Scientific Research Publishing

Journal of Applied Mathematics and Physics, 2026, 14(9), 3367-3380
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Controlling Nonlinear Signal Amplification in Superconductors via Modulational Instability

Paldou Yaya Alexis[1,2*], Essebe Eba David[1,2], Kamdem Tamwa Romuald[3], Kol Guy Richard[1,2]

[1]Laboratory of Mining and Energy Resources Engineering, Advanced School of Mines Processing and Energy Resources, University of Bertoua, Bertoua, Cameroon
[2]Technology and Innovation Support Center, Advanced School of Mines Processing and Energy Resources, University of Bertoua, Bertoua, Cameroon
[3]Laboratory of Mechanics and Complex Systems, Department of Physics, Faculty of Science, University of Yaounde I, Yaounde, Cameroon
Email: *paldyayaalexis@gmail.com

How to cite this paper: Alexis, P.Y., David, E.E., Romuald, K.T. and Guy Richard, K. (2026) Controlling Nonlinear Signal Amplification in Superconductors via Modulational Instability. *Journal of Applied Mathematics and Physics*, **14**, 3367-3380.
https://doi.org/10.4236/jamp.2026.149168

Received: June 30, 2026
Accepted: September 11, 2026
Published: September 14, 2026

Abstract

We investigate nonlinear signal propagation in superconducting media using the complete complex Ginzburg-Landau equation. A perturbative modulational instability framework is employed to derive analytical expressions for the dispersion relation and growth rate. The analysis reveals well-defined instability domains where small perturbations are exponentially amplified, giving rise to high-amplitude localized breather structures. Group velocity dispersion and phase self-modulation govern the transition between stable and unstable regimes. The growth rate exhibits maxima corresponding to optimal conditions for energy localization and signal amplification; outside these regions, the system supports stable, low-amplitude periodic waves. This analytical framework opens perspectives for the design of superconducting devices, photonic technologies, and advanced signal processing applications.

Keywords

Modulational Instability, Superconductors, Nonlinear Waves, Signal Dynamics, Complex Ginzburg-Landau Equation

1. Introduction

Superconducting media provides a rich framework for the study of nonlinear wave propagation and complex dynamical behaviors. Their theoretical description is commonly based on the complex Ginzburg-Landau equation (CGLE), a universal nonlinear model that arises in a wide range of physical systems, including nonlinear optics, fluid dynamics, plasma physics, and Bose-Einstein condensates [1]-[11]. Owing to its ability to incorporate dispersion, nonlinearity, gain, and dissipation, the CGLE has become a cornerstone for investigating pulse propagation and amplification processes in optical fibers, laser systems, and superconducting structures [12]-[17].

Despite its importance, the CGLE generally does not admit exact analytical solutions for arbitrary parameter values, which makes the prediction of signal dynamics particularly challenging [11] [18]-[20]. As a result, various analytical and semi-analytical approaches have been developed, among which modulational instability (MI) stands out as a powerful tool for examining the stability of continuous wave (CW) solutions and the emergence of localized nonlinear structures such as solitons, breathers, and rogue waves [21]-[31]. Originally identified in hydrodynamics, MI has since been extensively investigated in optical fibers, plasmas, and condensed matter systems, where it governs energy localization and pattern formation [32]-[38].

In nonlinear optical systems, MI plays a central role in pulse breakup, supercontinuum generation, and soliton formation [39]-[43], while in plasmas and Bose-Einstein condensates it is associated with wave collapse and coherent structure formation [44] [45]. The extension of these concepts to superconducting media has attracted increasing attention in recent years due to their potential applications in high-speed communication, quantum devices, and nanoscale energy transport [46]-[51]. However, most existing studies on MI in superconducting systems rely on simplified versions of the CGLE or on coupled models that neglect important physical effects such as nonlinear gain saturation, higher-order nonlinearities, and normalized linear gain contributions [52] [53]. These simplifications may limit the accuracy of the predicted dynamics and obscure key mechanisms governing instability development. Furthermore, conventional perturbation techniques used in MI analysis are not always suitable for capturing the full complexity of superconducting environments [54]-[56].

Despite these significant advances, several important issues remain insufficiently explored. Most previous studies have primarily focused on deriving modulational instability criteria or investigating localized nonlinear structures without systematically examining how the combined effects of dispersion and nonlinear phase modulation govern the amplification of unstable perturbations in superconducting media. Furthermore, the identification of parameter regions leading to maximum instability growth and their implications for nonlinear signal amplification have received limited attention. These unresolved issues motivate the present study.

In this work, we derive explicit analytical expressions for the modulation instability dispersion relation and growth rate, establish the stability boundaries in the (α, β) parameter plane, and demonstrate how these parameters can be tuned to control nonlinear signal amplification in dissipative superconducting media. The obtained results provide useful physical insights into the design of superconducting systems requiring controllable amplification and stable nonlinear wave propagation.

2. Model and Modulational Instability Analysis

Among the various nonlinear evolution equations proposed to describe wave propagation in dissipative media, the complex Ginzburg-Landau equation provides one of the most comprehensive macroscopic models because it simultaneously accounts for dispersion, Kerr nonlinearity, gain, dissipation and nonlinear saturation. These combined mechanisms are essential for describing the nonequilibrium dynamics of superconducting systems, where the competition between amplification and losses strongly influences the stability of nonlinear excitations. [8]-[10]

$$i\frac{\partial\psi}{\partial x}-\left(\frac{\alpha}{2}+iF\right)\frac{\partial^2\psi}{\partial t^2}+\left(\beta-i\gamma\right)\left|\psi\right|^2\psi+i\mu\left|\psi\right|^4\psi-iG\psi=0, \tag{1}$$

The governing equation (Equation (1)) describes the evolution of the slowly varying envelope of a nonlinear excitation propagating in a dissipative superconducting medium operating close to the superconducting transition, where the complex Ginzburg-Landau formalism provides an appropriate macroscopic description. In this normalized model, the group-velocity dispersion coefficient α governs the temporal spreading or compression of the propagating wave packet, whereas the self-phase modulation coefficient β characterizes the nonlinear phase shift induced by the intensity-dependent response of the superconducting condensate. The parameter F accounts for the spectral filtering associated with the finite gain bandwidth of the medium, while γ represents the nonlinear amplitude modulation responsible for dissipative effects. The quintic coefficient μ models the saturation of the nonlinear gain, preventing unlimited field growth at high intensities, and the parameter G denotes the normalized linear gain required to compensate intrinsic losses. Consequently, the interplay among these physical mechanisms determines whether small perturbations remain stable or evolve into strongly localized nonlinear structures through modulational instability.

