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Another Proof of the Truth of the Collatz Conjecture, Using Gaussian Arithmetic, Probability, and Statistics

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Abstract

In my previous article on the Collatz conjecture, I proved the conjecture without analyzing the structure of the trajectories in detail. It was sufficient to show that they all consisted of movements on a directed graph with six nodes. I used the principles of contradiction and mathematical induction to complete the proof. However, despite the mathematical proof in the aforementioned article, the human imagination can conceive of a realistic trajectory that exhibits phases of growth and decline, but whose overall trend is upward. This paper thoroughly analyzes the structure of the trajectories, using the directed graph mentioned above, a probabilistic decision tree, and statistical concepts, to prove that this trajectory, while appearing realistic, is not feasible. The Collatz conjecture has been proven to be accurate.

Keywords

Collatz Conjecture, Gaussian Arithmetic, Planar Graphs, Binomial Distribution

1. Introduction

The German mathematician Lothar Collatz is known for proposing, in 1937, the conjecture that bears his name, also known as the $3n + 1$ conjecture. The conjecture uses the following linear functions.

Definition 1 (Collatz Function).

We will refer to the following function as the Collatz function:

$C: N \to N$ defined by

$$C(k) := \begin{cases} 3k+1 & \text{if } k \text{ odd}, \\ \frac{k}{2} & \text{if } k \text{ even}. \end{cases}$$

Note that both functions are linear.

Definition 2 (Collatz Conjecture).

The Collatz Conjecture is defined as follows:

For any starting point $k \in N$, *the sequence obtained by successive applications of the function* C *would eventually arrive at* 1 (*so that the cycle* 4, 2, 1 *is repeated indefinitely*).

We will refer to these sequences as "trajectories."

For a historical view of the study of the Collatz conjecture, see [1] and [2].

Numerical computation was used to prove that all numbers less than or equal to 2.95147×10^{20} satisfy the Collatz conjecture; see [3].

The article [4] uses the concept of "congruences in general," which was introduced by Carl F. Gauss in Chapter 1 of his *Disquisitiones Arithmeticae* [5], to define a directed graph. This graph constitutes the Collatz trajectory space, since all trajectories are a continuous movement within the graph. The graph contains three loops that are essential for determining whether a counterexample trajectory exists. By contradiction, it is proven that only one of them could generate it, but by induction, it is proven that it is not feasible. Therefore, the conjecture is confirmed to be true.

In this paper, we will examine the detailed structure of the trajectories and address the conjecture from a different perspective. Our analysis will demonstrate that the conclusion reached in [4]—that the Collatz conjecture is true—is indeed valid.

This paper is organized as follows: In Section 2, we will study the structure of Collatz trajectories and their representation as a directed graph. We will also see that we only need to study the trajectories $T([4])$. In Section 3, we will see the structure of trajectories $T([4])$. In Section 4, we will define the probabilístic model. In Section 5, we will study $T([4])$ and the counterexample trajectories. Finally, in Section 6, we will prove the Collatz conjecture.

Notation

1) $\mathbb{Z}$: the set of integers.

2) $\mathbb{N} = \{k \mid k \in \mathbb{Z}, k > 0\}$: the set of natural numbers.

3) $\mathbb{N}_0 = \{k \mid k \in \mathbb{Z}, k \geq 0\}$: the set $\mathbb{N}$ including zero.

4) $[r_i]_6 = \{n \mid n \equiv r_i, \mathrm{mod}.6, n \in \mathbb{N}\}$; or also. $[r_i] = \{n \mid n = 6k + r_i, k \in \mathbb{N}_0\}$, $0 \leq r_i < 6$, where we do not explicitly state the value modulo 6.

5) $T([r_i])$ is any trajectory that starts at n, with $n \in [r_i]$.

6) $T([0]) = \{[0],[0],[3],[4],[2],[4],[2],[1],[4],[5],[4],\cdots\}$, is a generic expression of $T(k)$, with $k \in [0]$ and $[r_i]$ represents any number of that residue class.

7) $L5$ = A simplified way to refer to the loop, $[4] \to [5] \to [4]$.

8) $L2$ = A simplified way to refer to the loop, $[4] \to [2] \to [4]$.

9) $L1$ = A simplified way to refer to the loop, $[4] \to [2] \to [1] \to [4]$.

