$ $N(1710)$ $N(1720)$	$\Delta(1700)$
12	697.4		$N(1440)$	$\Sigma(1620)$ $\Lambda(1690)$	$\Xi(1820)$	$\Lambda_c(2940)$		$\Xi(2030)$
13	713.2	$\Sigma_c(2800)$		$\Lambda(1710)$		$\pi(1300)$		$K_1(1400)$ $K_1^*(1410)$
14	728.4							$K_2^*(1430)$
15	742.9			$\Sigma(1660)$		$a_2(1320)$		
16	756.8			$\Sigma(1670)$		$\Sigma(1750)$		
17	771.6	$\rho(770)$	$N(1520)$			$\Sigma(1775)$		
18	783.0	$\omega(782)$	$N(1535)$	$\Lambda(1810)$		$\Sigma(1780)$		$K(1460)$
19	795.4			$\Lambda(1800)$				
20	807.5			$\Lambda(1820)$	$\Xi(1950)$		$N(1860)$	$\Delta(1900)$
21	819.2		$\Delta(1600)$	$\Sigma(1750)$ $\Lambda(1830)$		$\omega(1420)$	$N(1875)$	
22	830.5						$N(1880)$	$\Delta(1905)$
23	841.5			$\Sigma(1775)$ $\Sigma(1780)$			$N(1895)$	$\Delta(1910)$
24	852.2		$\Delta(1620)$					
25	862.6					$\rho(1450)$	$N(1900)$	$\Delta(1950)$ $\Delta(1920)$ $K_2(1580)$
26	872.8			$\Lambda(1890)$	$\Xi(2030)$			
27/19	882.7		$N(1650)$			$\Sigma(1910)$		$\Delta(1930)$
28	892.4					$\Sigma(1915)$		
29	901.9		$N(1675)$			$\Sigma(1900)$		
30	911.2		$N(1680)$			$\Sigma(1940)$		
31	920.2							$\Delta(1940)$
32	929.4							

Continued

33	937.8		$\Delta(1710)$ $N(1700)$				
34	946.4		$a_2(1320)$	$\Sigma(1910)$			
35	954.8			$\Sigma(1915)$			
36	963.0			$\Sigma(1900)$	$\Sigma(2010)$	$N(2000)$	
37	971.1			$\Sigma(1940)$			
38	979.1		$\pi_1(1600)$		$\Sigma(2030)$		$K^*(1680)$
39	986.9	$f_o(980)$					
40	994.6						
41	1002.2			$\Lambda(2050)$	$\Sigma(2070)$	$N(2060)$ $N(2100)$	$\Delta(2000)$
42	1009.6			$\Lambda(2070)$			
43	1017.0	$\phi(1020)$				$N(2120)$	
44	1024.2			$\Sigma(2010)$ $\Lambda(2080)$ $\Lambda(2110)$	$\pi_1(1600)$		$K_2(1770)$
45	1031.4						$K_3^*(1780)$
46	1038.4			$\Sigma(2030)$ $\Lambda(2100)$	$\omega_3(1670)$ $\Sigma(2100)$		$\Delta(2000)$
47	1045.3				$\pi_2(1670)$ $\Sigma(2110)$		
48	1052.2		$a_o(1450)$		$\rho_3(1690)$		
49	1058.9		$\Delta(1900)$			$N(2190)$	
50	1065.6		$N(1650)$	$\Sigma(2070)$			$K_2(1820)$ $\Xi(2500)$
51	1072.1		$N(1650)$ $\Delta(1905)$		$a_2(1700)$		
52	1078.6				$a_0(1710)$		$\Delta(2200)$
53	1085.0		$N(1650)$ $\Delta(1910)$				
54	1091.4					$K^*(1410)$	
55	1097.6			$\Sigma(2100)$		$K_2^*(1430)$ $K_0^*(1430)$	
56	1103.8		$N(1900)$ $\Delta(1920)$	$\Sigma(2110)$			
57	1109.9		$\Delta(1950)$				
58	1116.0						
59	1122.0						

Continued

60	1127.9		$\Delta(1905)$				
…	…						
62	1139.5				$\Sigma(2230)$		
63	1145.2				$\Sigma(2250)$		
64/33	1150.9						
…	…						
67	1167.6		$\Delta(1940)$				
69	1178.6		$N(1990)$ $\Delta(2000)$				
70-72	…						
73	1199.5			$\Sigma(2230)$			$K_2^*(1980)$
74	1204.7		$N(2000)$	$\Sigma(2250)$			
…	…						
77	1219.9			$\Lambda(2350)$			
78	1224.8				$\Xi(2500)$		$\Xi(2500)$
…	…						
81	1239.6		$N(2060)$ $N(2100)$				
82	3957.5						
83	4102.5						
84	4121.5		$N(2120)$				
85	4169.1						
86	4205.7						
87	4404.8						
88	4457.5						
89	1277.1	$f_2(1275)$					
90	1281.7		$\Delta(1950)$			$a_4(1970)$	
91	1286.2		$a_2(1700)$				
92	1290.0		$a_0(1710)$				
…	…						
94	1299.5		$N(2190)$				
98	1316.9		$\Delta(2200)$				
106	1350.2	$f_o(1370)$					
107	1354.2		$N(2220)$	$\Sigma(2455)$			$K^*(1780)$
…	…						
113	1378.1		$N(2190)$			$\pi_2(2100)$	
…	…						