It is worth emphasizing that the adopted complex Ginzburg-Landau equation extends the conservative nonlinear Schrödinger equation by incorporating dissipative mechanisms that are unavoidable in realistic superconducting devices. In particular, the combined effects of spectral filtering, linear gain, nonlinear gain saturation and cubic nonlinear interactions enable the model to describe the competition between amplification and dissipation governing the evolution of superconducting wave packets. Such a generalized formulation therefore provides a more realistic framework for investigating modulational instability and nonlinear signal amplification than conservative models, especially in driven superconducting systems operating under nonequilibrium conditions.

We perform a linear stability analysis of the superconducting medium by introducing infinitesimal perturbations around the continuous-wave solution. This methodology has been successfully applied in nonlinear optical systems, plasmas, Bose-Einstein condensates, and exciton-polariton condensates to investigate modulational instability and the formation of localized nonlinear structures.

The continuous wave (CW) solution given by the relation

$$\psi_0\left(x,t\right)=a\mathrm{e}^{i(bx+ct)}, \tag{2}$$

verifies the system (1) under the constraint

$$\beta a^2-b+\frac{\alpha}{2}c^2+i\left(-\gamma a^2+\mu a^4+Fc^2-G\right)=0, \tag{3}$$

Since Equation (3) is a complex-valued constraint, its real and imaginary parts must vanish independently. Therefore, the continuous-wave solution exists only if ($-b+\beta a^2+\frac{\alpha}{2}c^2=0$), and ($-\gamma a^2+\mu a^4+Fc^2-G=0$). The first condition determines the propagation constant (b) as a function of the wave amplitude (a), the dispersion parameter (α), the modulation frequency (c), and the self-phase modulation coefficient (β). The second condition expresses the balance between nonlinear losses (γ), gain saturation (μ), spectral filtering (F), and linear gain (G). Consequently, the continuous-wave background used in the modulational instability analysis exists only for parameter combinations satisfying these two relations simultaneously. These conditions define the admissible operating regime of the superconducting medium and ensure the physical consistency of the stability analysis. For compactness of notation, the quantities $\left(A_1\right)$, $\left(A_2\right)$, $\left(B_1\right)$, and $\left(B_2\right)$ introduced later in the dispersion relation are retained in their general form. Under the continuous-wave existence conditions given above, $\left(A_2\right)$ and $\left(B_2\right)$ vanish identically; however, they are kept in the analytical expressions to preserve the compact structure of the derivation.

We add perturbations $\varepsilon\left(x,t\right)$ with an infinitesimal amplitude χ in order to study the modulation instability of the continuous wave solution Equation (2), subject to constraint Equation (3),

$$\psi\left(x,t\right)=\left(a+\chi\varepsilon\left(x,t\right)\right)\mathrm{e}^{i(bx+ct)}, \tag{4}$$

which is followed by the derivation of the linearized equation for the perturbation:

$$\begin{aligned}&-\left(\frac{\alpha}{2}+iF\right)\varepsilon_{tt}-2ic\left(\frac{\alpha}{2}+iF\right)\varepsilon_t+i\varepsilon_x\\&+\left[-b+c^2\left(\frac{\alpha}{2}+iF\right)+2a^2\left(\beta-i\gamma\right)+3i\mu a^4-iG\right]\varepsilon\\&+\left[a^2\left(\beta-i\gamma\right)+2i\mu a^4\right]\varepsilon^*=0,\end{aligned} \tag{5}$$

Relevant solutions to Equation (5) we take $\varepsilon\left(x,t\right)$ in the form:

$$\varepsilon\left(x,t\right)=\alpha_1\cos\left(kx-\omega t\right)+i\alpha_2\sin\left(kx-\omega t\right), \tag{6}$$

where α_1 and α_2 are real amplitudes, the substitution of Ansatz Equation (6) into Equation (5) leads to the following dispersion relation for the perturbations

$$\begin{cases}\omega^2\left(\dfrac{\alpha}{2}A_1+FA_2-\dfrac{\alpha}{2}B_2-B_1F\right)=-A_1A_2+B_1B_2\\ k=-c\alpha\omega+\dfrac{\left(F\omega^2+A_1\right)\left(\dfrac{\alpha}{2}\omega^2+A_2\right)}{2cF\omega}\end{cases}, \tag{7}$$

with $A_1 = Fc^2 - 3\gamma a^2 + 5\mu a^4 - G$, $A_2 = -b + \beta a^2 + \frac{\alpha}{2}c^2$, $B_1 = -b + 3\beta a^2 + \frac{\alpha}{2}c^2$, $B_2 = Fc^2 - \gamma a^2 + \mu a^4 - G$. It follows from Equation (7) that we know the frequencies w is complex when the following criterion is verified

$$\left(\frac{\alpha}{2}A_1 + FA_2 - \frac{\alpha}{2}B_2 - B_1F\right)\left(-A_1A_2 + B_1B_2\right) \prec 0, \tag{8}$$

thus, the phenomenon of modulational instability is observed in superconductor media under the criterion Equation (8). We obtain the wave number and frequency pairs corresponding to the breather wave function of the envelope of the signal propagating in the superconductor medium

$$\begin{cases} \omega_{1,2} = \pm\sqrt{\dfrac{-A_1A_2 + B_1B_2}{\frac{\alpha}{2}A_1 + FA_2 - \frac{\alpha}{2}B_2 - B_1F}} \\ k_{1,2} = -c\alpha\omega_{1,2} + \dfrac{\left(F\omega_{1,2}^2 + A_1\right)\left(\frac{\alpha}{2}\omega_{1,2}^2 + A_2\right)}{2cF\omega_{1,2}} \end{cases}, \tag{9}$$

with $\omega_1 = \omega_+ ; \omega_2 = \omega_-$.

