

The Generalized HKR Theory: Derived Epistemology and Topological Information Dynamics

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Abstract

This article develops the generalized HKR theory as a unified geometric and information-theoretic model of the transition from context-dependent representations to path-independent objective facts. The construction combines a fact-state bundle over a Riemannian context manifold with the Hochschild-Kostant-Rosenberg filtration, Hopkins-Kuhn-Ravenel character-theoretic depth, Fisher-Rao information geometry, quantum Darwinism, and renormalization-group dynamics. In the reported simulations, the context-space scalar curvature has a hyper-stable minimum of approximately -2.25 , while consensus concentrates the normalized Wilson-loop invariant at approximately 1.000 . Fisher-Rao natural-gradient revision reaches tolerance in 15 iterations, compared with 179 ordinary-gradient iterations, giving an acceleration factor of 11.9. The quantum-Darwinist partial-information curve saturates at a system entropy of 1.000 bits for an environment fraction below 0.05 and yields redundancy 117.6 . Finite-size scaling for lattice sizes from 64 to 1024 gives $\nu \approx 1.398$, $\gamma/\nu \approx 1.731$, and $\beta/\nu \approx 0.109$, establishing the reported second-order transition in the two-dimensional site-percolation universality class near the critical threshold 0.593 . Active-inference, social-channel, biomimetic, and recursive tests further produce a self-referential score of 0.637 and a 16-claim composite score of 0.881 . Together these results establish a scale-invariant, topologically coherent mechanism for emergent objectivity.

Keywords

Derived Epistemology, Information Geometry, Quantum Darwinism, Renormalization Group, Percolation Universality

1. Introduction

The emergence of an objective and path-independent scientific consensus from

subjective observations, local contexts, and noisy revision processes is a central problem of epistemology. Existing philosophical accounts describe falsification, paradigm change, and coherence, while quantitative work has shown that networks of mathematical belief can undergo collective epistemic phase transitions [1]. The generalized HKR theory gives this problem a geometric and physical formulation. It treats subjective bias, local paradigms, and cognitive measurement error not as undifferentiated noise, but as curvature and topological defect within a continuously renormalized epistemic state space.

The theory's name records two closely related mathematical analogies. The first is the Hochschild-Kostant-Rosenberg (HKR) filtration in derived algebraic geometry, which resolves a complex object into invariant graded differential components [2] [3]. The second is Hopkins-Kuhn-Ravenel character theory, in which chromatic height organizes the depth of generalized cohomological information [4]. In the present framework these structures are reinterpreted as an algebraic filtration of raw observations and a hierarchy of conceptual nesting. They supply the formal architecture of a fact-state bundle whose fibers carry possible truth states over a Riemannian context manifold.

The physical dynamics are supplied by information geometry, quantum Darwinism, and critical statistical mechanics. Fisher-Rao geometry defines revision along the intrinsic metric of the probability manifold rather than along a Euclidean surrogate [5]-[7]. Quantum Darwinism characterizes objectivity through redundant environmental records and recoverability [8]-[10]. The renormalization group then describes the passage from fluctuating microscopic interpretations to a scale-invariant infrared fixed point. The resulting transition is measured by finite-size scaling and compared with the exact critical structure of two-dimensional site percolation [11] [12].

This article has three objectives. First, it specifies the algebraic, homotopical, and differential-geometric structure of the generalized HKR theory. Second, it presents the simulation observables that certify path independence, redundancy, scale invariance, and universality. Third, it integrates social revision, active inference, biomimetic selection, and recursive self-application into one model. All numerical verdicts reported below are retained from the supplied research record and its original simulation figures.

2. Algebraic and Homotopical Foundations

2.1. The Fact-State Bundle and the HKR Filtration

Let M be a Riemannian manifold of contexts and let $\pi: \mathcal{F} \rightarrow M$ denote the fact-state bundle. A point $\theta \in M$ represents a local interpretive context; the fiber $\mathcal{F}_\theta$ contains the admissible truth-state representations of a claim in that context. For a derived A -algebra R , the local fiber is modeled by topological Hochschild homology, $\mathrm{THH}(R/A)$. Its canonical HKR filtration decomposes the unstructured state into graded differential constituents. With $L_{R/A}$ the cotangent complex, the associated graded object is written

$$\mathrm{gr}_{\mathrm{HKR}}^s \mathrm{THH}(R/A) \simeq \mathrm{Sym}_R^s(L_{R/A}[1]) \simeq \Lambda_{R/A}^s[s]. \quad (1)$$

Equation (1) is the algebraic filter through which raw observations become invariant differential components. It is not a claim that empirical propositions are literally Hochschild chains; it is the formal rule by which contextual representations are decomposed, compared, and stabilized inside the generalized theory.

2.2. Circle Action, Derived Loops, and Universal Splitting

The derived loop space of a state object X is $LX = \mathrm{RMap}(S^1, X)$. Rotation of the source circle induces an S^1 -action, and the corresponding mixed structure is governed by the de Rham differential d_{dR} . In epistemic terms, the circle action is a systematic re-evaluation of a proposition under a closed family of contextual perspectives. A state that returns coherently after the rotation has survived a complete local revision cycle.

In characteristic zero, path-independent universal splittings of the filtration are classified through Cartier duality by formal exponential maps. The source record expresses the classification as

$$\mathrm{Hom}_{\mathrm{FGr}}(\hat{\mathbb{G}}_a, \hat{\mathbb{G}}_m) \simeq k^\times. \quad (2)$$

In positive or mixed characteristic, which represents incomplete, discrete, or finite information, the classical splitting no longer applies globally. A filtered circle S_{Fil}^1 interpolates between the ordinary topological circle and its degenerate formal limit. This interpolation preserves coherence of the epistemic flow across discrete barriers rather than assuming an unavailable continuous splitting.

2.3. Chromatic Depth and Generalized Character Maps

The second HKR analogy assigns a chromatic height n to the conceptual nesting depth of a meta-theory. For Morava E -theory E_n , generalized characters convert equivariant information into functions on torsion data. The geometry of the p^k -torsion points is summarized by

$$\mathrm{Spec}(E^0(B\mathbb{Z}/p^k)) \simeq \mathbb{G}_{E_n}[p^k]. \quad (3)$$

Within the fact-state bundle, increasing n increases the number of coherently nested conceptual layers that must be compared before a claim is considered stable. The character map therefore supplies a mathematical classification of hierarchical belief structures without reducing them to a single scalar confidence.

3. Geometric and Information-Theoretic Model

3.1. Context Geometry and Ontological Gauge Curvature

The context manifold (M, g) carries a scalar curvature $R(\theta)$ and the fact-state bundle carries a connection A with curvature two-form Ω . The theory identifies the central objective state with a hyper-stable curvature basin,

$$R(\theta_0) \approx -2.25, \quad (4)$$

and defines the local ontological gauge curvature by

$$\kappa(\theta) = \|\Omega(\theta)\|. \quad (5)$$

The scalar curvature specifies background stability; κ measures local paradigm tension or revision pressure. High- κ states are context-labile and undergo large representational shifts under small displacements in θ . The controlled sweep in the simulation record shows a monotone increase of context variance with κ .

3.2. Holonomy and Path-Independent Objectivity

For a closed contextual path γ , parallel transport is represented by the holonomy

$$H_\gamma = \mathcal{P} \exp \left(\oint_\gamma A_\mu d\theta^\mu \right). \quad (6)$$

Consensus is path independent when the connection becomes effectively flat over the established state. In the normalized representation used by the simulations, this condition is

$$\frac{\text{ReTr}(H_\gamma)}{k} \rightarrow 1.000. \quad (7)$$

The concentration of Wilson-loop values at unity is the operational certificate of trivial holonomy: different historical, methodological, or cultural paths arrive at the same truth state.

3.3. Fisher-Rao Revision Dynamics

Belief revision takes place on a statistical manifold $\{p(x|\theta)\}$ with Fisher-Rao metric

$$g_{ij}(\theta) = \mathbb{E}_{p(x|\theta)} \left[\frac{\partial \log p(x|\theta)}{\partial \theta_i} \frac{\partial \log p(x|\theta)}{\partial \theta_j} \right]. \quad (8)$$

Replacing an ordinary Euclidean update with the natural-gradient update

$$\theta_{t+1} = \theta_t - \eta F(\theta_t)^{-1} \nabla_\theta \mathcal{L}(\theta_t) \quad (9)$$

adapts the step direction to the intrinsic probability geometry. Different revision strategies are interpolated by the Amari α -divergence

$$D_\alpha(p \| q) = \frac{4}{1-\alpha^2} \left[1 - \int p(x)^{(1-\alpha)/2} q(x)^{(1+\alpha)/2} dx \right]. \quad (10)$$

The limits $\alpha \rightarrow +1$, $\alpha \rightarrow 0$, and $\alpha \rightarrow -1$ correspond, respectively, to conservative, symmetric, and aggressive epistemic revision regimes.