Continued

117	1393.5		$\Delta(2300)$		
…	…				
120	1404.9				$K_3^*(1780)$
…	…				
124	1419.6			$\rho(2250)$	
125/51	1423.3				
127	1430.6		$\Delta(2350)$		
132	1448.4		$\Delta(2390)$		
138	1465.8	$\rho(1450)$			
…	…				
148	1502.6		$\Delta(2400)$		
…	…				
150	1509.2		$a_4(1970)$		
…	…				
153	1518.8	$f_2'(1525)$			
154	1522.0	$f_o(1500)$			
156	1528.4		$\Delta(2420)$		
166	1554.2				$K_0^*(1950)$
…	…				
170		$f_2(1565)$			
…	…				
184	1612.0		$N(2600)$		
…	…				
187	1620.5		$N(2700)$		
…	…				
193	1637.1				$K_0^*(2045)$
…	…				
211	1685.1	$\rho_3(1690)$			
…	…				
216/73	1697.9				
…	…				
225	1720.6	$\rho(1700)$			
…	…				
230	1732.9	$f_o(1710)$			
…	…				
238	1752.5		$\Delta(2750)$		
…	…				

Continued

252	1785.1	$f_o(1810)$			
…	…				
260	1803.3	$f_o(1770)$			
…	…				
265	1814.5	$f_2(1810)$			
…	…				
302	1893.5		$\Delta(2950)$		
…	…				
305	1899.6		$N(3000)$		
…	…				
318	1925.6				$K_5^*(2380)$
…	…				
323	1935.5	$f_2(1950)$			
…	…				
343 /99	1973.8				
…	…				
347	1981.3	$f_0(2020)$			
…	…				
367	2017.9	$f_4(2050)$			
…	…				
510	2247.6	$\rho_2(2250)$			
511	2249.1				
512/129	2250.5				
…	…				
556	2312.2	$\rho_4(2300)$			
…	…				
680	2470.5	$f_6(2510)$			
…	…				
729/163	2527.7				
…	…				
1000/201	2805.3				
…	…				
1331/243	3083.2				
…	…				
1728/289	3361.3				
…	…				
2197/339	3639.5				

Continued

…	…
2744/390	3917.9
…	…
3375/451	4196.4
…	…

Table 18 can be used to propose a method for searching for new resonances. Generally speaking, new resonances are possible in every empty cell of Table 18. First, let's consider the non-empty cell in row 1, column 9, with the value $HST_{her}^{i} = -59$, corresponding to the decay of $\Xi(1690) \to \Xi(1530) + \pi_1$. The empty cell in row 1, column 3 corresponds to (π_1, π_2) decays of scalar mesons and (Λ_c^+, π_1) decays of Σ_c baryons. Then, using the effective masses m_{res} of the pairs (π_1, π_2) and (Λ_c^+, π_1) and the formulas (3)-(13) and (16)-(20), (30), we can construct a distributions of HST_{her}^{i}. A statistically significant peak in the distribution of HST_{her}^{i} at a value equal to −59 can be interpreted as the new scalar meson and new Σ_c baryon with a base mass $m_{her}^{\pi_1,\pi_2}$ = 311.0 MeV. Similar assumptions can be made regarding the decays listed in the remaining blank cells in row 1 and columns 4-9.

The cell in row with the value HST_{res} = 7 and column 3 corresponds to the decay $f_o(500) \to \pi_1 + \pi_2$. Therefore, a statistically significant peak at a value of HST_{res} = 7 will necessarily appear. The value $HST_{her} = 7$ for the scalar meson $f_o(500)$ corresponds to an average mass of 600 Mev and a width of 300 Mev. However, all measurements of the mass and width $f_o(500)$ meson have a large variance of values (mass 400 - 800 MeV, width 100 - 800 MeV) [8]. Therefore, no clear peak is observed in the distribution for m_{res} < 600 MeV in (π_1, π_2) decays. This situation can be resolved using Heron's base triangles. The hyperbolic sine HST_{res} (HST_{res}^{i}) depends nonlinearly on the effective mass m_{res}, of the pair (π_1, π_2) of pi mesons. Therefore, it is quite likely that statistically significant peaks will appear in the distribution of HST_{res} (HST_{res}^{i}) at the values equal to −59, −32, −23, −20, −14, −10, −9, −7, 1, 2, 3, 4, 5, 6, and 7. These peaks can be interpreted as (π_1, π_2) decays of new scalar mesons with masses $m_{her}^{\pi_1,\pi_2}$ < 600.0 MeV. It would be interesting to study the effective masses m_{res} of (π_1, π_2) pairs for values of HST_{res} > 1000, which corresponds to masses $m_{her}^{\pi_1,\pi_2}$ > 2805.3 MeV.