10) $L0$ = A simplified way to refer to the loop, $[0] \to [0]$.

11) $F5$ = A simplified way to refer to the multiplier factor of $L5$.
12) $F2$ = A simplified way to refer to the multiplier factor of $L2$.
13) $F1$ = A simplified way to refer to the multiplier factor of $L1$.
14) F_O = A simplified way to refer to the overall multiplier factor.

2. Structure of Collatz Trajectories

In this Section, we will examine the definition of a trajectory and its structural elements, and how to represent them as a directed graph, using modulo 6 arithmetic, according to [4].

Finally, we prove that we only need to study the trajectories $T([4])$, see Notation (5).

Definition 3 (Collatz Trajectory).

For $k \in N$, the trajectory of k is the sequence obtained by successively applying the function C to k. We will denote it by $T(k)$. Hence,

$$T(k) := \left\{C^n(k)\right\}_{n=0}^{\infty}, \text{ where } C^n(k) := \begin{cases} k & \text{if } n = 0, \\ \overbrace{C \circ C \cdots \circ C}^{n \text{ times}}(k) & \text{for } n \geq 1. \end{cases}$$

And we can also write $T(k) = \{n_1, n_2, n_3, \cdots, n_j, \cdots\}$, where $n_1 = k$ and n_j is the number of position j in the sequence, with $j = 1, 2, 3, \cdots$.

At this juncture, I would like to recall Proposition 1 from my previous article on the Collatz Conjecture, as it serves as a fundamental reference point for the proof.

Proposition 1. (E.A. Diarte-Carot [4]. The reference to the Proof).

Let be $n \in T(k)$, *and let* $T(n)$ *be the trajectory starting from* n, *if* $T(n)$ *satisfies the conjecture, then so does* $T(k)$.

Proof. By the definition of trajectory, 3, if $n \in T(k)$, $\exists\, i \in \mathbb{N}$ such that $C^i(k) = n$ and, from n, it follows that $C^{i+1}(k) = C(n)$ and $C^{i+2}(k) = C^2(n)$ and $C^{i+3}(k) = C^3(n)$ and so on.

Then, from n, $T(k)$ and $T(n)$ are equal. Thus, if $T(n)$ satisfies the conjecture, then $1 \in T(n)$ and also $1 \in T(k)$. Therefore, $T(k)$ also satisfies the conjecture. □

Now, by Definition 2, we can state the following.

Remark 1 (Condition to satisfy the conjecture).

$T(k)$, with $k \in N$, *satisfies the Collatz conjecture* whenever $1 \in T(k)$.

Remark 2 (Directed graph as a Collatz trajectory space).

Collatz trajectories are continuous movements within the directed graph shown in **Graph 1**.

According to article [4], the transition of each residue class can be calculated by applying the Collatz function to the corresponding general term, as outlined in Notation (4):

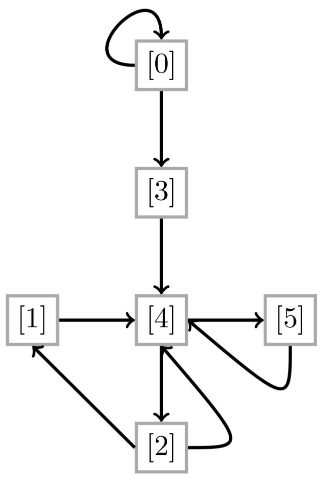
1) $[0]_6 \to [0]_6 \cup [3]_6$.
2) $[1]_6 \to [4]_6$.

3) $[2]_6 \to [1]_6 \cup [4]_6$.

4) $[3]_6 \to [4]_6$.

5) $[4]_6 \to [2]_6 \cup [5]_6$.

6) $[5]_6 \to [4]_6$.



Graph 1. A directed graph in which all trajectories represent continuous motion.

The directed graph shows four loops:

$$L0 = [0] \to [0]; \quad L1 = [4] \to [2] \to [1] \to [4].$$

$$L2 = [4] \to [2] \to [4]; \quad L5 = [4] \to [5] \to [4].$$

The directed graph also shows that all possible trajectories correspond to one of these 9 patterns.

Definition 4 (Trajectory Patterns).