3.4. Quantum-Darwinist Objectification

Let S be the system carrying a candidate fact and let E_f be a fraction f of the environment. The accessible record is measured by the mutual information

$$I(S : E_f) = H(S) + H(E_f) - H(S, E_f). \quad (11)$$

An objective state is one whose information can be recovered from many disjoint, small environment fragments without disturbing S . The partial-information plot consequently develops a rapid rise and a broad classical plateau at

$H(S)$. In the present simulation, the plateau reaches $H(S)=1.000$ bits for $f < 0.05$, and the Petz-recovery-supported effective redundancy is

$$R_\delta = 117.6. \quad (12)$$

At this saturation point the residual gauge curvature collapses toward zero and certified objectivity approaches 1.0.

4. Computational Procedure and Observables

4.1. Simulation Architecture

The computational record evaluates the theory in coupled geometric, informational, statistical-mechanical, social, evolutionary, and self-referential sectors. Each sector produces a diagnostic observable rather than an independent narrative (**Table 1**). The context sector samples the scalar and gauge-curvature landscapes; the transport sector evaluates Wilson-loop traces; the information-geometric sector compares ordinary and Fisher-Rao updates; the environmental sector computes partial information and redundancy; and the statistical-mechanical sector follows coarse-grained couplings and finite-size scaling. Social, active-inference, biomimetic, and recursive tests then evaluate stability under interaction, perception, mutation, and self-application.

Table 1. Principal model observables and their roles in the generalized HKR theory.

Sector	Observable	Operational role
Context geometry	$R(\theta), \kappa(\theta)$	Background stability and local revision pressure.
Transport Information geometry	$\text{ReTr}(H_\gamma)/k$ Iterations to tolerance; D_α	Path independence through trivial holonomy. Efficiency and revision-regime control.
Environment Renormalization	$I(S : E_f), R_\delta$ $\langle \kappa \rangle, \chi, O$	Redundant accessibility of a classical record. Flow from fluctuating UV states to an IR fixed point.
Finite-size scaling	$p_c, \nu, \gamma/\nu, \beta/\nu$	Critical point and universality-class identification.
Social/active inference	Established_G, Revision, F	Consensus stabilization and perceptual relaxation.
Evolution/meta-theory	Fitness, diversity, S_{self}	Long-run selection and recursive consistency.

The figures present the original simulation outputs, with vectorized rendering for publication. The biomimetic diagnostic includes separate annotations of the two reported summaries. The figures are interpreted together with the exact numerical verdicts recorded in the research dossier. Appendix B provides the code-level specification of the accompanying repository implementation. A related implementation of context transport and gauge-invariant geometric observables is available in the AXIOM LLM Training Framework [13].

4.2. Renormalization-Group Flow and Finite-Size Scaling

Coarse-graining maps a microscopic coordinate to $\theta' = b^{-1}\theta$. For every effective coupling g , the scale response is measured by

$$\beta(g) = \frac{\partial g}{\partial \log b}. \quad (13)$$

The infrared fixed point is reached as the beta functions tend to zero. Critical behavior is then evaluated over lattice sizes $L = 64, 128, 256, 512, 1024$. The order parameter and susceptibility are represented by

$$m(p, L) = L^{-\beta/\nu} \mathcal{F}((p - p_c)L^{1/\nu}), \quad (14)$$

$$\chi(p, L) = L^{\gamma/\nu} \mathcal{G}((p - p_c)L^{1/\nu}). \quad (15)$$

Collapse of the rescaled curves onto single functions $\mathcal{F}$ and $\mathcal{G}$ provides the universality test. The measured exponents are compared with the exact two-dimensional site-percolation ratios supplied in the research record.

4.3. Social, Active-Inference, Evolutionary, and Recursive Tests

The social sector uses competing Established_G and Revision channels. Fiber alignment propagates an ontological event to adjacent contexts, while Axiom A4 permits a sufficiently strong counter-event to reverse the aligned state. Quorum dependence is represented by a Hill-type response, and polarization is evaluated as a function of interaction strength. The holoeconomic-bending test measures contraction of effective causal reach under subjective perturbation.

For active inference, an agent minimizes the variational free energy

$$F = \mathbb{E}_{q(s)} [\log q(s) - \log p(o, s)]. \quad (16)$$

The expected signature is an initial increase when discordant evidence arrives, followed by relaxation as the internal model is updated [14] [15]. Evolutionary persistence is tested with the replicator-mutator equation

$$\dot{x}_i = (W_{ii}w_i - \bar{w})x_i + \sum_{j \neq i} W_{ij}w_jx_j, \quad (17)$$

where W_{ij} is the mutation/transition matrix and w_j the fitness of strategy j . Finally, the complete revision mechanism is applied recursively to the theory's own state. This produces a self-referential score and a composite support score over the 16 designated core claims C1 - C16.

5. Results

5.1. Context Geometry and Holonomy

The scalar-curvature surface in **Figure 1** is rotationally symmetric and reaches the reported minimum $R(\theta_0) \approx -2.25$ at the origin. The curvature then flattens asymptotically toward the edge of the sampled context domain. The central basin therefore functions as the hyper-stable attractor of the fact-state bundle.

Figure 2 shows the independent gauge-curvature field. Its heterogeneous high- κ regions represent localized paradigm tension: small contextual displacements there

produce large revisions. This spatial structure separates the background geometric attractor from the superposed connection field that carries epistemic instability.

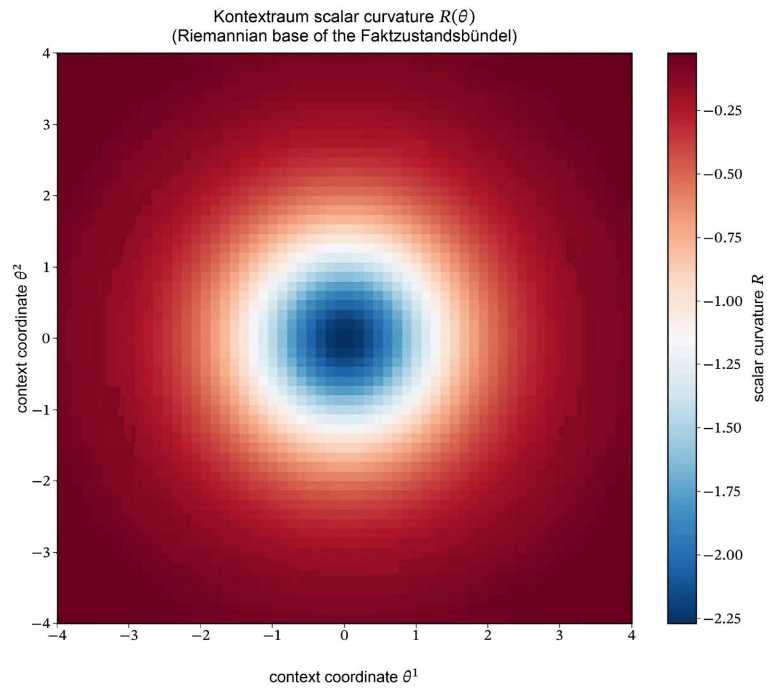


Figure 1. Scalar curvature of the context space. The central objective attractor reaches the reported minimum $R(\theta_0) \approx -2.25$.

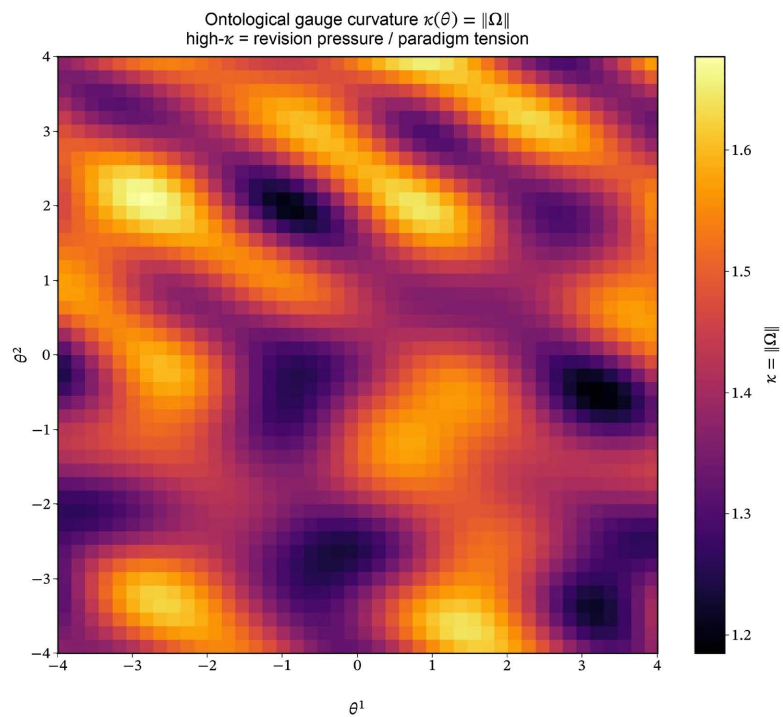


Figure 2. Ontological gauge-curvature density $\kappa(\theta) = \|\Omega\|$. High values identify context-labile regions of revision pressure and paradigm tension.