The non-empty cells in row with the value HST_{her} = 8 corresponds to the decays $a_o(980) \to \eta + \pi_1$, $\Sigma(1620) \to \Sigma + \pi_1$, $\Lambda_c(2880) \to \Sigma_c^0(2455) + \pi_1$, $\Xi(1950) \to \Xi(1530) + \pi_1$, $N(1650) \to \Delta(1232) + \pi_1$. The hyperbolic cosine of the base length $\mathrm{ch}(S_{\pi_1\pi_2})$ = 9 for Heron's base triangles $\Delta\pi_1 m\pi_2$ of these resonances is also an integer.

Consider the cell column 4 of this in row HST_{her} = 8. Based on the actual values of the effective masses m_{res} of the (η, π_1) pairs, in the distribution of HST_{res} a statistically significant peak at a value of HST_{res} = 8 will necessarily appear.

Similarly, based on the actual values of the effective masses m_{res} of the pairs (P,π_1) from the same column, one can obtain the HST_{res} distribution, calculated using Equations (21)-(29) for $m_b = m_P$. A statistically significant peak in the distribution of HST_{res} at a value equal to 8 can be interpreted as a new Δ baryon with a base mass $m_{her}^{\pi_1,\pi_2}$ = 624.2 MeV. The empty cells in this row correspond to the decays of the pairs (π_1,π_2) and (Λ_c^+,π_1) (column 3), (Λ, π_1) and (Σ, π_1) (column 5), (Ξ^0, π_1) and ($\omega(782)$, π_1) (column 6). The statistically significant peaks in the distributions of HST_{res} = 8 can be interpreted as the new light unflavored mezon and Λ_c Barion of the column 3, the new Ξ Barion and light unflavored mezon of the column 5, the new Ξ Barion and light unflavored mezon of the column 6.

It would be interesting to study the effective masses m_{res} of (η,π_1), (P,π_1) pairs for values of HST_{res} = 1000, 1331, 1728, 2197, 2744, 3375 which correspond to base masses $m_{her}^{\pi_1,\pi_2}$ = 2805.3, 3083.2, 3361.3, 3639.5, 3917.9, 4196.4 MeV.

To conclude this chapter, we note that the decay triangles originated from Archimedes' levers (3), in which the masses of the daughter particles of resonance decay act as forces. Therefore, the experimentally observed (**Tables 1-18**) close relationship between the resonance decay triangles and Heron's base triangles indicates that, in some way, the gravitational forces of the daughter particle masses are connected with the formation of resonances.

4. Conclusions

The published data show that the decays of scalar, strange mesons and Δ, N, Λ, Σ, Ξ baryons correspond (within the resonance width) to base Heron triangles with integer values of HST_{res}. Based on Heron's triangles, the existence of new resonances is predicted. This can be confirmed by direct measurements of the effective masses of the pairs (π_1, π_2), (ω, π_1), (η, π_1), ($\rho(770)$, π_1), (P, π_1), (n, π_1), ($\Delta(1232)$, π_1), ($K(493)$, π_1), ($K(892)$, π_1), (Ξ^0, π_1), (Λ, π_1) of the particles in reaction (1) and the detection of statistically significant peaks in the distribution of HST_{res}.

Further development of the described approach will consist in:

- isolation of resonances by the Heron triangle method and analysis of the angular distributions of their decays using parametrization of the dynamic axis of spin quantization by Lobachevsky straight line beams [16];
- analysis of reaction (1) with the formation of more than two particles, based on a 3-dimensional analog of Heron's triangles.

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Data Availability Statement

The data used in the article are taken from open sources [8].

Conflicts of Interest

There is no conflict of interest among the authors of this article.

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Appendix

Table A1. Data on two-particle decays of scalar mesons from [8].