Under the constraint Equation (8) the localized signal in the superconductor media takes the form:

$$\psi_{1,2}(x,t) = \left[a + \chi\left(\alpha_1 \cos\left(k_{1,2}x - \omega_{1,2}t\right) + i\alpha_2 \sin\left(k_{1,2}x - \omega_{1,2}t\right)\right)\right]\mathrm{e}^{i(bx+ct)}, \tag{10}$$

The modulational instability growth rate is defined by the relation:

$$\Gamma = 2\left[\left|\mathrm{Im}(\omega)\right| + \left|\mathrm{Im}(k)\right|\right]. \tag{11}$$

The growth rate defined in Equation (11) accounts for both temporal and spatial amplification mechanisms. The imaginary part of the perturbation frequency, $\mathrm{Im}(\omega)$, characterizes the temporal growth of disturbances, while the imaginary part of the wave number, $\mathrm{Im}(k)$, describes their spatial amplification during propagation. Since the complete complex Ginzburg-Landau equation represents a dissipative medium where instability can develop simultaneously in space and time, both contributions are included in the definition of Γ to provide a global measure of the instability strength.

3. Dynamical Analysis of the Signal

In this section we will analyze the evolution of the signal emitted by superconductors Equation (10) solution of the equation Equation (1) following parameter α first and parameter β second; the other parameters of the system and construction constants fixed; which will be accompanied for each of these cases by the analysis of the diagram energy gain.

The baseline parameter values adopted in this section correspond to a normalized reference configuration chosen to emphasize the influence of the dispersion parameter α and the self-phase modulation parameter β. Additional tests performed with moderate variations of γ, μ, *F*, and G show that these parameters mainly affect the quantitative values of the instability thresholds and growth rates, while the overall qualitative behavior remains unchanged. In particular, the existence of stable and unstable propagation regimes, the formation of localized breather structures, and the occurrence of amplification peaks are preserved.

3.1. Study of the Signal as a Function of the Parameter α

The modulational-instability analysis is performed using the following parameter values: $\beta = 0.1$, $\gamma = 0.1$, $F = 0.1$, $\mu = 0.1$, $G = 0.1$, $a = 0.1$, $A = 0.1$, $B = 0.1$, $\chi = 0.0001$. Under these conditions, we obtain $\forall \alpha \in [-10;10]$ the relation $\alpha \notin]-0.37;0[$ translating the criterion Equation (8).