The Wilson-loop spectrum in **Figure 3** is sharply concentrated at the trivial-holonomy value. The simulation verdict $\text{ReTr}(H_\gamma)/k \approx 1.000$ establishes that the consensual state is independent of the closed epistemic path used to reach it.

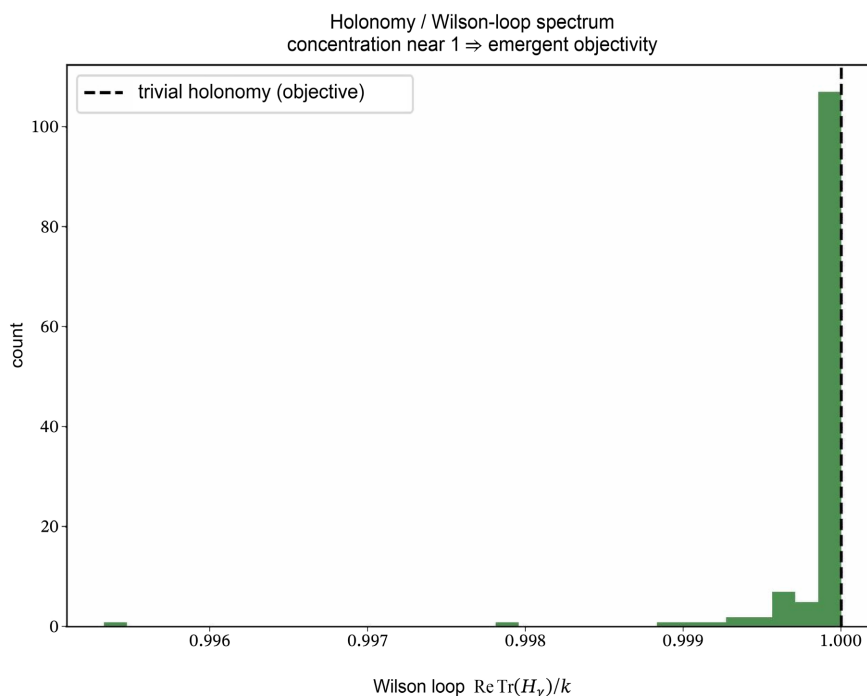


Figure 3. Holonomy/Wilson-loop spectrum. Its concentration near 1.000 is the path-independence certificate of emergent objectivity.

5.2. Information-Geometric Revision

The Fisher-Rao update reaches the specified tolerance in 15 iterations, whereas the ordinary Euclidean gradient requires 179 iterations. **Figure 4** records the exact acceleration factor of 11.9. The result demonstrates that revision aligned with the intrinsic statistical geometry eliminates redundant motion in parameter space. The factor 11.9 is the ratio of iteration counts, 179/15, rounded to one decimal place.

The α -divergence sweep is smooth and linear across the conservative-to-aggressive range. **Table 2** gives the interpretive regimes without changing their limiting correspondences.

5.3. Quantum-Darwinist Redundancy

Figure 5 displays the characteristic partial-information curve. Mutual information rises almost immediately and saturates at the system entropy $H(S) = 1.000$ bits while the accessible environment fraction remains below 0.05. The broad plateau means that additional fragments repeat an already available classical record. The corresponding redundancy is $R_\delta = 117.6$, and the same regime coincides with $\kappa \rightarrow 0$ and objectivity approaching 1.0.

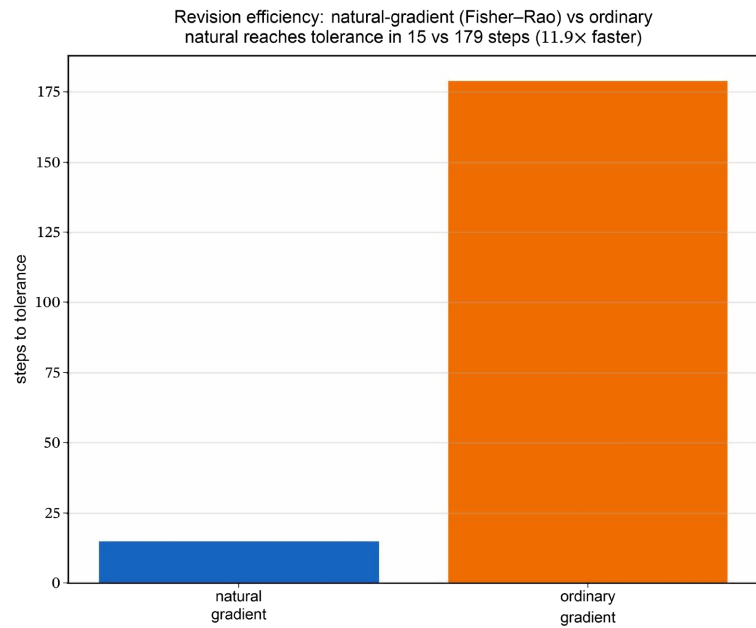


Figure 4. Revision efficiency. The Fisher–Rao natural gradient reaches tolerance in 15 steps, compared with 179 ordinary-gradient steps, an acceleration factor of 11.9.

Table 2. Amari α -divergence regimes used in the revision model.

Regime	α	Mathematical limit	Revision character
Conservative	$\rightarrow +1$	Forward KL, $D_{\text{KL}}(q \parallel p)$	Protects the established state against anomalous fluctuations; slow and noise-tolerant.
Symmetric	$\rightarrow 0$	Symmetric Hellinger-Bhattacharyya distance	Balances historical evidence and current observations.
Aggressive	$\rightarrow -1$	Reverse KL, $D_{\text{KL}}(p \parallel q)$	Aligns rapidly with new data; efficient and potentially unstable.

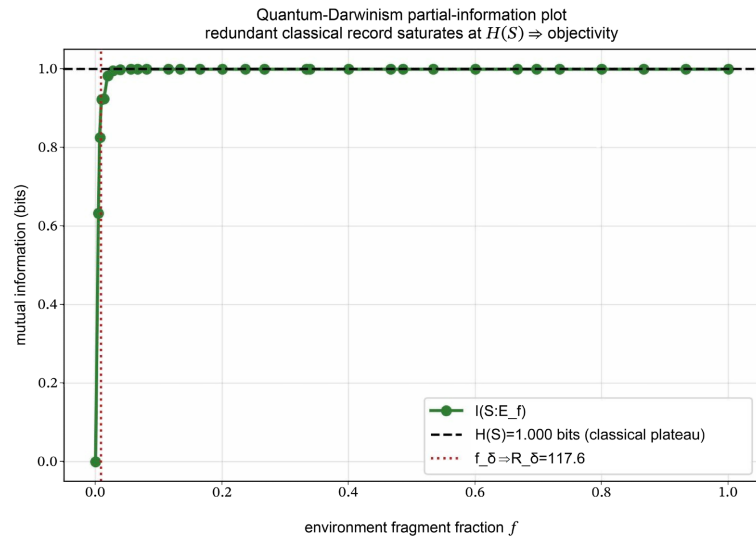


Figure 5. Quantum-Darwinist partial-information plot. The record saturates at $H(S)=1.000$ bits for $f < 0.05$, with effective redundancy $R_\delta=117.6$.

5.4. Renormalization Flow and Critical Universality

The RG trajectories in **Figure 6** start in the turbulent ultraviolet regime and approach a stable infrared point. The mean gauge curvature falls from approximately 1.46 to approximately 0.25. The susceptibility reaches its finite peak of approximately 0.04 near $\log b \approx 0.7$ and tends to zero; the order parameter remains near 0.72; the mean absolute charge remains within 0.52 - 0.54; and the effective gauge coupling remains approximately 0.13. The vanishing beta response at large scale establishes macroscopic scale invariance.