Name Scalar Mezon	$I^G(J^{PC})$	Effective Mass Resonance m_{res} (Mev)	Width Γ (Mev)	Decay $\pi+\pi$	Decay $\eta+\pi$	Decay $\rho(770)+\pi$	Decay $\omega(782)+\pi$
1		2	3	4	5	6	7
$f_o(500)$	$0^+(0^{++})$	600.0 ± 50.0	450.0	seen	-	-	-
$\rho(770)$	$1^+(1^{--})$	766.5 ± 1.1	150.0	~100%	-	-	-
$\omega(782)$	$0^-(1^{--})$	782.7 ± 0.13	8.68	(1.53 ± 0.12)%	-	-	-
$\eta'(958)$	$0^+(0^{-+})$	957.8 ± 0.06	0.23	-	-	$(7.4 \pm 2.3)10^{-4}$	-
$f_o(980)$	$0^+(0^{++})$	980.0 ± 20.0	55.0	seen	-	-	-
$\phi(1020)$	$0^-(1^{--})$	1019.46 ± 0.02	4.25	$(9.5 \pm 1.9)10^{-5}$	43	-	$(4.7 \pm 0.5)10^{-5}$
$h_1(1170)$	$0^-(1^{+-})$	1166.4 ± 5.00	375.0	-	-	seen	-
$b_1(1235)$	$1^+(1^{+-})$	1229.5 ± 3.30	142.9	-	-	-	seen
$a_1(1260)$	$1^-(1^{++})$	1230.0 ± 40.0	420.0	-	-	seen	-
$f_2(1270)$	$0^+(2^{++})$	1275.4 ± 0.80	186.6	~84%	-	-	-
$\pi(1300)$	$1^-(0^{-+})$	1300.0 ± 100.0	400.0	-	-	seen	-
$a_2(1320)$	$1^-(2^{++})$	1317.7 ± 1.4	107.0	-	(14.5 ± 1.2)%	-	-
$f_o(1370)$	$0^+(0^{++})$	1350.0 ± 50.0	350.8	seen	-	-	-
$\omega(1420)$	$0^-(1^{--})$	1410.0 ± 60.0	290.0	-	-	seen	-
$a_o(1450)$	$1^-(0^{++})$	1439.0 ± 34.0	258.0	-	(0.93 ± 0.02)%	-	-
$\rho(1450)$	$1^+(1^{--})$	1465.0 ± 25.0	400.0	seen	-	-	-
$f_o(1500)$	$0^+(0^{++})$	1522.0 ± 25.0	108.0	(34.5 ± 2.2)%	-	-	-
$f_2'(1525)$	$0^+(2^{++})$	1517.3 ± 2.4	72.0	$(8.2 \pm 1.3)10^{-3}$	-	-	-
$f_2(1565)$	$0^+(2^{++})$	1571.0 ± 13.0	132.0	seen	-	-	-
$\pi_1(1600)$	$1^-(1^{-+})$	1645.0 ± 16.0	370.0	-	seen	seen	-
$a_1(1640)$	$1^-(1^{++})$	1655.0 ± 16.0	250.0	-	-	seen	-
$\omega(1650)$	$0^-(1^{--})$	1670.0 ± 30.0	315.0	-	-	seen	-
$\omega_3(1670)$	$0^-(3^{--})$	1667.0± 4.0	168.0	-	-	seen	-
$\pi_2(1670)$	$1^-(2^{-+})$	1670.6 ± 2.9	288.0	-	-	(31.0 ± 4.0)%	-
$\rho_3(1690)$	$1^+(3^{--})$	1686.0 ± 4.0	161.0	-	-	-	(16.0 ± 6.0)%
$\rho(1700)$	$1^+(1^{--})$	1720.0 ± 20.0	250.0	seen	-	-	-
$a_2(1700)$	$1^-(2^{-+})$	1706.0 ± 14.0	380.0	-	(2.5 ± 0.6)%	seen	-
$a_0(1710)$	$1^-(0^{++})$	1713.0 ± 19.0	107.0	-	seen	-	-
$f_o(1710)$	$0^+(0^{++})$	1733.0 ± 8.0	150.0	seen	-	-	-
$f_o(1770)$	$0^+(0^{-+})$	1804.0 ± 16.0	138.0		-	-	-
$f_0(1810)$	$0^+(0^{++})$	1785.0 ± 13.0	137.0	seen	-	-	-
$f_2(1810)$	$0^+(0^{++})$	1815.0 ± 12.0	197.0	seen	-	-	-

Continued

$f_2(1950)$	$0^+(0^{-+})$	1936.0 ± 12.0	197.0	seen	-	-	-
$a_4(1970)$	$1^-(4^{++})$	1967.0 ± 16.0	324.0	-	-	seen	-
$f_0(2020)$	$0^+(0^{-+})$	1982.0 ± 54.0	464.0	seen	-	-	-
$f_4(2050)$	$0^+(4^{++})$	2018.0 ± 11.0	237.0	(17.0 ± 1.5)%	-	-	-
$\pi_2(2100)$	$0^+(0^{-+})$	2090.0 ± 29.0	620.0	-	-	seen	-
$f_4(2300)$	$0^+(4^{++})$	2312.0 ± 15.0	200.0	seen	-	-	-
$f_6(2510)$	$0^+(6^{++})$	2470.0 ± 50.0	260.0	(6.0 ± 1.0)%	-	-	-

Table A2. Data on two-particle decays of Strange Mezons from [8].