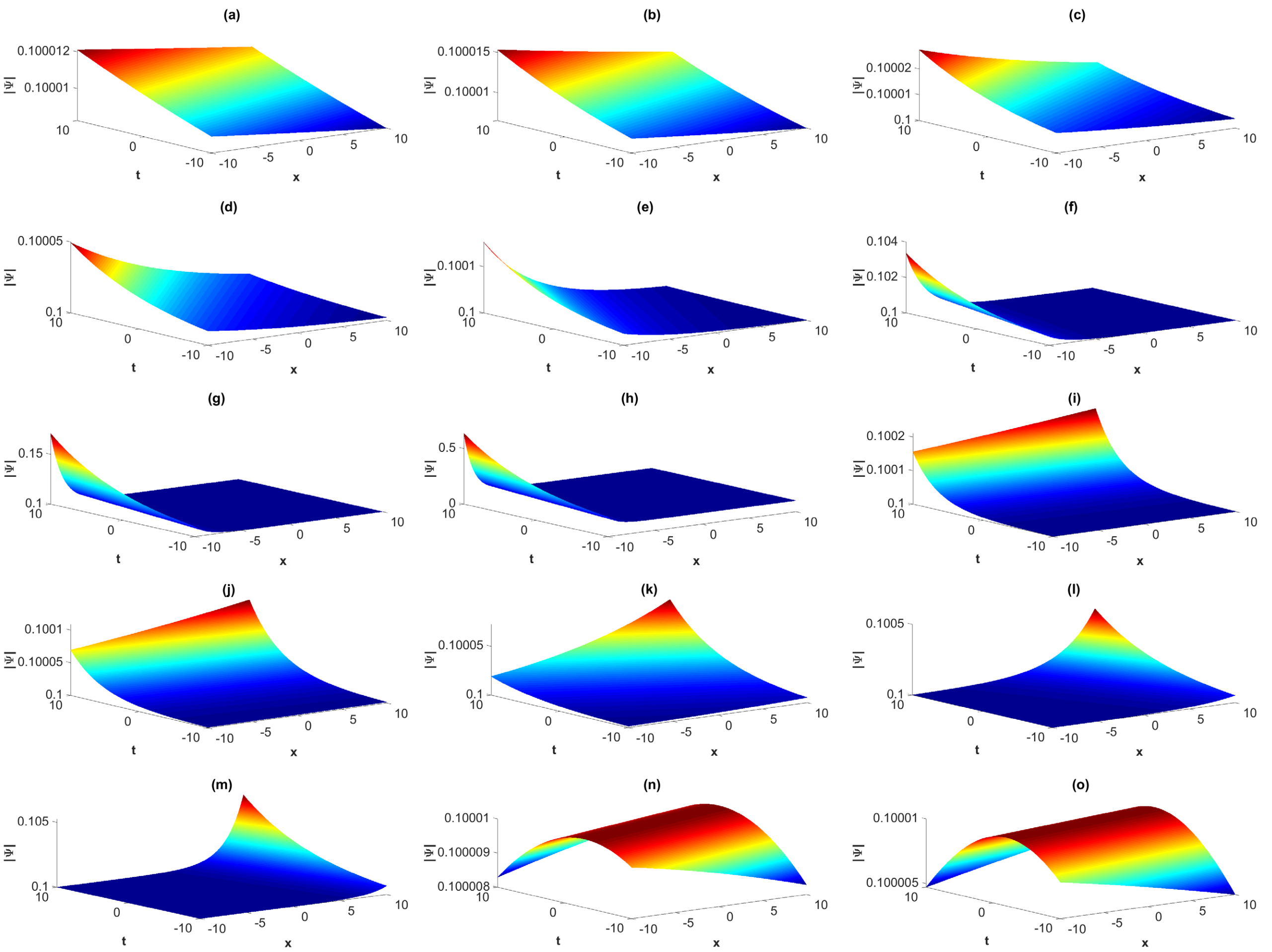

Figure 1. Evolution of the nonlinear signal envelope in superconducting media for different values of the group velocity dispersion parameter α with $\omega = \omega_1$; (a) $\alpha = -0.37$, (b) $\alpha = -0.40$, (c) $\alpha = -0.60$, (d) $\alpha = -1.00$, (e) $\alpha = -2.00$, (f) $\alpha = -5.00$, (g) $\alpha = -8.00$, (h) $\alpha = -10.00$, (i) $\alpha = 0.05$, (j) $\alpha = 0.1$, (k) $\alpha = 0.5$, (l) $\alpha = 2.50$, (m) $\alpha = 5.00$, (n) $\alpha = -0.30$, (o) $\alpha = -0.20$.

Figure 1 illustrates the evolution of the signal propagating in the superconducting medium for different values of the group-velocity dispersion parameter α. When α satisfies the modulational instability criterion (Equation (8)), the signal exhibits a strongly localized breather-type structure, characterized by a high amplitude and a strong spatial concentration of energy. This reflects an efficient amplification of the initial perturbations, resulting from the constructive interplay between dispersion and nonlinearity. In contrast, for values of α lying outside the instability region, the signal adopts a quasi-sinusoidal form with relatively low amplitude. In this regime, the introduced perturbations do not grow significantly, indicating a stable state of the system. The parameter α thus acts as a control parameter for the stability-instability transition, determining whether localized structures form in the superconducting medium.

Figure 2 shows that the signal dynamics are symmetric with respect to the origin of the space-time frame, reflecting the intrinsically symmetric nature of the solutions derived from the dispersion relation. The obtained structures retain the same amplitudes as those associated with (ω_1, k_1), but exhibit spatial or temporal inversion. The system therefore admits conjugate or symmetric solutions, confirming the mathematical consistency of the model and the conservation of the signal's energy properties.

The diagram presented in **Figure 3** highlights the existence of a well-defined gain peak corresponding to a critical value of α ($\alpha \approx -0.37$). At this point, the growth rate of perturbations is maximal, meaning that the system is in a strongly amplified instability regime. On either side of this critical value, the gain gradually decreases until it vanishes in the stable regions. This result shows that there exists an optimal dispersion value at which signal amplification is maximized, which is crucial for applications in signal amplification and transmission.

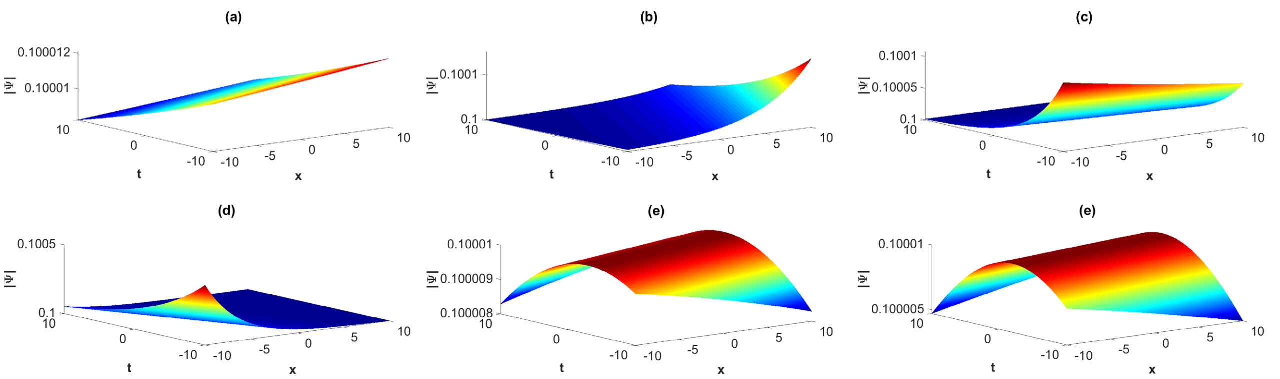

Figure 2. Symmetry of the nonlinear signal envelope corresponding to the two frequency branches ω_1 and ω_2 with $\omega = \omega_2$; (a) $\alpha = -0.37$, (b) $\alpha = -2.00$, (c) $\alpha = 0.1$, (d) $\alpha = 2.50$, (e) $\alpha = -0.30$, (f) $\alpha = -0.20$.

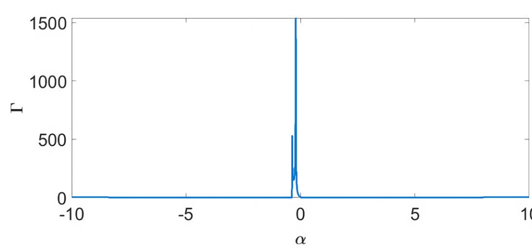

Figure 3. Instability growth rate as a function of the dispersion parameter α.