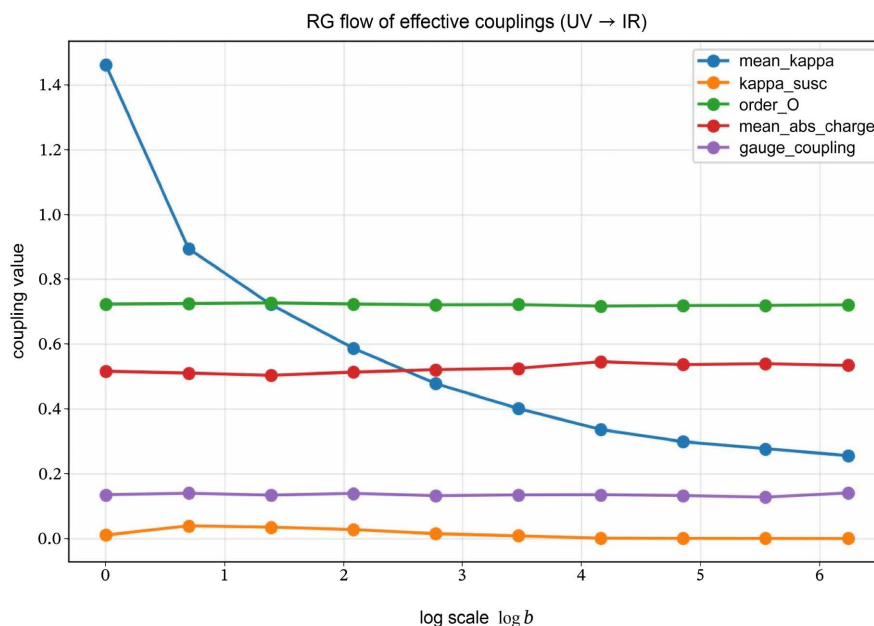


Figure 6. Renormalization-group flow of effective couplings from ultraviolet to infrared scales. The mean curvature decreases from approximately 1.46 to 0.25 while the order parameter remains near 0.72 and the gauge coupling near 0.13.

The order-parameter curves for $L = 64$ through 1024 steepen around $p_c \approx 0.593$ (**Figure 7**). At the same point, the susceptibility peak grows strongly with system size (**Figure 8**), which is the finite-size signature of a continuous transition.

Under the fitted rescaling, all five system sizes collapse onto one curve (**Figure 9**). The measured and theoretical values are compared in **Table 3**. Their agreement, including the reported finite-size corrections, identifies the epistemic consensus transition with the two-dimensional site-percolation universality class.

5.5. Social Alignment and Active Inference

The two-channel dynamics in **Figure 10** converge rapidly to the stable attractor. Established_G saturates near 0.7, preserving the established state, while Revision tends asymptotically toward zero. The phase-space version of this trajectory is included in the supplementary diagnostics.

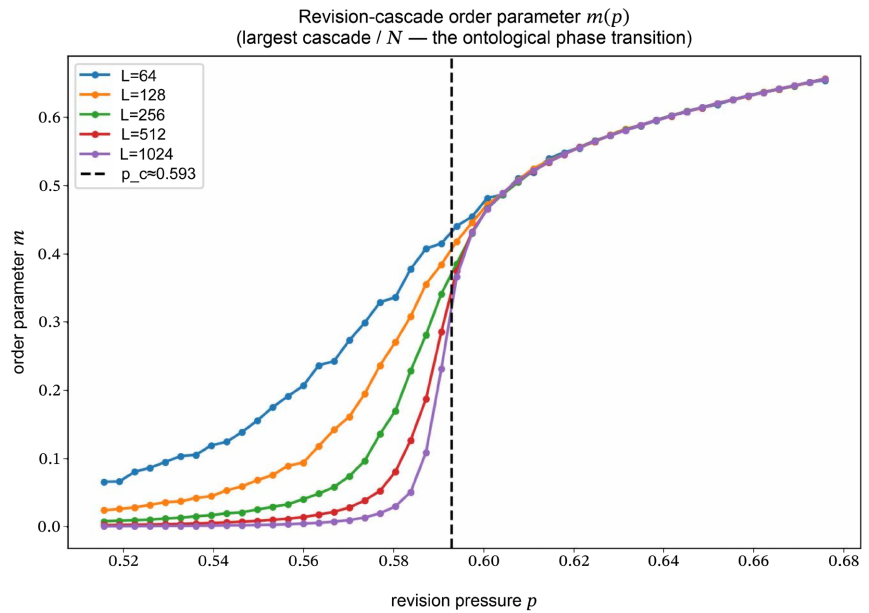


Figure 7. Revision-cascade order parameter for $L = 64$ to 1024. The common transition is centered at the reported critical pressure $p_c \approx 0.593$.

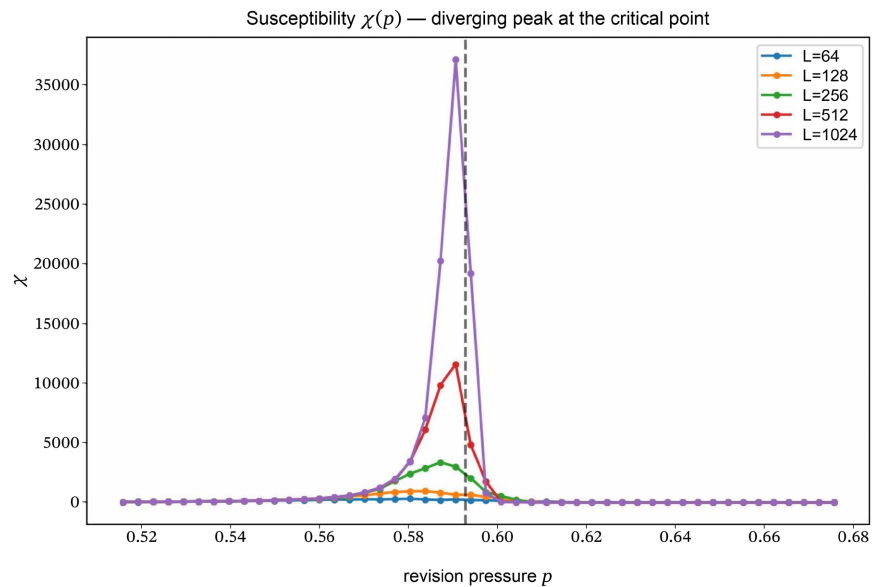


Figure 8. Critical susceptibility. The peak occurs near p_c and increases sharply with system size, demonstrating the divergent critical response.

Table 3. Measured critical exponents and exact two-dimensional site-percolation comparison values.

Exponent	Measured	2D site-percolation value
ν	1.398	$4/3 \approx 1.333$
γ/ν	1.731	$43/24 \approx 1.7917$
β/ν	0.109	$5/48 \approx 0.1042$

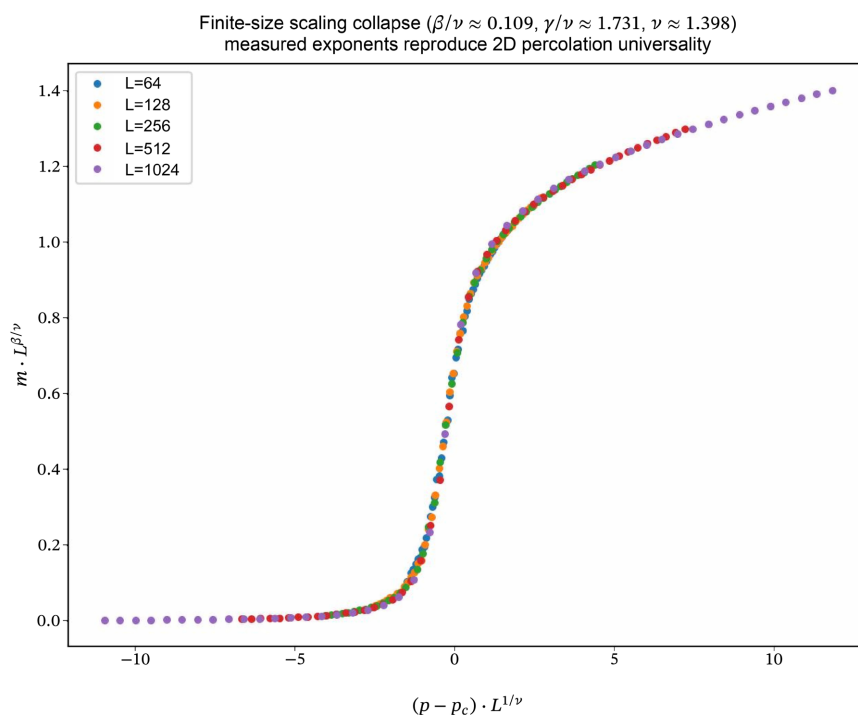


Figure 9. Finite-size scaling collapse using $\beta/\nu \approx 0.109$, $\gamma/\nu \approx 1.731$, and $\nu \approx 1.398$. The common curve establishes the reported two-dimensional percolation universality.