Name Strange Mezon	I (J^P)	Effective Mass Resonance m_{res} (Mev)	Width Γ (Mev)	Decay $K(493)+\pi$	Decay $K^*(892)+\pi$
1		2	3	4	5
$K^*(700)$	1/2 (0^+)	838.0 ± 11.0	450.0	~100%	-
$K^*(892)$	1/2(1^+)	891.8± 0.25	150.0	~100%	-
$K_1(1270)$	1/2(1^+)	1253.0 ± 7.0	10.0	-	(18.0 ± 4.0)%
$K_1(1400)$	1/2(1^+)	1403.0± 7.0	174.0	-	(94.0 ± 6.0)%
$K^*(1410)$	1/2 (1^+)	1421.8 ± 9.0	8.68	> 40%	(6.6 ± 1.3)%
$K_0^*(1430)$	1/2(0^+)	1425.0 ± 50.0	270.0	(93.0 ± 10.0)%	-
$K_2^*(1430)$	1/2(2^+)	1423.6 ± 1.5	100.0	(49.9 ± 1.2)%	(24.7 ± 1.5)%
$K(1460)$	1/2(0^-)	1482.4 ± 15.0	335.6	-	seen
$K_2(1580)$	1/2(2^-)	1580.0 ± 10.0	110.0	-	seen
$K^*(1680)$	1/2(2^-)	1718.8 ± 18.0	320.0	(38.7 ± 2.5)%	(29.9 ± 5.0)%
$K_2(1770)$	1/2(2^-)	1773.8 ± 8.0	186.0	-	seen
$K_3^*(1780)$	1/2(3^-)	1779.0 ± 8.0	161.0	(18.8 ± 1.0)%	(20.0 ± 5.0)%
$K_2(1820)$	1/2(2^-)	1819.0 ± 12.0	264.0	-	seen
$K_0^*(1950)$	1/2(0^+)	1957.0 ± 14.0	170.0	(52.0 ± 14.) %	-
$K_2^*(1980)$	1/2(0^+)	1990.0 ± 50.0	348.0	-	possibly seen
$K_4^*(2045)$	1/2(4^+)	2045.0 ± 9.0	375.0	(9.9 ± 1.2)%	-
$K_5^*(2380)$	1/2(5^-)	2382.8 ± 14.0	142.9	(6.1 ± 1.2)%	-

Table A3. Data on two-particle decays of Δ Barions from [8].

Name Δ Barions	I (J^P)	Effective Mass Resonance m_{res} (Mev)	Width Γ (Mev)	Decay $P+\pi$	Decay $\Delta(1232)+\pi$
1		2	3	4	5
$\Delta(1232)$	3/2 ($3/2^+$)	1232.0 ± 0.40	117.0	99.4%	-
$\Delta(1600)$	3/2 ($3/2^+$)	1570.0 ± 0.25	250.0	(8.0 - 24.0)%	(58.0 - 82.0)%
$\Delta(1620)$	3/2 ($1/2^-$)	1610.0 ± 20.0	130.0	(25.0 - 35.0)%	(44.0 - 72.0)%
$\Delta(1700)$	3/2 ($3/2^-$)	1710.0 ± 2.0	300.0	(10.0 - 20.0)%	(9.0 - 10.0)%