The trajectory patterns are as follows:

1) $T([0]) = [[0], \cdots, L0(m \text{ times}), [3], [4], \cdots] = [[0], \cdots, L0(m \text{ times}), [3], T([4])]$.

Note that the loop $L0 = [0] \to [0]$ can only be repeated a finite number of times, since every $k \in [0]$ can be factored as $k = 2^m \times 3 \times (\text{odd number})$ and it is not repeated after m repetitions, then, the trajectories transition to $[3]$.

2) $T([3]) = [[3], [4], \cdots] = [[3], T([4])]$.

3) $T([1]) = [[1], [4], \cdots] = [[1], T([4])]$.

4) $T([5]) = [[5], [4], \cdots] = [[5], T([4])]$.

5) $T([2]) = [[2], [4], \cdots] = [[2], T([4])]$.

6) $T([2]) = [[2], [1], [4], \cdots] = [[2], [1], T([4])]$.

7) $T([4]) = [[4], [5], [4], \cdots] = [L5, T([4])]$.

8) $T([4]) = [[4], [2], [4], \cdots] = [L2, T([4])]$.

9) $T([4]) = [[4],[2],[4],[4],\cdots] = [L1, T([4])]$.

In these patterns, $[r_i]$ does not represent the entire class of residues, but rather a number within that class; see Notation (6).

Proposition 2 (The key to the proof).

We only need to study the trajectory structure $T([4])$.

Proof. By Definition 4, all trajectories $T([r_i]) = T(k) = \{n_1, n_2, n_3, \cdots, n_j, \cdots\}$, with $n_1 = k$, reach $n_j \in [4]$ for the first time, they end up following a $T([4])$ trajectory.

So, if all trajectories $T([4])$ decay to 1, then, by Proposition 1, all Collatz trajectories, $T(k)$, decay to 1 and satisfy the conjecture. □

3. The Structure of the Trajectories *T*([4])

From this Section onward, all of the studied trajectories will be $T(n)$, where $n \in [4]$. If none of these trajectories are counterexamples, then, by Proposition 2, the conjecture holds.

Remark 3. (Estructure of $T([4])$)

Figure 1 shows the elementary structure with which all $T([4])$ trajectories begin.

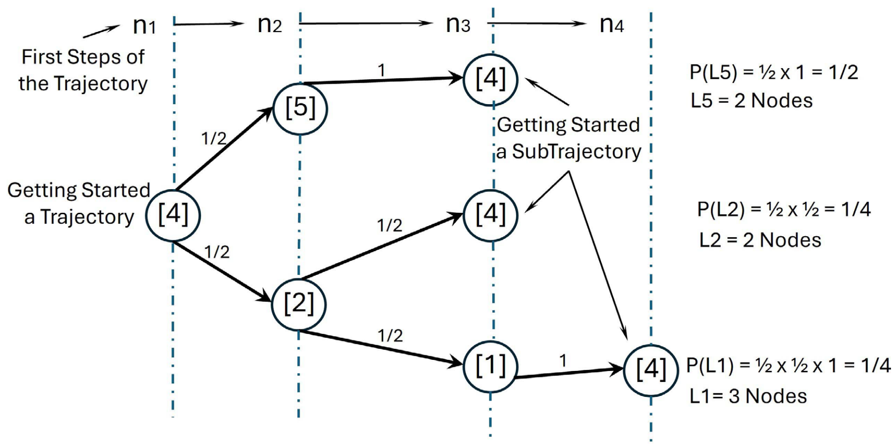


Figure 1. Initial structure of the *T*([4]) Collatz trajectory.

This structure will be repeated throughout the trajectory, and its most significant elements are summarized as follows:

A) Half of all $T([4])$ trajectories begin with an $L5 = [4] \to [5] \to [4]$ loop, and each of them is made up of two trajectory numbers (2 nodes).

B) A quarter of them with an $L2 = [4] \to [2] \to [4]$ loop, and each of them is made up of two trajectory numbers (2 nodes).

C) The remaining quarter with an $L1 = [4] \to [2] \to [1] \to [4]$ loop, and each of them is made up of three trajectory numbers (3 nodes).

D) At the end of each loop, a new subtrajectory $T([4])$ begins, and the process repeats.

Therefore, any $T([4])$ trajectory is a sequence of loops L5, L2, and L1. Or, in other words, it is a sequence of subtrajectories $T([4])$.