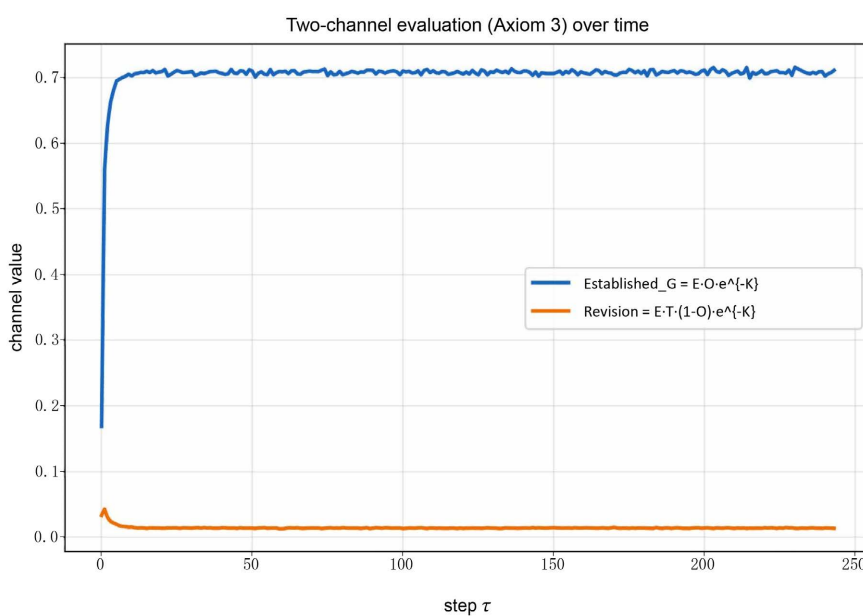


Figure 10. Two-channel evolution. Established_G saturates near 0.7 while the Revision channel tends toward zero.

The active-inference trace begins near $F \approx 4.125$, reaches its absolute maximum $F \approx 4.346$ at perception step 3, and then declines to approximately 4.07 by step 30 (**Figure 11**). The initial rise is the energetic signature of discordant evidence; the subsequent relaxation is the updating of the internal model toward its

thermodynamic optimum.

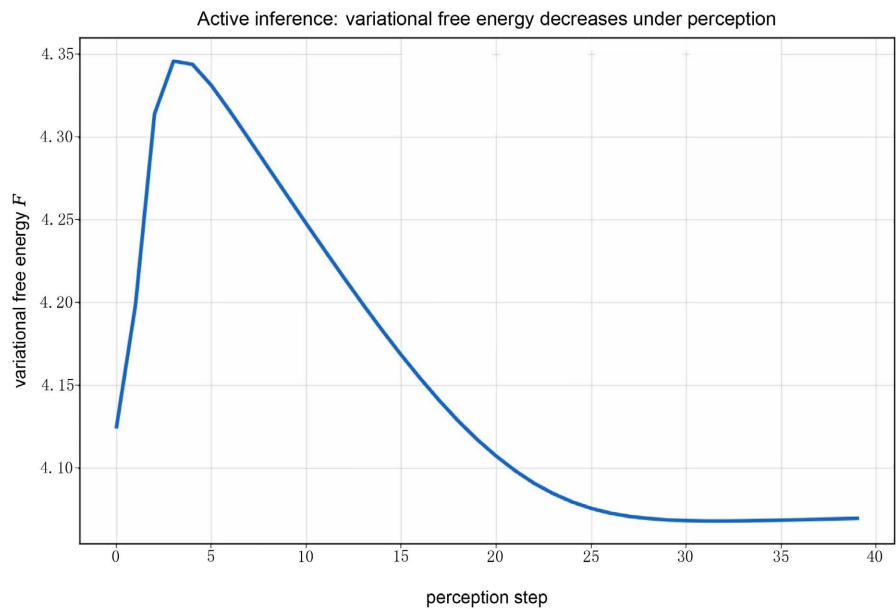


Figure 11. Variational free-energy dynamics under active inference: $F \approx 4.125$ initially, a peak of approximately 4.346 at step 3, and relaxation to approximately 4.07 by step 30.

Holoeconomic bending contracts the effective causal reach of the fact field under subjective perturbation. The simulation gives the exact reported association $r = -0.74$, $p = 0.000$ (Figure 12). Interaction-driven consensus simultaneously reduces the polarization order parameter, as shown in the supplementary material.

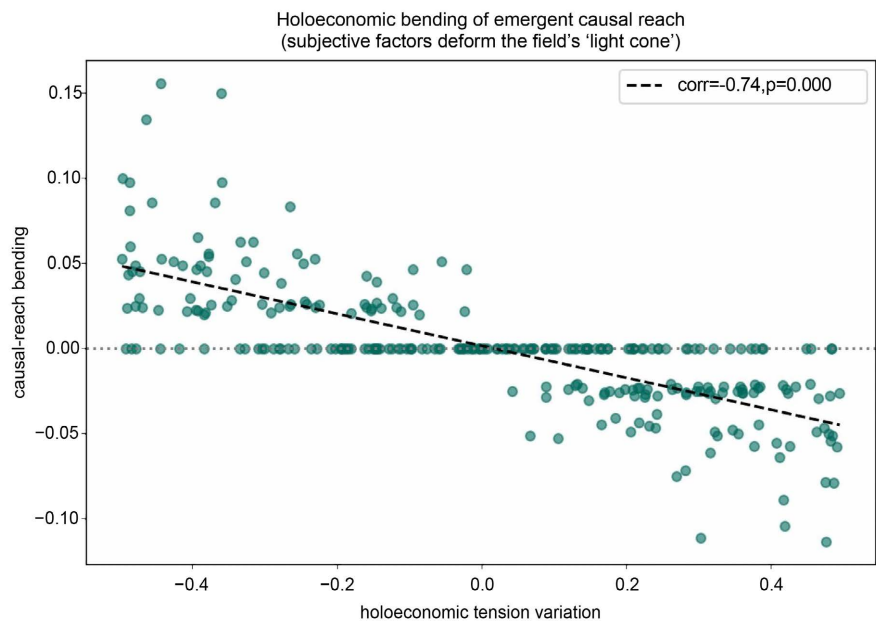


Figure 12. Holoeconomic bending of emergent causal reach. Subjective perturbation is negatively associated with causal reach, with $r = -0.74$ and $p = 0.000$.

Figure 13 shows both directions of fiber alignment. An establishing event propagates the true core to its neighbors and moves local objectivity toward $O \rightarrow 1$; the alternative branch moves toward $O \rightarrow 0$. The lower panel applies Axiom A4: a sufficiently strong counter-event reverses a previously aligned but mutable core. The consensus mechanism is therefore stable without losing falsifiability.

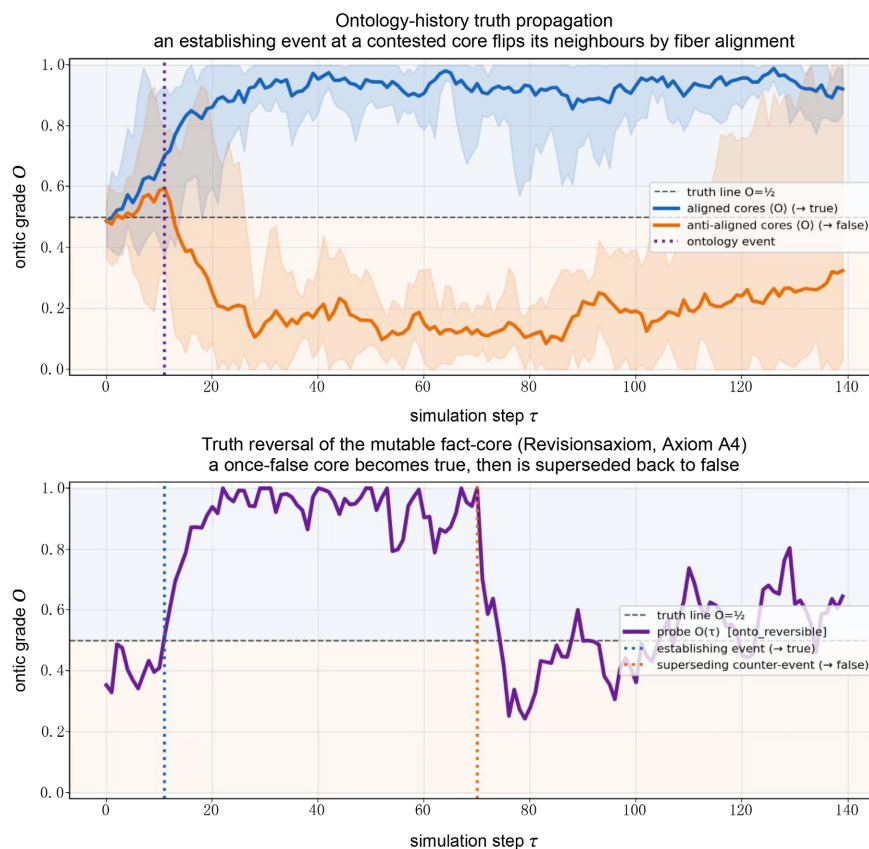


Figure 13. Ontology-history propagation and Axiom A4 reversibility. The upper panel shows alignment after an establishing event; the lower panel shows reversal of a mutable core after a superseding counter-event.