3.2. Study of the Signal as a Function of the Parameter β

The modulational-instability analysis is performed using the following parameter values: $\alpha = 0.1$, $\mu = 0.1$, $\gamma = 0.1$, $F = 0.1$, $G = 0.1$, $a = 0.1$, $A = 0.1$, $B = 0.1$, $\chi = 0.0001$. For these parameters, we demonstrate that the criterion Equation (8), reduces to the relation $\beta \in [0;5] \quad \forall \beta \in [-5;5]$.

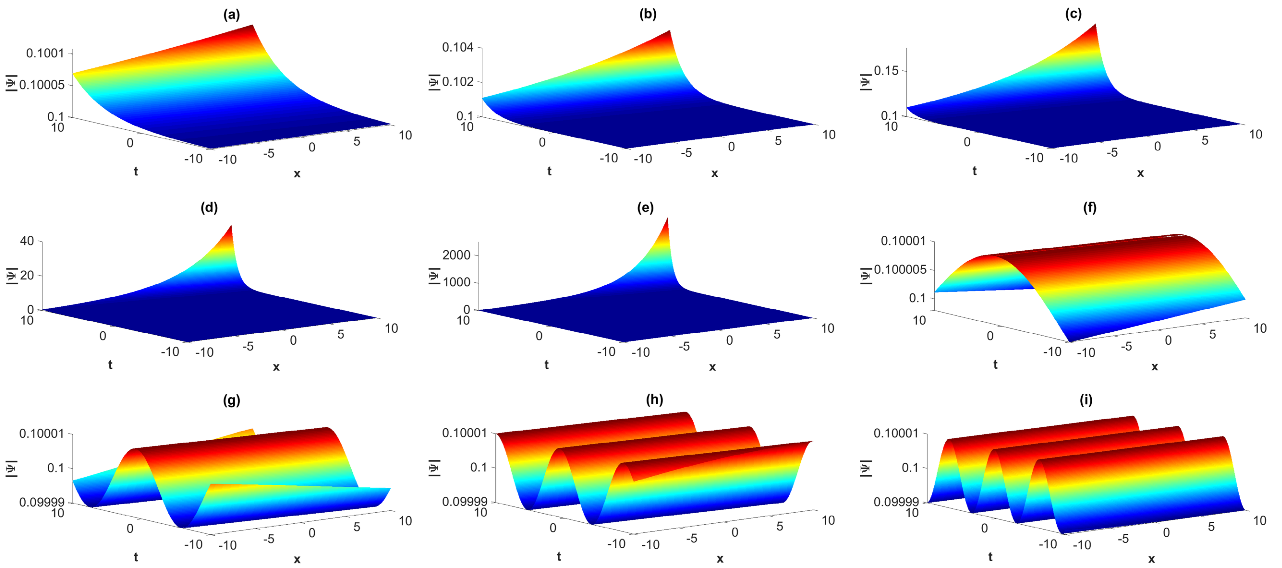

Figure 4. Nonlinear signal evolution for different values of the phase self-modulation parameter β with $\omega = \omega_1$; (a) $\beta = 0.1$, (b) $\beta = 0.7$, (c) $\beta = 1.5$, (d) $\beta = 3.5$, (e) $\beta = 5.0$, (f) $\beta = -0.1$, (g) $\beta = -0.7$, (h) $\beta = -1.5$, (i) $\beta = -4$.

Figure 4 presents an analysis of the effect of the self-phase modulation parameter β on the signal dynamics. When β lies within the instability interval, the signal evolves into high-amplitude localized structures, similar to those observed for α. This confirms that phase nonlinearity plays a determining role in the growth of perturbations. Outside this interval, the signal becomes periodic again with low amplitude, indicating a stable regime. This result shows that the parameter β controls the strength of the nonlinearity and acts as a triggering factor for modulational instability.

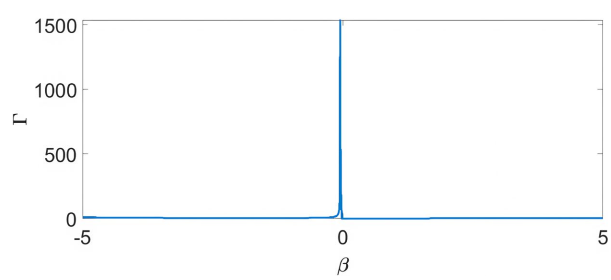

Figure 5. Instability growth rate as a function of the phase self-modulation parameter β.

Figure 5 shows that the instability growth rate reaches its maximum in the vicinity of $\beta \approx 0$, corresponding to the most efficient amplification of infinitesimal perturbations during the early stage of modulational instability. This maximum should not be interpreted as the largest amplitude of the fully developed nonlinear structures shown in **Figure 4**. Indeed, the latter result from the subsequent nonlinear evolution of the system, where gain saturation, dissipation, and nonlinear interactions may significantly modify the final waveform amplitude. Therefore, **Figure 5** characterizes the onset of instability, whereas **Figure 4** illustrates its nonlinear outcome.

4. Discussion

The present results provide a clear physical picture of nonlinear signal dynamics in superconducting media governed by the complete complex Ginzburg-Landau equation. The emergence of modulational instability highlights the intrinsic tendency of the system to amplify small perturbations and redistribute energy into localized structures. Unlike conventional MI analyses of the CGLE, which mainly focus on instability criteria and dispersion relations, the present perturbative framework allows the simultaneous derivation of explicit analytical expressions for the wave envelope, the instability domains, and the associated amplification characteristics. This approach provides a direct connection between instability growth and nonlinear signal amplification in superconducting media.

A central outcome of this study is the identification of a well-defined transition between stable and unstable propagation regimes. In the stable regime, the system supports low-amplitude periodic waves, indicating that dispersion and dissipation dominate the dynamics. In contrast, within instability domains, nonlinear effects overcome dispersive spreading, leading to the formation of high-amplitude breather structures.

The role of key physical parameters is particularly significant. The group velocity dispersion governs the spectral broadening and stability of the wave, while phase self-modulation enhances nonlinear focusing mechanisms. Their interplay determines the onset of instability and the subsequent energy localization process.