E) The probability that a trajectory will start with a given loop is:

$$P(L5)=\frac{1}{2},\ P(L2)=\frac{1}{4} \text{ and } P(L1)=\frac{1}{4}.$$

4. The Probabilistic Model

The probabilistic model defined below is based on the elementary tree structure with three branches shown in **Figure 1**.

Any trajectory $T([4])$, which is the one we need to study, see Proposition 2, is a random and independent sequence of this structure, executing, also randomly, one of the three branches.

Definition 5. (Elementary Probabilistic Structure)

This is a definition, by description, of the Elementary Probabilistic Structure.

From a probabilistic point of view, **Figure 1** presents the following elements:

1) In the first stage, n_1

a) a Bernoulli trial, as we will see later, with sample space $\{[5],[2]\}$ and probabilities $(1/2,1/2)$.

2) In the second stage, n_2

a) a new Bernoulli trial with sample space $\{[1],[2]\}$ and probabilities $(1/2,1/2)$ if branch $[2]$ is followed.

b) a transition $[5]\rightarrow[4]$ with probability 1, therefore, with no probabilistic impact.

3) In the third stage, n_3

a) in addition to ending what we will call loop $L5$ and initiating a new elementary structure.

b) a transition $[2]\rightarrow[4]$ with probability 1/2, which ends loop $L2$ and initiates a new elementary structure.

c) a transition $[2]\rightarrow[1]$ with probability 1/2.

4) In the fourth stage, n_4

a) a transition $[1]\rightarrow[4]$ with probability 1, therefore, with no probabilistic impact, but which ends what we will call loop $L1$ and initiates a new elementary structure.

$L5, L2$ and $L1$ as outlined in Notations (7), (8), and (9).

Definition 6. (Bernoulli Trial and Binomial Distribution)

Let $\in$ be an experiment and let A be an event associated with $\in$. Let $P(A)=p$, the probability of the event A occurring, and $P(\bar{A})=(1-p)$, the probability of $\bar{A}$. Performing this experiment only once constitutes a Bernoulli Trial.

A coin toss is an example of a Bernoulli trial. Event A is defined as getting "heads" with probability $P(A)=p$.

The random variable X defined, for example, as "gets heads" k times, associated with n independent repetitions of Bernoulli Trials, with the same probability for all repetitions, has a Binomial Distribution: $B(n, p)$. See [6].

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}.$$

Remark 4. Experiments involve the probabilistic model.

The proposed probabilistic model contains two identical experiments: The experiments $\in_1$ in the first stage and $\in_2$ in the second stage.

These experiments, both $\in_1$ and $\in_2$, are entirely similar to the coin toss in Definition 6, as follows:

- In a coin toss, there is an object (the coin) with two inherent characteristics: "Heads" and "Tails".
  In these experiments $\in$, we have an object, a number $n = 6k + r$ (see Notation (4)), with a parameter, k, that characterizes the number as "even" or "odd".
- The procedure involves tossing the coin and noting whether the result is "heads" or "tails".
  When we apply the Collatz function to n and note the result, which is either "even" or "odd".
- These experiments, coin toss or applying the Collatz function to n, are repeated N times with fixed probabilities p and 1/2, respectively, and the repetitions are independent.
  In particular, for the case at hand, $\in_2$ acts on $C(n)$, regardless of its numerical value, in the same way that $\in_1$ acts on n.
- Therefore, the Binomial Distribution, $B(n, p)$, applies.

So, repeating the elementary structure of Remark 3, N times. The experiment $\in_1$ has a sample spaces $\{[5],[2]\}$ and $\in_2$ has $\{[1],[2]\}$, as outlined in Definition 5. The binomial distribution corresponding to the random variables $L5, L2$, and $L1$ will be:

- $B(N, 1/2)$ for $L5$.
- $B(N, 1/2)$, followed by $B(N', 1/2)$, for $L2$ and $L1$. Here, N' is the number of times $[2]$ was obtained in experiment $\in_1$.

Note that, as outlined in Remark 3:

N_A = the number of times $L5 = [5]$ occurs, and N' = the number of times $[2]$ occurs in N repetitions of trial $\in_1$.