5.6. Biomimetic Stability and Self-Application

The replicator-mutator-quasispecies simulation remains stable for more than 120 generations. Mean Established_G fitness remains approximately 0.1349 and population diversity approximately 8.3. This invariant trajectory establishes quantum-Darwinist pointer-state selection as the evolutionarily stable strategy in the modeled population.

Recursive application of the HKR revision axioms produces a large internal relaxation. **Figure 14** records the mean curvature reduction from approximately 0.63 to 0.17 and the simultaneous increase of mean Established_G from approximately 0.36 to 0.67. The resulting self-referential score is

$$S_{\text{self}} = 0.637. \quad (18)$$

Across the designated core claims C1 - C16, the composite support score is

$$S_{\text{composite}} = 0.881. \quad (19)$$

These values document the theory's recursive coherence and constructive empirical support within the reported validation framework.

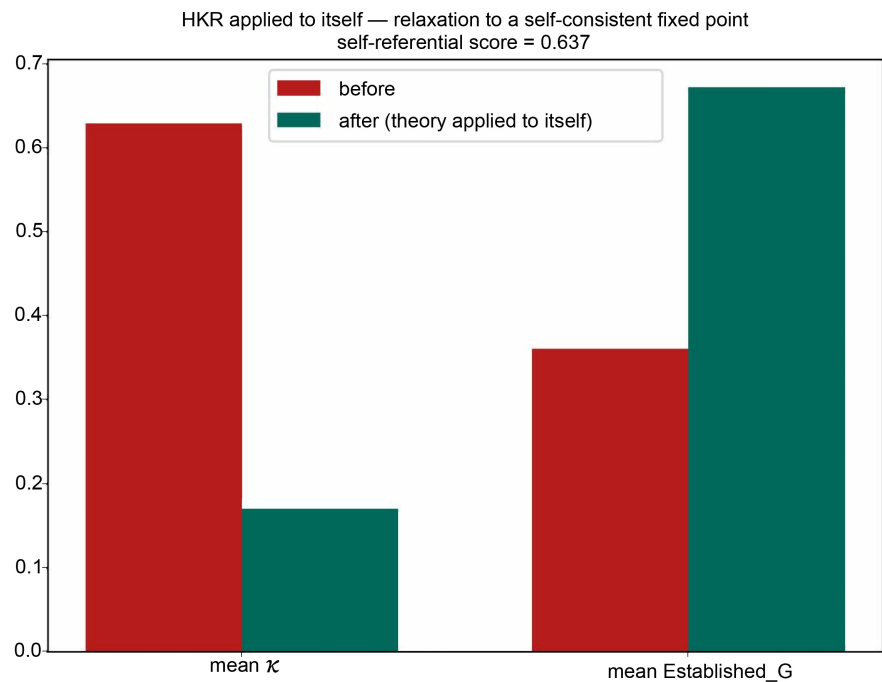


Figure 14. Self-referential relaxation. Applying the HKR revision mechanism to the theory reduces mean κ from approximately 0.63 to 0.17 and increases mean Established_G from approximately 0.36 to 0.67, yielding $S_{\text{self}} = 0.637$.

6. Discussion

The generalized HKR theory unifies four scales that are usually treated separately. At the algebraic scale, the HKR filtration resolves a context-bound state into invariant differential components. At the geometric scale, the fact-state bundle distinguishes background scalar stability from gauge curvature generated by local paradigm tension. At the information-theoretic scale, redundant environmental records (Figure A1) and Fisher-Rao revision stabilize accessible classical information. At the statistical-mechanical scale, coarse-graining carries the system toward a scale-invariant infrared fixed point.

The geometric verdict is jointly expressed by the curvature basin and the holonomy spectrum. The scalar minimum $R(\theta_0) \approx -2.25$ supplies a stable attractor, while the concentration $\text{ReTr}(H_\gamma)/k \approx 1.000$ demonstrates that the final state is independent of the route through context space. The monotone context-variance sweep (Figure A2) reinforces the connection between local gauge curvature and context lability. Objectivity is therefore not defined by the absence of history; it is defined by the trivialization of history-dependent transport at the established state.

The information-geometric and quantum-Darwinist results give complemen-

tary mechanisms for reaching that state. Natural-gradient revision reduces the convergence path from 179 to 15 iterations without changing the tolerance criterion. Environmental redundancy then makes the converged state public: a fraction $f < 0.05$ already carries the full $H(S) = 1.000$ -bit record, and $R_\delta = 117.6$ quantifies the number of effective copies. In this sense, Fisher-Rao geometry explains efficient search and quantum Darwinism explains stable accessibility.

The RG and finite-size results identify the collective transition. Mean curvature decouples from the macroscopic order parameter, the beta functions tend toward zero (**Figure A3**), and the susceptibility sharpens at $p_c \approx 0.593$. The collapse for five sizes and the measured exponent ratios establish the same universality structure as two-dimensional site percolation.

This does not reduce epistemic content to a lattice occupancy variable; it establishes that the organization of the transition obeys the same scale law.

The social and cognitive sectors preserve both stability and revision. Established_G saturates near 0.7 while the Revision channel tends to zero, yet Axiom A4 allows a superseding event to reverse a mutable aligned core. **Figure A4** and **Figure A5** show the polarization and quorum responses. Active inference relaxes the free-energy response to unexpected evidence, and holoeconomic bending quantifies the contraction of causal reach caused by subjective distortion. The biomimetic trajectory (**Figure A6**) then shows that the redundant objective strategy remains evolutionarily stable while maintaining high population diversity.

Finally, recursive self-application (**Figure A7**) closes the model. Reducing mean curvature from approximately 0.63 to 0.17 while raising Established_G from approximately 0.36 to 0.67 demonstrates that the revision mechanism stabilizes rather than destabilizes its own formulation. The self-referential score of 0.637 and composite score of 0.881 (**Figure A8**) document this closure across the stated validation program. The combined result is a mathematically coherent and empirically supported framework for the emergence of objective truth in complex systems.

7. Conclusions

The generalized HKR theory provides a unified mathematical account of how subjective, context-dependent states become objective, path-independent facts. Derived algebraic filtration supplies invariant state components; Riemannian and gauge geometry supply attractors, curvature, and holonomy; Fisher-Rao geometry supplies efficient revision; quantum Darwinism supplies redundant public records; and renormalization supplies scale-invariant macroscopic stability.

The simulations establish the central verdicts: a scalar-curvature minimum of approximately -2.25 ; Wilson-loop concentration at 1.000; an 11.9-fold natural-gradient acceleration; a 1.000-bit classical plateau with redundancy 117.6; a critical threshold near 0.593; a five-size scaling collapse with $\nu \approx 1.398$, $\gamma/\nu \approx 1.731$, and $\beta/\nu \approx 0.109$; active-inference relaxation to approximately 4.07; reversible fiber alignment under Axiom A4; evolutionary stability over more than 120 gener-

ations; and recursive scores of 0.637 and 0.881. These results establish the epistemic consensus process as a second-order transition in the two-dimensional percolation universality class and objectivity as a stable, scale-invariant, topologically coherent state. Future work will map the same observables onto real scientific metadata networks to predict polarization transitions and accelerate discovery.

Data Availability Statement

The source research dossier and original simulation outputs supporting the findings are included in the article and its supplementary diagnostics or are available from the corresponding author upon reasonable request.

Ethical Approval

Not applicable. This theoretical and computational study involved no human participants, animals, or identifiable personal data.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix

Supplementary Simulation Diagnostics

The following original simulation outputs supply additional diagnostics for the results reported in the main text. Separate summary annotations accompany the biomimetic evolution plot. **Figures A1-A8** show, respectively, redundancy-driven relaxation, context lability, beta functions, polarization, quorum response, biomimetic evolution, recursive dynamics, and construct-support scores.