The observed peaks in the instability growth rate correspond to optimal amplification conditions, where energy transfer from the background wave to perturbations is maximized. This behavior can be interpreted as a resonance-like mechanism in nonlinear dissipative systems, providing a powerful tool for controlling signal amplification.

Importantly, the use of the complete complex Ginzburg-Landau equation allows us to capture effects that are often neglected in simplified models, such as nonlinear gain saturation and dissipative contributions. This leads to a more accurate and realistic description of superconducting media.

From an application perspective, these findings suggest that superconducting systems can be engineered to operate in controlled instability regimes, enabling efficient signal amplification and energy localization. This has potential implications for photonic devices, nonlinear signal processing, and quantum technologies.

5. Conclusion

In conclusion, this work investigated the non-linear propagation dynamics of signals within superconducting media, based on the complete complex Ginzburg-Landau equation. Through a perturbative approach applied to modulational instability, we established explicit analytical expressions for the dispersion relation and the growth rate. Our results highlight clearly delineated instability regions where small perturbations undergo exponential amplification, leading to the formation of high-amplitude localized structures of the breather type. The transition between stable and unstable regimes is intrinsically governed by group-velocity dispersion and self-phase modulation. The growth rate reaches maxima corresponding to optimal conditions for energy localization and signal amplification; outside these critical domains, the system only supports stable, low-amplitude periodic waves.

Data Availability

The data used are available under request to the corresponding author.

Author Contributions

All authors contributed equally.

Conflicts of Interest

The authors declare no potential conflict of interests/competing interests.

References

[1] Aranson, I.S. and Kramer, L. (2002) The World of the Complex Ginzburg-Landau Equation. *Reviews of Modern Physics*, **74**, 99-143. https://doi.org/10.1103/revmodphys.74.99

[2] Cross, M.C. and Hohenberg, P.C. (1993) Pattern Formation outside of Equilibrium. *Reviews of Modern Physics*, **65**, 851-1112. https://doi.org/10.1103/revmodphys.65.851

[3] Akhmediev, N. and Ankiewicz, A. (2001) Solitons of the Complex Ginzburg—Landau Equation. In: Trillo, S. and Torruellas, W.E., Eds., *Spatial Solitons*, Springer, 311-341. https://doi.org/10.1007/978-3-540-44582-1_12

[4] Kivshar, Y.S. and Malomed, B.A. (1989) Dynamics of Solitons in Nearly Integrable Systems. *Reviews of Modern Physics*, **61**, 763-915. https://doi.org/10.1103/revmodphys.61.763

[5] Yomba, E. and Kofané, T.C. (1999) On Exact Solutions of Modified Complex Ginzburg-Landau Equation. *Physica D*: *Nonlinear Phenomena*, **125**, 105-122. https://doi.org/10.1016/s0167-2789(98)00152-3

[6] Akhmediev, N., Soto-Crespo, J.M. and Town, G. (2001) Pulsating Solitons, Chaotic Solitons, Period Doubling, and Pulse Coexistence in Mode-Locked Lasers: Complex Ginzburg-Landau Equation Approach. *Physical Review E*, **63**, Article ID: 056602. https://doi.org/10.1103/physreve.63.056602

[7] Vansaarloos, W. (2003) Front Propagation into Unstable States. *Physics Reports*, **386**, 29-222. https://doi.org/10.1016/j.physrep.2003.08.001

[8] Malomed, B.A. and Nepomnyashchy, A.A. (1990) Kinks and Solitons in the Generalized Ginzburg-Landau Equation. *Physical Review A*, **42**, 6009-6014. https://doi.org/10.1103/physreva.42.6009

[9] Kodama, Y., Romagnoli, M., Midrio, M. and Wabnitz, S. (1994) Role of Third-Order Dispersion on Soliton Instabilities and Interactions in Optical Fibers. *Optics Letters*, **19**, 165-167. https://doi.org/10.1364/ol.19.000165

[10] Akhmediev, N., Królikowski, W. and Lowery, A.J. (1996) Influence of the Raman-Effect on Solitons in Optical Fibers. *Optics Communications*, **131**, 260-266. https://doi.org/10.1016/0030-4018(96)00283-0

[11] García-Morales, V. and Krischer, K. (2012) The Complex Ginzburg-Landau Equation: An Introduction. *Contemporary Physics*, **53**, 79-95. https://doi.org/10.1080/00107514.2011.642554

[12] Zhan, M., Zou, W. and Liu, X. (2010) Taming Turbulence in the Complex Ginzburg-Landau Equation. *Physical Review E*, **81**, Article ID: 036211. https://doi.org/10.1103/physreve.81.036211

[13] Correia, S. and Figueira, M. (2023) A Note on Bifurcations from Eigenvalues of the Dirichlet-Laplacian with Arbitrary Multiplicity. *Nonlinear Differential Equations and Applications NoDEA*, **30**, Article No. 37. https://doi.org/10.1007/s00030-023-00846-y

[14] Sadaf, M., Akram, G., Arshed, S. and Farooq, K. (2023) A Study of Fractional Complex Ginzburg-Landau Model with Three Kinds of Fractional Operators. *Chaos, Solitons & Fractals*, **166**, Article ID: 112976. https://doi.org/10.1016/j.chaos.2022.112976

[15] Conte, R., Musette, M., Ng, T.W. and Wu, C. (2022) New Solutions to the Complex Ginzburg-Landau Equations. *Physical Review E*, **106**, L042201. https://doi.org/10.1103/physreve.106.l042201

[16] Wellot, Y.A.S. and Nkaya, G.D. (2022) Analytical Solution of the Ginzburg-Landau Equation. *European Journal of Pure and Applied Mathematics*, **15**, 1750-1759. https://doi.org/10.29020/nybg.ejpam.v15i4.4551