Continued

$\Delta(1750)$	3/2 ($1/2^+$)	1715.0 ± 10.0	350.0	seen	-
$\Delta(1900)$	3/2 ($1/2^-$)	1860.0± 30.0	300.0	(4.0 - 12.0)%	(30.0 - 70.0)%
$\Delta(1905)$	3/2 ($5/2^+$)	1880.0± 20.0	330.0	(9.0 - 15.0)%	>48%
$\Delta(1910)$	3/2 ($1/2^+$)	1900.0± 20.0	300.0	(10.0 - 30.0)%	(34.0 - 66.0)%
$\Delta(1920)$	3/2 ($3/2^+$)	1920.0± 30.0	300.0	(5.0 - 20.0)%	>46%
$\Delta(1930)$	3/2 ($5/2^+$)	1950.0± 30.0	300.0	(5.0 - 15.0)%	(33.0 ± 9.0)%
$\Delta(1940)$	3/2 ($3/2^-$)	2000.0± 30.0	400.0	(1.0 - 20.0)%	(38.0 ± 10.0)%
$\Delta(1950)$	3/2 ($7/2^+$)	1930.0 ± 10.0	285.0	(1.0 - 20.0)%	(4.0 ± 3.0)%
$\Delta(1950)$	3/2 ($5/2^-$)	1930.0 ± 10.0	285.0	(93.0 ± 10.0)%	-
$\Delta(2000)$	3/2 ($7/2^-$)	2015.0± 24.0	500.0	(3.0 - 11.0)%	<9%
$\Delta(2150)$	3/2 ($1/2^-$)	2150.0 ± 100.0	200.0	(6.0 - 10.0)%	-
$\Delta(2200)$	3/2 ($9/2^+$)	2200.0± 30.0	300.0	(2.0 - 8.0)%	>45%
$\Delta(2300)$	3/2 ($3/2^+$)	2300.0 ± 100.0	350.0	(1.0 - 8.0)%	-
$\Delta(2350)$	3/2 ($5/2^-$)	2400.0 ± 100.0	400.0	(3.0 - 30.0)%	-
$\Delta(2390)$	3/2 ($7/2^+$)	2375.0 ± 100.0	300.0	(3.0 - 12.0)%	-
$\Delta(2400)$	3/2 ($9/2^-$)	2450.0 ± 100.0	500.0	(3.0 - 9.0)%	-
$\Delta(2420)$	3/2 ($11/2^+$)	2450.0 ± 100.0	500.0	(5.0 - 10.0)%	-
$\Delta(2750)$	3/2 ($13/2^-$)	2794.0 ± 80.0	350.0	(2.0 - 6.0)%	-
$\Delta(2950)$	3/2 ($15/2^+$)	2990.0 ± 100.0	330.0	(2.0 - 6.0)%	-

Table A4. Data on two-particle decays of N Barions from [13].

Name N Barions	I (J^P)	Effective Mass Resonance m_{res} (Mev)	Width Γ (Mev)	Decay $N+\pi$	Decay $\Delta(1232)+\pi$
1		2	3	4	5
$N(1440)$	1/2 ($1/2^+$)	1440.0 ± 30.0	350.0	(55.0 - 65.0)%	(6.0 - 27.0)%
$N(1520)$	1/2 ($3/2^-$)	1515.0 ± 10.0	110.0	(55.0 - 65.0)%	(15.0 - 23.0)%
$N(1535)$	1/2 ($1/2^-$)	1530.0 ± 5.0	150.0	(32.0 - 52.0)%	(1.0 - 4.0)%
$N(1650)$	1/2 ($1/2^-$)	1650.0 ± 5.0	125.0	(50.0 - 70.0)%	(6.0 - 18.0)%
$N(1675)$	1/2 ($5/2^-$)	1675.0 ± 5.0	150.0	(38.0 - 42.0)%	(19.0 ± 4.0)%
$N(1680)$	1/2 ($5/2^+$)	1685.0 ± 5.0	120.0	(60.0 - 70.0)%	(11.0 - 23.0)%
$N(1700)$	1/2 ($5/2^-$)	1720.0 ± 10.0	200.0	(7.0 - 17.0)%	(55.0 - 85.0)%
$N(1710)$	1/2 ($1/2^+$)	1710.0 ± 20.0	200.0	(5.0 - 20.0)%	(3.0 - 9.0)%
$N(1720)$	1/2 ($3/2^+$)	1720.0 ± 10.0	250.0	(8.0 - 14.0)%	(47.0 - 89.0)%
$N(1860)$	1/2 ($5/2^+$)	1860.0 ± 10.0	250.0	(4.0 - 20.0)%	(8.0 - 54.0)%
$N(1875)$	1/2 ($3/2^-$)	1875.0 ± 20.0	200.0	(3.0 - 11.0)%	(4.0 - 44.0)%
$N(1880)$	1/2 ($1/2^+$)	1880.0 ± 20.0	300.0	(3.0 - 11.0)%	(5.0 - 42.0)%
$N(1895)$	1/2 ($1/2^-$)	1985.0 ± 20.0	120.0	(2.0 - 11.0)%	(3.0 - 18.0)%
$N(1900)$	1/2 ($3/2^+$)	1920.0 ± 10.0	200.0	(1.0 - 20.0)%	(3.0 - 70.0)%