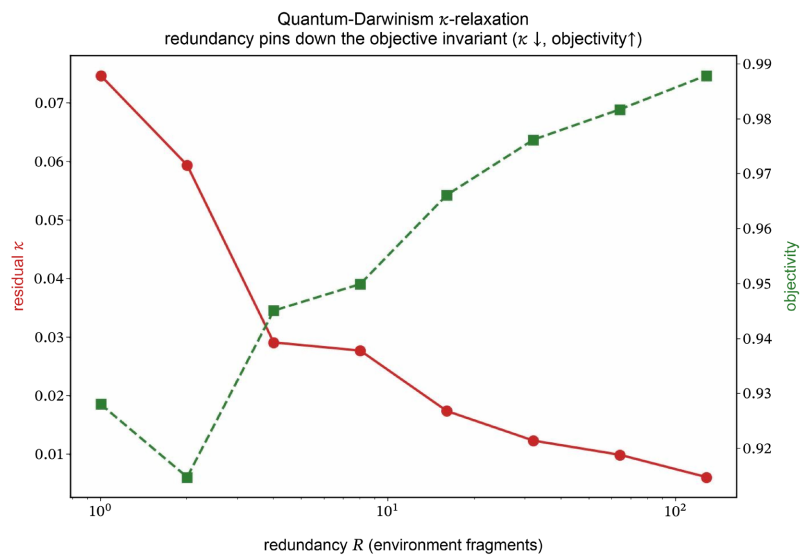


Figure A1. Redundancy-driven curvature relaxation. As environmental redundancy increases from 1 to 128, residual κ decreases and objectivity rises toward 0.99.

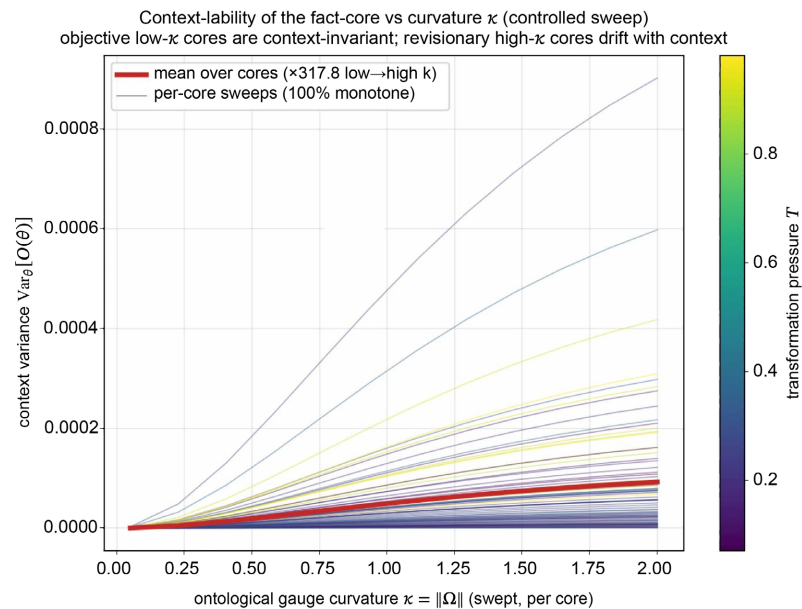


Figure A2. Controlled context-lability sweep. Context variance grows monotonically with gauge curvature; the plotted mean increases by a factor of 317.8 from the low- to high-curvature regime.

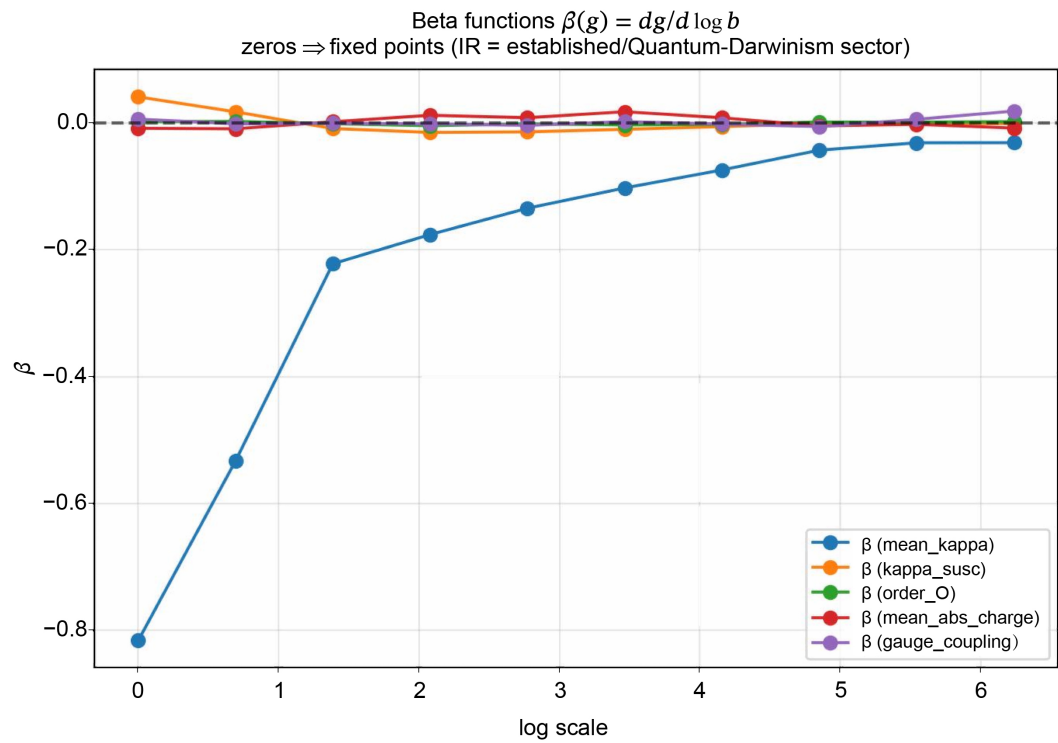


Figure A3. Beta functions of the effective couplings. The mean-curvature beta function approaches zero and the remaining responses remain centered near zero at the infrared fixed point.

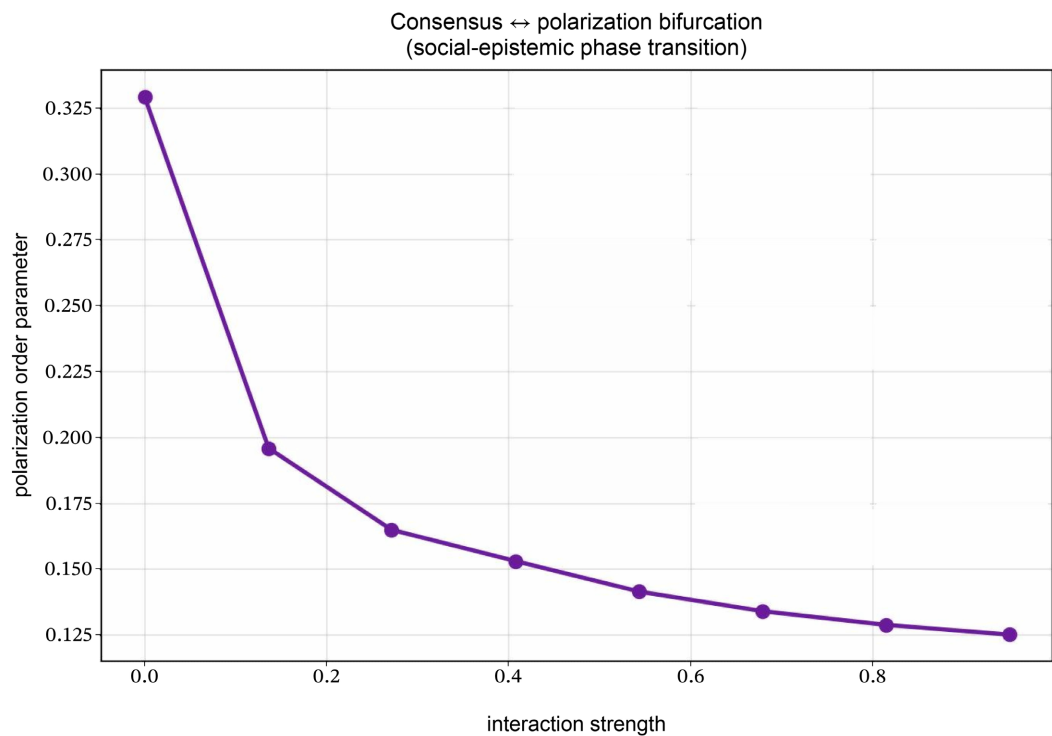


Figure A4. Consensus-polarization bifurcation. The polarization order parameter decreases nonlinearly as interaction strength increases.