N_B be the number of times $L2$ occurs, and N_C the number of times $L1$ occurs after N' repetitions of trial $\in_2$.

Then, $N = N_A + N' = N_A + N_B + N_C$.

5. The Trajectories $T([4])$ and the Counterexample Trajectories

From this point onward, we will focus our study on trajectories, $T(n)$, with $n \in [4]$ and $n > 2.95147 \times 10^{20}$, see [3]. This will streamline the process and allow us to simplify expressions at a later stage.

Remark 5. The effect of the elementary structure on the $T([4])$.

1) After implementing the elementary structure of the model, its effect on the numerical values of the trajectory $T([4])$ is always one of the following sequences of terms,

$$\text{either } n,\frac{n}{2},\frac{n}{4},\cdots \text{ or } n,\frac{n}{2},\frac{n}{4},3\frac{n}{4}+1,\cdots \text{ or } n,3n+1,\frac{3n+1}{2},\cdots$$

This corresponds to performing the loops. to $L2$, $L1$ and $L5$, respectively.

We observe that $L2$ and $L1$ reduce n, while $L5$ increases it.

2) The increase or decrease in the value of n can be evaluated as a multiplier factor.

Thus,

$L5$ has a factor $F5=\frac{3}{2}+\frac{1}{2n}\to F5=\frac{3}{2}$ $(n\gg 1)$,

$L2$ has a factor $F2=\frac{1}{4}$,

$L1$ has a factor $F1=\frac{3}{4}+\frac{1}{n}\to F1=\frac{3}{4}$ $(n\gg 1)$.

3) The sequential application of loops along a trajectory results in a successive multiplication of factors.

Thus, if we have an application sequence with N_A *times* $L5$, N_B *times* $L2$, N_C *times* $L1$, then $n\to n_j$, with $n_j=n\cdot F_O$; where F_O is the overall multiplier factor

$$F_O=\left(\frac{3}{2}\right)^{N_A}\left(\frac{1}{4}\right)^{N_B}\left(\frac{3}{4}\right)^{N_C} \text{ and } j=2N_A+2N_B+3N_c+1.$$

Note that F_O is a product of real numbers, which has the associative and commutative properties; therefore, the exact position of the loops in that sequence is completely irrelevant, only the value of factors and the number of times each loop appears matter.

Now, we will begin by defining a counterexample trajectory and the criteria for identifying whether a trajectory is actually a counterexample to the conjecture.

Corollary 1. *A counterexample trajectory.*

A counterexample trajectory, $T(n)$, *is one that does not converge to* 1, *that is,* $1\notin T(n)$. *So, it will be a divergent sequence.*

Proof. This assertion is a direct consequence of the definition of conjecture 2 and of Remark 1. □

Proposition 3. (Criteria for $T([4])$ trajectory as a counterexample).

For a trajectory $T([4])$ to be a counterexample to the Collatz conjecture, it must satisfy the following:

1) *It is a sequence of infinitely many loops* $L5, L2$, *and* $L1$. *So,* $N_A+N_B+N_C\to\infty$; *where* $N_A=\#L5\to\infty$, $N_B=\#L2\to\infty$ *and* $N_C=\#L1\to\infty$ *are random variables.*

2) *All elements at the end of any loop in the sequence,* $n_j\in[4]$, *must be greater than the initial element,* n_1 *of* $T([4])$.

This implies that $F_O = \left(\frac{3}{2}\right)^{N_A} \left(\frac{1}{4}\right)^{N_B} \left(\frac{3}{4}\right)^{N_C} > 1$

Proof.

1) To prove this point, it suffices to recall that every $T([4])$ is an infinite sequence of loops $L5, L2$, and $L1$, Point D of Remark 3. Additionally, for it to serve as a counterexample, the number of loops of each type must be infinite.

In a trajectory that satisfies the conjecture, only the $L1$ loops, when they first reach 1, repeat indefinitely, as outlined in Definition 2.

2) We have seen that the structure of any trajectory starting at $n \in [4]$ becomes a random succession of loops ending at some $n_j \in [4]$, hence $n_j \in T(n)$.

Suppose, by way of contradiction, that n is the smallest term, belonging to $[4]$, that starts a trajectory, $T(n)$, that does not satisfy the Collatz conjecture.