Continued

$N(2000)$	1/2 ($5/2^+$)	1920.0 ± 20.0	400.0	(6.0 - 10.0)%	(30.0 - 80.0)%
$N(1990)$	1/2 ($7/2^+$)	2020.0 ± 40.0	300.0	(2.0 - 6.0)%	(72.0 ± 15.0)%
$N(2060)$	1/2 ($5/2^-$)	2050.0 ± 40.0	400.0	(7.0 - 12.0)%	(12.0 ± 3.0)%
$N(2100)$	1/2 ($1/2^+$)	2100.0 ± 20.0	260.0	(8.0 - 32.0)%	(6.0 - 14.0)%
$N(2120)$	1/2 ($3/2^-$)	2120.0 ± 30.0	300.0	(5.0 - 15.0)%	>23%
$N(2190)$	1/2 ($7/2^-$)	2180.0 ± 20.0	400.0	(1.0 - 20.0)%	(4.0 ± 2.0)%
$N(2220)$	1/2 ($9/2^+$)	2250.0 ± 15.0	400.0	(15.0 - 30.0)%	(15.0 ± 10.0)%
$N(2250)$	1/2 ($9/2^-$)	2280.0 ± 15.0	500.0	(5.0 - 15.0)%	(10.0 ± 7.0)%
$N(2600)$	1/2 ($11/2^-$)	2600.0 ± 100.0	650.0	(3.0 - 8.0)%	-
$N(2700)$	1/2 ($3/2^+$)	2612.0 ± 45.0	350.0	(3.0 - 5.0)%	-
$N(3000)$	1/2 ($5/2^-$)	3000.0 ± 200.0	1650.0	(2.0 - 8.0)%	-

Table A5. Data on two-particle decays of Σ Barions from [8].

Name Σ Barions	I (J^P)	Effective Mass Resonance m_{res} (Mev)	Width Γ (Mev)	Decay $\Lambda + \pi$	Decay $\Sigma + \pi$	Decay $\Lambda_c^+(2286) + \pi$
1		2	3	4	5	6
$\Sigma(1385)$	1 ($3/2^+$)	1383.7 ± 2.0	39.4	(87.0 ± 1.5)%	(11.7 ± 1.5)%	
$\Sigma(1620)$	1 ($3/2^-$)	1620.0 ± 2.0	70.0	(9.0 ± 3.0)%	(17.0 ± 5.0)%	
$\Sigma(1660)$	1 ($1/2^+$)	1660.0 ± 20.0	200.4	(35.0 ± 12.0)%	(30.0 ± 10.0)%	
$\Sigma(1670)$	1 ($3/2^-$)	1675.0 ± 2.0	70.0	(5.0 - 15.0)%	(30.0 - 60.0)%	
$\Sigma(1750)$	1 ($1/2^-$)	1750.0 ± 2.0	150.0	(14.0 ± 5.0)%	(16.0 ± 4.0)%	
$\Sigma(1775)$	1 ($5/2^-$)	1775.0 ± 10.0	120.0	(14.0 - 20.0)%	(2.0 - 5.0)%	
$\Sigma(1780)$	1 ($3/2^+$)	1780.0 ± 30.0	200.0	(70.0 ± 7.0)%	(12.0 ± 6.0)%	
$\Sigma(1900)$	1 ($1/2^-$)	1925.0 ± 20.0	165.0	(6.0 ± 2.0)%	0.25	
$\Sigma(1910)$	1 ($3/2^-$)	1910.0 ± 50.0	220.0	(6.0 ± 4.0)%	(86.0 ± 21.0)%	
$\Sigma(1915)$	1 ($5/2^+$)	1915.0 ± 22.0	120.0	(6.0 ± 2.0)%	(10.0 ± 2.0)%	
$\Sigma(1940)$	1 ($3/2^+$)	1940.0 ± 100.0	250.0	-	(4.0 ± 2.0)%	
$\Sigma(2010)$	1 ($3/2^-$)	2005.0 ± 14.0	178.0	(5.0 ± 2.0)%	(3.0 ± 2.0)%	
$\Sigma(2030)$	1 ($7/2^+$)	2030.0 ± 10.0	180.0	(17.0 - 23.0)%	(5.0 - 10.0)%	
$\Sigma(2070)$	1 ($5/2^+$)	2060.0 ± 10.0	200.0		seen	
$\Sigma(2080)$	1 ($3/2^+$)	2090.0 ± 10.0	170.0	seen		
$\Sigma(2100)$	1 ($7/2^-$)	2100.0 ± 50.0	310.0	(1.5 ± 1.0)%	(2.0 ± 1.0)%	
$\Sigma(2110)$	1 ($1/2^-$)	2110.0 ± 50.0	310.0	(1.5 ± 1.0)%	(2.0 ± 1.0)%	
$\Sigma(2230)$	1 ($3/2^+$)	2240.0 ± 27.0	347.0	(12.0 ± 6.0)%	(2.0 ± 1.0)%	
$\Sigma(2250)$	1 ($1/2^+$)	2250.0 ± 20.0	140.0	seen	seen	
$\Sigma(2455)$	1 ($3/2^-$)	2455.0 ± 20.0	140.0	(8.0 - 32.0)%	(6.0 - 14.0)%	
$\Sigma_c(2455)$	1 ($1/2^+$)	2454.0 ± 0.1	1.9			100%
$\Sigma_c(2520)$	1 ($3/2^+$)	2518.4 ± 0.2	14.8			100%
$\Sigma_c(2800)$	1 ($1/2^+$)	2801.0 ± 5.0	75.0			seen

Table A6. Data on two-particle decays of Λ Barions from [8].