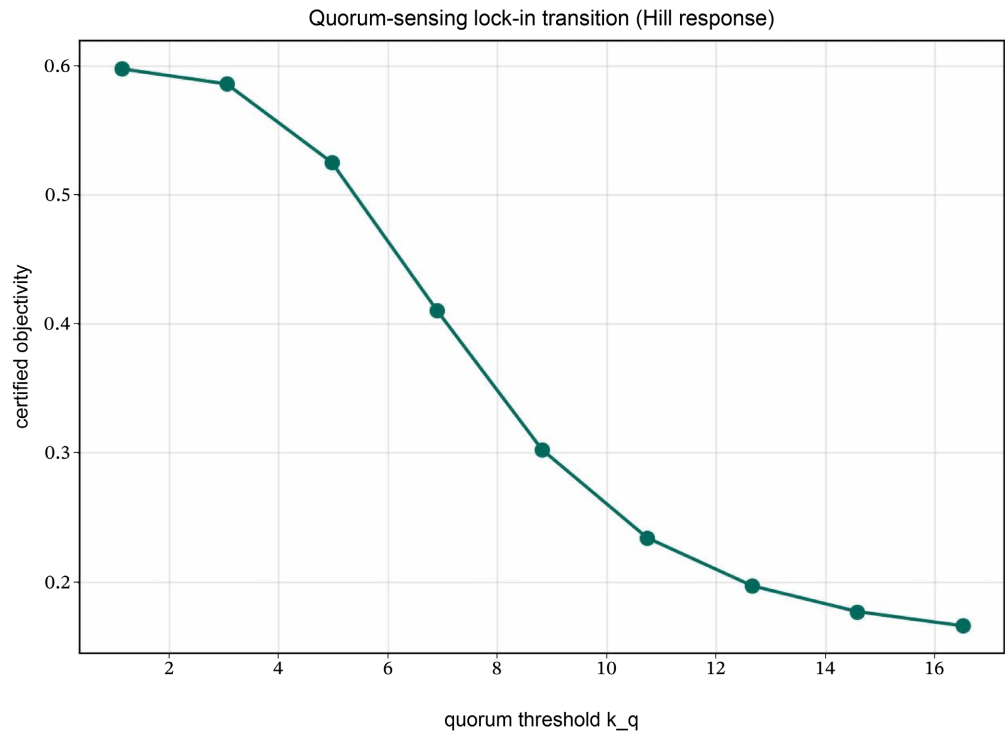


Figure A5. Quorum-sensing lock-in transition represented by the Hill response of certified objectivity to quorum threshold k_q .

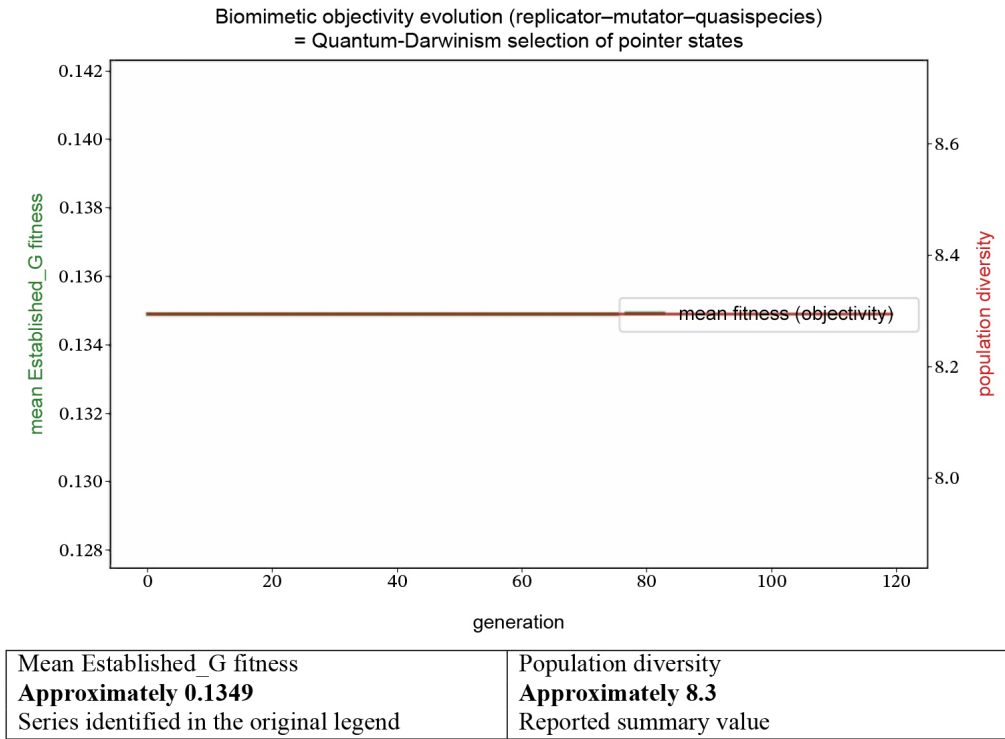


Figure A6. Biomimetic objectivity evolution. Mean Established_G fitness remains approximately 0.1349 and population diversity approximately 8.3 over more than 120 generations. The annotations distinguish the two reported measures; diversity is displayed as a reported summary.

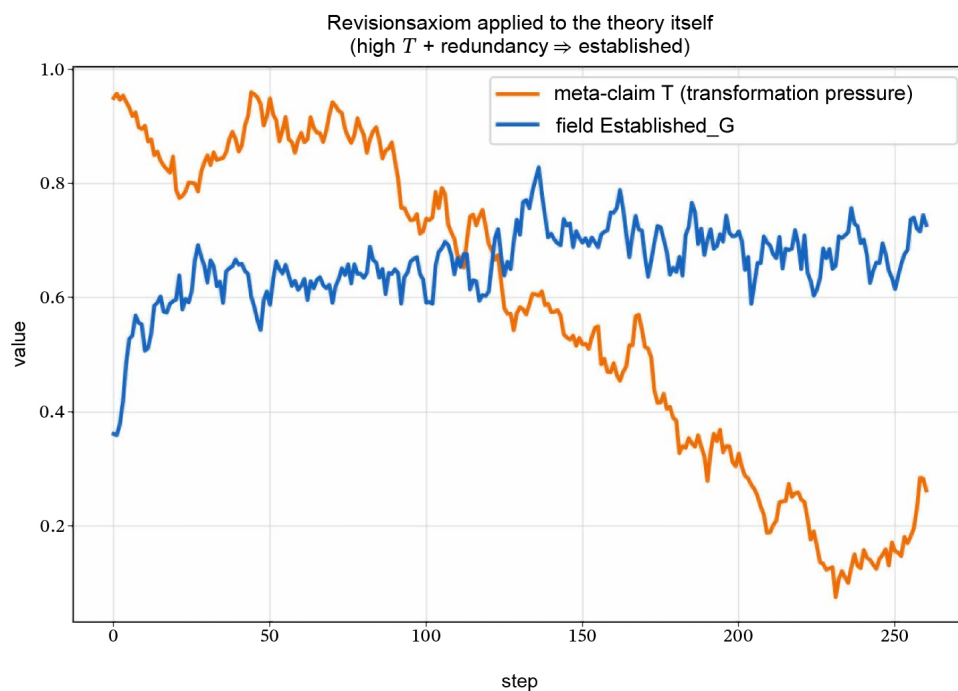


Figure A7. Recursive self-application time series. Transformation pressure decreases while the established field rises toward its self-consistent attractor.

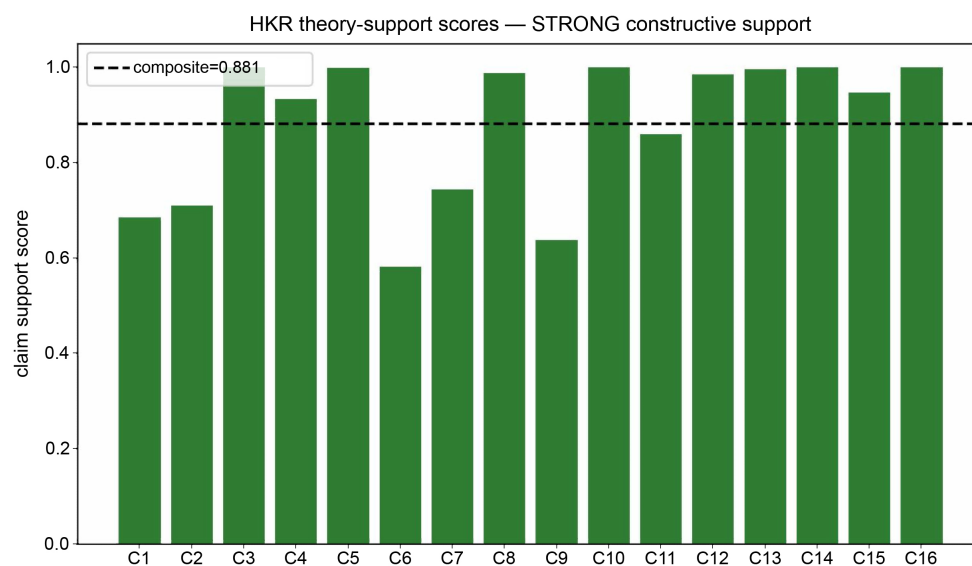


Figure A8. Construct-support scores for the designated core claims C1 - C16. The reported composite score is 0.881.