[17] Forbes, A., Dudley, A. and McLaren, M. (2016) Creation and Detection of Optical Modes with Spatial Light Modulators. *Advances in Optics and Photonics*, **8**, 200-227. https://doi.org/10.1364/aop.8.000200

[18] Dudley, J.M., Genty, G. and Coen, S. (2006) Supercontinuum Generation in Photonic Crystal Fiber. *Reviews of Modern Physics*, **78**, 1135-1184. https://doi.org/10.1103/revmodphys.78.1135

[19] Caliari, M. and Cassini, F. (2024) Efficient Simulation of Complex Ginzburg-Landau Equations Using High-Order Exponential-Type Methods. *Applied Numerical Mathematics*, **206**, 340-357. https://doi.org/10.1016/j.apnum.2024.08.009

[20] Southgate, H.N. (2024) Free Evolution in the Ginzburg-Landau Equation and Other Complex Diffusion Equations. *Physica Scripta*, **100**, Article ID: 015261. https://doi.org/10.1088/1402-4896/ad8aa0

[21] Zakharov, V.E. and Ostrovsky, L.A. (2009) Modulation Instability: The Beginning. *Physica D: Nonlinear Phenomena*, **238**, 540-548. https://doi.org/10.1016/j.physd.2008.12.002

[22] Alexander, V.V., Kulkarni, O.P., Kumar, M., Xia, C., Islam, M.N., Terry, F.L., *et al.* (2012) Modulation Instability Initiated High Power All-Fiber Supercontinuum Lasers and Their Applications. *Optical Fiber Technology*, **18**, 349-374. https://doi.org/10.1016/j.yofte.2012.07.014

[23] Kol, G.R. (2018) Exact Solutions of the Complex Cubic-Quintic Ginzburg-Landau

Equation. *The European Physical Journal B*, **91**, 167.

[24] Kol, G.R. (2017) Rogue Waves in Exciton-Polariton Condensates under Homogeneous Pumping. *Optical and Quantum Electronics*, **49**, Article No. 385. https://doi.org/10.1007/s11082-017-1215-0

[25] Kol, G.R., Kamdem, J. and Kofané, T.C. (2016) Routes to Various Breather Solitons in Whispering-Gallery-Mode Resonators through the Variational Method. *Journal of Modern Optics*, **63**, 2259-2264. https://doi.org/10.1080/09500340.2016.1193639

[26] Kol, G.R. (2017) Rogue Waves with Multiple Compression Points in Nonlinear Optical Cavities. *The European Physical Journal Plus*, **132**, Article No. 331. https://doi.org/10.1140/epjp/i2017-11588-3

[27] Zahran, E.H.M. and Bekir, A. (2023) Optical Soliton Solutions to the Perturbed Biswas-Milovic Equation with Kudryashov's Law of Refractive Index. *Optical and Quantum Electronics*, **55**, Article No. 1211. https://doi.org/10.1007/s11082-023-05453-w

[28] Al-Ghafri, K.S. (2021) Soliton Structures in Optical Fiber Communications with Kundu-Mukherjee-Naskar Model. *Open Physics*, **19**, 679-682. https://doi.org/10.1515/phys-2021-0074

[29] Younis, M., Sulaiman, T.A., Bilal, M., Rehman, S.U. and Younas, U. (2020) Modulation Instability Analysis, Optical and Other Solutions to the Modified Nonlinear Schrödinger Equation. *Communications in Theoretical Physics*, **72**, Article ID: 065001. https://doi.org/10.1088/1572-9494/ab7ec8

[30] Kamel, N.M., Ahmed, H.M. and Rabie, W.B. (2025) Solitons Unveilings and Modulation Instability Analysis for Sixth-Order Coupled Nonlinear Schrödinger Equations in Fiber Bragg Gratings. *AIMS Mathematics*, **10**, 6952-6980. https://doi.org/10.3934/math.2025318

[31] Boubekeur, G., Ciancio, A., Moussa, A., Alhakim, L. and Mati, Y. (2023) New Analytical Solutions and Modulation Instability Analysis for the Nonlinear (1 + 1)-Dimensional Phi-Four Model. *International Journal of Mathematics and Computer in Engineering*, **1**, 79-90.

[32] Kamdem Tamwa, R., Paldou Yaya, A. and Kol, G.R. (2024) Modulational Instability Mechanisms in Exciton-Polariton Condensates. *The European Physical Journal Plus*, **139**, Article No. 355. https://doi.org/10.1140/epjp/s13360-024-05105-2

[33] Houwe, A., Abbagari, S., Akinyemi, L., Doka, S.Y. and Crépin, K.T. (2023) Modulation Instability Gain and Localized Waves in the Modified Frenkel-Kontorova Model with High-Order Nonlinearities. *Chaos, Solitons & Fractals*, **173**, Article ID: 113744. https://doi.org/10.1016/j.chaos.2023.113744

[34] Megne, L.T., Tabi, C.B. and Kofane, T.C. (2020) Modulation Instability in Nonlinear Metamaterials Modeled by a Cubic-Quintic Complex Ginzburg-Landau Equation beyond the Slowly Varying Envelope Approximation. *Physical Review E*, **102**, Article ID: 042207. https://doi.org/10.1103/physreve.102.042207

[35] Veni, S.S., Mani Rajan, M.S., Tabi, C.B. and Crépin Kofané, T. (2026) Propagation Dynamics and Modulation Instability Control in Inhomogeneous Nonlinear Schrödinger Equation with Two-Photon Absorption. *Chaos, Solitons & Fractals*, **202**, Article ID: 117605. https://doi.org/10.1016/j.chaos.2025.117605

[36] Abbagari, S., Houwe, A., Akinyemi, L. and Doka, S.Y. (2023) Modulation Instability and Modulated Wave Patterns in a Nonlinear Electrical Transmission Line with the Next-Nearest-Neighbor Coupling. *Chinese Journal of Physics*, **85**, 722-732. https://doi.org/10.1016/j.cjph.2023.07.019