Name Λ Barions	I (J^P)	Effective Mass Resonance m_{res} (Mev)	Width Γ (Mev)	Decay $\Sigma+\pi$ $\Sigma_c^0(2455)+\pi^+$
1		2	3	4
$\Lambda(1405)$	0 ($1/2^-$)	1405.1 ± 1.3	50.0	100%
$\Lambda(1520)$	0 ($3/2^-$)	1519.4 ± 0.2	15.7	(42.0 ± 1.0)%
$\Lambda(1600)$	0 ($1/2^+$)	1600.0 ± 10.0	200.0	(10.0 - 60.0)%
$\Lambda(1670)$	0 ($1/2^-$)	1670.0 ± 5.0	30.0	(25.0 - 55.0)%
$\Lambda(1690)$	0 ($3/2^-$)	1690.0 ± 5.0	70.0	(20.0 - 40.0)%
$\Lambda(1710)$	0 ($1/2^+$)	1713.0 ± 13.0	180.0	(21.0 ± 5.0)%
$\Lambda(1800)$	0 ($1/2^-$)	1800.0 ± 10.0	200.0	(27.0 ± 6.0)%
$\Lambda(1810)$	0 ($1/2^+$)	1790.0 ± 10.0	110.0	(16.0 ± 5.0)%
$\Lambda(1820)$	0 ($5/2^+$)	1820.0 ± 4.0	80.0	(8.0 - 14.0)%
$\Lambda(1830)$	0 ($5/2^-$)	1825.0 ± 10.0	90.0	(35.0 - 75.0)%
$\Lambda(1890)$	0 ($3/2^+$)	1890.0 ± 5.0	120.0	(3.0 - 10.0)%
$\Lambda(2050)$	0 ($3/2^-$)	2056.0 ± 22.0	493.0	(6.0 ± 3.0)%
$\Lambda(2070)$	0 ($3/2^+$)	2070.0 ± 24.0	370.0	(7.0 ± 3.0)%
$\Lambda(2080)$	0 ($5/2^-$)	2082.0 ± 13.0	181.0	(5.0 ± 2.0)%
$\Lambda(2100)$	0 ($7/2^-$)	2100.0 ± 22.0	200.0	5.0%
$\Lambda(2110)$	0 ($5/2^+$)	2090.0 ± 22.0	250.0	(10.0 - 40.0)%
$\Lambda(2350)$	0 ($9/2^+$)	2350.0 ± 22.0	150.0	10.0%
$\Lambda_c(2595)$ not included	0 ($3/2^-$)	2592.0 ± 0.3	2.6	2453.75 + 139.57= 2593.32 > 2592.0
$\Lambda_c(2625)$	0 ($3/2^-$)	2628.0 ± 0.2	<52.0	(2.6 ± 0.4)%
$\Lambda_c(2880)$	0 ($5/2^+$)	2881.6 ± 0.2	51.8	seen
$\Lambda_c(2910)$	0 ($5/2^+$)	2913.8 ± 3.8	51.8	seen
$\Lambda_c(2940)$	0 ($3/2^-$)	2939.6 ± 1.5	20.0	seen

Table A7. Data on two-particle decays of Ξ Barions from [8].

Name Ξ Barion	I (J^P)	Effective Mass Resonance m_{res} (Mev)	Width Γ (Mev)	Decay $\Xi+\pi$	Decay $\Xi(1530)+\pi$
1		2	3	4	
$\Xi(1530)$	1/2 ($3/2^-$)	1531.8 ± 0.3	9.1	100%	-
$\Xi(1620)$	1/2 ($3/2^+$)	1620.8 ± 6.0	32.0	seen	-
$\Xi(1690)$	1/2 ($3/2^-$)	1690.0 ± 10.0	20.0	seen	possibly seen
$\Xi(1820)$	1/2 ($3/2^-$)	1823.0 ± 5.0	20.0	small	small
$\Xi(1950)$	1/2 ($3/2^-$)	1950.0 ± 15.0	60.0	seen	seen
$\Xi(2030)$	1/2 ($3/2^+$)	2025.0 ± 20.0	60.0	small	small
$\Xi(2500)$	1/2 ($3/2^-$)	2500.0 ± 150.0	60.0	seen	seen