Note that under this assumption, no term in $T(n)$ satisfies the Collatz conjecture, according to Proposition 1 and Corollary 1. So, $T(n_j)$ does not satisfy the conjecture and $n_j \in [4]$.

But, if $n_j < n$ then the hypothesis is contradicted. So, n_j must always be greater than n.

Furthermore, this implies that F_O must be greater than 1, since $n_j = n \cdot F_O$

□

6. Proof of the Collatz Conjecture

Let's prove that no $T([4])$ trajectory is a counterexample to the Collatz conjecture.

To this end, we will show that no trajectory $T(n) = \{n, n_1, n_2, n_3, \cdots, n_j, \cdots\}$, with $n \in [4]$, satisfies the second criterion of Proposition 4, that is, we will show that all these trajectories reach a $n_j = n \cdot F_O$ less than n. So,

$$F_O = \left(\frac{3}{2}\right)^{N_A} \left(\frac{1}{4}\right)^{N_B} \left(\frac{3}{4}\right)^{N_C} \text{ must be less than 1.}$$

The problem is determining the value of N_A, N_B and N_C.

Note that we try to determine "a priori" how many of the results of the N repetitions will be favorable to $L5$, how many to $L2$ and how many to $L1$, knowing their respective probabilities (1/2, 1/4, 1/4).

The key to this matter lies in a fundamental concept: Determining an equitable distribution of profits between two parties a priori. This issue began to be addressed mathematically in the mid-17th century, around 1654.

Our objective is to distribute the results of repeating N random processes among the three loops in a fair and "a priori" manner.

The mathematical solution was based on the principle: "*The value of a future gain must be directly proportional to the possibility of getting it*".

In updated mathematical language, this would be expressed as follows: It is essential to recognize that the potential value of a future gain is directly proportional to the likelihood of its realization.

In 1657, see [7], a solution based on the same principle was extended to 3 or more parties.

Finally, in 1814, see [8], the concept of the expected value of a random variable was explicitly defined as the answer to this question.

Definition 7 (P.L. Meyer [6]. Expected value).

Let X be a discrete random variable with values X_i *and probabilities p(* X_i *). The expected value is*:

$$E(X)=\sum_{i=1}^{N} X_i p(X_i).$$

Remark 6. Expected Value of the Binomial variable.

Let X be a random variable with a binomial probability distribution, $B(n, p)$, its expected value is $E(X)=np$. See [6].

Proposition 4 (Values of N_A, N_B and N_C).

The N_A, N_B and N_C *are the number of times the loops* $L5$, $L2$, *and* $L1$ *appear in* N *repetitions of the elementary structure. So, assigning their expected value, we get*:

$$N_A=E(L5)=N/2 \quad N_B=E(L2)=N/4 \quad N_C=E(L1)=N/4$$

Proof.

According to Remark 4 of the probabilistic model,

1) the random variables $L5$ has a binomial distribution, $B(N,1/2)$. So, by Remark 6, $N_A=E(L5)=N/2$.

2) The variables $L2$ and $L1$ have a final binomial distribution, $B(N',1/2)$. So, by Remark 6, $N_B=E(L2)=N/4$ and $N_C=E(L1)=N/4$.

□

Theorem 1. (Final Theorem).

No trajectory $T([4])$ is a counterexample to the Collatz conjecture. So, the Collatz conjecture is true.

Proof. Bringing the values of the Proposition 4 to,

$$F_O=\left(\frac{3}{2}\right)^{N_A}\left(\frac{1}{4}\right)^{N_B}\left(\frac{3}{4}\right)^{N_C}$$

We get:

$$F_O=\left(\frac{3}{2}\right)^{\frac{N}{2}}\left(\frac{1}{4}\right)^{\frac{N}{4}}\left(\frac{3}{4}\right)^{\frac{N}{4}}=0.80592745^N \ll 1$$

The Point (2) of Proposition 4 is not satisfied, then all trajectories $T([4])$ hold the conjecture and, by Proposition 2, the Collatz conjecture is true.

Furthermore, as N approaches infinity, F_O approaches zero and $T([4])$ decreases to 4, the minimum of the residue class $[4]$. From that point on, the trajectory repeats the loop $4 \to 2 \to 1 \to 4$ indefinitely, thus fulfilling the Collatz conjecture. □

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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