[37] Yasin, M.W., Baber, M.Z., Munir, M., Hassaballa, A.A., Inc, M., Iqbal, M.S., *et al.*

(2024) Modulation Instability and Optical Wave Profiles for the Conformable Schrödinger-Poisson Dynamical System. *Optical and Quantum Electronics*, **56**, Article No. 1294. https://doi.org/10.1007/s11082-024-07015-0

[38] Yao, S., Tariq, K.U., Inc, M. and Tufail, R.N. (2023) Modulation Instability Analysis and Soliton Solutions of the Modified BBM Model Arising in Dispersive Medium. *Results in Physics*, **46**, Article ID: 106274. https://doi.org/10.1016/j.rinp.2023.106274

[39] P., V., Trabelsi, Y., A., S., M. M., N. and T., L. (2023) Modulation Instability Induced Supercontinuum Generation in Barium Zinc Borate Glass Doped with Praseodymium Based Photonic Crystal Fiber. *Optical and Quantum Electronics*, **55**, Article No. 1087. https://doi.org/10.1007/s11082-023-05272-z

[40] Raja, R.V.J., Porsezian, K. and Nithyanandan, K. (2010) Modulational-Instability-Induced Supercontinuum Generation with Saturable Nonlinear Response. *Physical Review A*, **82**, Article ID: 013825. https://doi.org/10.1103/physreva.82.013825

[41] Wang, P., Meng, F.C., Zhang, Z.L., Wang, D.Z., Wang, S., Yao, E.T., Wang, L. and Ye, R.Q. (2022) A Bump Height Measurement Method Based on Optical Triangulation. *Acta Photonica Sinica*, **51**, Article ID: 0512001. (In Chinese)

[42] Ghosh, S., Drouillard, N.G. and Hammond, T.J. (2024) Supercontinuum Amplification by Kerr Instability. *Physical Review A*, **109**, Article ID: 043508. https://doi.org/10.1103/physreva.109.043508

[43] Demircan, A. and Bandelow, U. (2005) Supercontinuum Generation by the Modulation Instability. *Optics Communications*, **244**, 181-185. https://doi.org/10.1016/j.optcom.2004.09.049

[44] Thomas, N., Bowen, N.E.J.A. and Rose, H. (2021) A Diachronic Analysis of Explicit Definitions and Implicit Conceptualizations of Language Learning Strategies. *System*, **103**, Article ID: 102619. https://doi.org/10.1016/j.system.2021.102619

[45] Li, B., Chen, S., Song, J., Li, J., Kang, G. and Wang, Z. (2026) First Observation of Transverse Mode Instability in a High-Power Linearly Polarized Supercontinuum Source. *High Power Laser Science and Engineering*, **14**, e4. https://doi.org/10.1017/hpl.2025.10100

[46] Etourneau, J. (1986) Les matériaux supraconducteurs: Problèmes et perspectives d'utilisation. Nouvelles orientations des recherches. *Revue de Physique Appliquée*, **21**, 649-657. https://doi.org/10.1051/rphysap:019860021011064900

[47] Bardeen, J., Cooper, L.N. and Schrieffer, J.R. (1957) Theory of Superconductivity. *Physical Review*, **108**, 1175-1204. https://doi.org/10.1103/physrev.108.1175

[48] Huebener, R.P. (2019) The Path to Type-II Superconductivity. *Metals*, **9**, Article 682. https://doi.org/10.3390/met9060682

[49] Krantz, P., Kjaergaard, M., Yan, F., Orlando, T.P., Gustavsson, S. and Oliver, W.D. (2019) A Quantum Engineer's Guide to Superconducting Qubits. *Applied Physics Reviews*, **6**, Article ID: 021318. https://doi.org/10.1063/1.5089550

[50] Pickett, W.E. (2023) *Colloquium*: Room Temperature Superconductivity: The Roles of Theory and Materials Design. *Reviews of Modern Physics*, **95**, Article ID: 021001. https://doi.org/10.1103/revmodphys.95.021001

[51] Weitering, H.H. and Wu, J.Z. (2017) Superconductivity: How the Unconventional Became the New Norm. *Superconductor Science and Technology*, **30**, Article ID: 040301. https://doi.org/10.1088/1361-6668/aa5c49

[52] Mohamadou, A., Jiotsa, A.K. and Kofané, T.C. (2005) Pattern Selection and Modulational Instability in the One-Dimensional Modified Complex Ginzburg-Landau Equation. *Chaos, Solitons & Fractals*, **24**, 957-966.

https://doi.org/10.1016/j.chaos.2004.09.106

[53] Mohammed, W.W., Ozdemir, N., Secer, A., Ozisik, M., Bayram, M., Radwan, T., *et al*. (2025) Modulation Instability and Soliton Families of the Complex Ginzburg-Landau Equation Having the Parabolic with Nonlocal Law of Self-Phase Modulation. *AIMS Mathematics*, **10**, 18321-18336. https://doi.org/10.3934/math.2025818

[54] Kengne, E. and Vaillancourt, R. (2006) Ginzburg-Landau System of Complex Modulation Equations for Distributed Nonlinear Dissipative Transmission Lines. *Nonlinear Oscillations*, **9**, 438-477. https://doi.org/10.1007/s11072-006-0055-9

[55] Zakeri, G. and Yomba, E. (2015) Modulational Instability Regions for Coupled Ginzburg-Landau Equations with Higher Order of Nonlinearities. *Physical Review E*, **91**, Article ID: 062904. https://doi.org/10.1103/physreve.91.062904

[56] Fu, L., Han, X., Dong, H. and Yang, H. (2023) The Modulation Instability of Shallow Wake Flows Based on the Higher-Order Generalized Cubic-Quintic Complex Ginzburg-Landau Equation. *Physics of Fluids*, **35**, Article ID: 026603. https://doi.org/10.1063/5.0138566