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A Link Merges Classical Mechanics to Quantum Theory (Part III)

Alaa S. Bayoumi

Independent Mathematician, Giza, Egypt
Email: alaa.bayoumi2000@gmail.com

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Abstract

This part of the series "a link merges classical mechanics to quantum theory" is dedicated to finding the mathematical equations that govern the relationships between electromagnetic waves, thermal radiation waves, and photons, based on the results obtained in the first two parts of this series. The physical implications of these relationships are then presented within a coherent framework. The process begins by finding a solution to maxwell's equation in both external and internal vacuum. From this solution, the kinetic energy of the photon is extracted. To relate these results to thermal radiation, Wien's displacement law is derived from kinetic theory. However, kinetic theory, in its current form, cannot be used to derive Wien's displacement law. Therefore, kinetic theory is re-derived using an innovative mathematical method specifically designed to solve complex mathematical problems. This method has not been published in any scientific journal, periodical, or conference. As a first consequence, it became possible to define the kinetic temperature of the particle and the temperature of the system, and to find the relationship between them. As a second consequence, a natural wave with an electromagnetic wavelength similar to that observed by radio telescopes was discovered. This wave is believed to originate from deep space and is accompanied by two long waves in the normal radio frequency range, thus providing an explanation for the hiss of radio waves. The generation of the discovered electromagnetic waves is attributed to the exchange of energy between the vacuum and the particles, and vice versa. Then, a connection was made between two of the most famous laws in physics: the ideal gas law and coulomb's law of electrostatic attraction. This connection led to the discovery of a new physical constant related to the fine-structure constant. The relationship of this constant to Planck's constant and Boltzmann's constant was then presented, leading to the concept of the thermal volume of the photon and its relationship to the photon's spatial volume, as proposed by Heisenberg. From this relationship, the wavelength of the photon was calculated. Therefore, the relationship between electromagnetic waves, thermal radiation waves, and the photon can be mathematically understood, allowing for easy transitions between the mathematical relationships of these three concepts. At the end of this article, a mathematical and physical explanation was presented for the phenomenon of the relationship between x-ray and free electron region. The content of this article is based on the concepts, equations, mathematical relationships, and mathematical formulas that were introduced, developed, and deduced in the previous parts, part one and part two. If you are not quite familiar with their contents, you will not be able to follow the content of this article.

Keywords

Maxwell's Wave Equation, Internal & External Vacuum, Photon's Kinetic Energy, Kinetic Theory, Kinetic Temperature, Wien's Displacement Law, Cosmic Microwave Background, Hydrogen Line 21, Radio Long Wavelength, Fine Structure Constant, Universal Photon Gas Modulus, Photon Thermal Volume, Free Electron Region & X-Ray

1. Introduction

In 1857, physicist Rudolf Clausius published a research paper that is considered the first serious mathematical attempt to correlate the kinetic energy of molecules to thermal energy, through a mathematical relationship that connects kinetic energy to temperature. This temperature is called kinetic temperature, and it differs from the temperature of the system as a whole. Later, in 1877, Boltzmann modified this relationship by introducing a constant called Boltzmann's constant, and this relationship became the foundation of kinetic theory. This relationship applies only to a hypothetical gas called an ideal gas, whose molecules meet specific criteria. While the kinetic theory was being developed to find a mathematical relationship between the temperature of a gas and the speed of its molecules, the problem arose that temperature is a macroscopic phenomenon, while speed is a microscopic one. The solution was to calculate the average speed of the gas molecules to represent the motion of all the molecules. To achieve this, certain criteria, postulates, and assumptions were established, which came to be known as the ideal gas theory.

Among these assumptions are that the velocity of the particles does not change upon collision with other particles or with the walls of the container, and that the number of particles is large enough to obtain the best representation of the particle velocity and satisfy Newton's second law. Subsequently, distribution functions of particles velocities, such as the Boltzmann's distribution and the Maxwell distribution, were used to calculate the average velocity. All the assumptions made by the ideal gas theory, necessary for calculating the average speed of particles, are consistent with the mean velocity of each particle obtained in Part (I) under the same title. Therefore, the mean velocity of each particle can be substituted into the mathematical expression of the kinetic theory to calculate the kinetic temperature without hesitation. Consequently, each particle acquires a kinetic temperature, and kinetic temperature becomes a microscopic phenomenon rather than a macroscopic one, while the system's temperature remains a macroscopic variable, allowing for a new and deeper understanding of thermal phenomena. This resulted in a new formulation of the first law of thermodynamics. In this article, an innovative mathematical approach is developed to deduce the main equation of the kinetic theory.

In 1893, the physicist Wilhelm Wien published the results of his theoretical and experimental researches, represented by Wien's law of displacement. At the time when Wien's law was derived, the focus was on the thermal radiation of blackbody matter, attempting to find a mathematical relationship between the properties of the particles in the blackbody material and the thermal radiation produced by their motion. This article presents two different approaches to deriving Wien's law. The first approach relies on the properties of the particle motion and its relationship to the resulting radiation. The second approach calculates the kinetic energy of a photon in space after it is emitted from the radiating body. Both approaches utilize kinetic theory. Consequently, it is recommended that Wien's law be refined to explain the wavelength of all electromagnetic waves, as the different approaches to deriving this law contribute to a broader understanding of its applications.

The evolution of the understanding of the concept of the "Free Electron Region" has witnessed three historical steps, which will be presented below:

In 1895, Wilhelm Röntgen discovered X-rays by accident. He saw a continuous spectrum and sharp lines but didn't know why. He just called them "X-rays" because the cause was unknown. The "free electron region" was there, but not named or explained yet.

In 1911, Arnold Sommerfeld was the first to study the continuous part of X-ray spectrum seriously. He named it "Bremsstrahlung", German for "braking radiation". His idea: Fast electrons slow down near the atomic nucleus and lose energy as radiation. That's the "free electron region" because electrons are free, not bound to atomic energy levels. His explanation used classical physics, but it failed to explain the shortest wavelength limit.

In 1923, Hendrik Kramers fixed Sommerfeld's model using quantum theory.

He explained that electrons lose energy in packets called photons, not smoothly.

This quantum idea explained 2 key things Sommerfeld missed:

1) Why there's a minimum wavelength, an electron can't give more energy than it has.

2) Why does the intensity drop to zero at the shortest wavelength?

Kramers' quantum picture matched experiments perfectly.

This article provides an understanding of the nature of the relationship between the "Free Electron Region" and hard X-rays via the concept of a two-sided surface layer which is based on the classical concept of free electrons on the surfaces of materials, acting as conductors of electricity and heat, in a mathematical and physical way, and that electron jumping is not the only way to generate electromagnetic waves.

2. Applied Methodology

2.1. Incident Angle θ_{ij} - Exit Angle $\tilde{\theta}_{aj}$ - Association Angle $\tilde{\theta}_j$

In this section, the relation between θ_j, which is an auxiliary angle obtained in Part (I) [1] and the association angle $\tilde{\theta}_j$, which is a principle angle obtained in Part (II) [2] will be deduced.

Auxiliary Angle θ_j is the angle between the direction of the mean velocity $\tilde{v}_j$ and the deviation velocity d_j.

The incident angle θ_{ij} is defined as the angle between the incident ray and the normal to the surface.

The relationship between the auxiliary angle θ_j and the incident angle θ_{ij} is given as

$$\theta_{ij} = \theta_j + \frac{\pi}{2} - \pi = \theta_j - \frac{\pi}{2} \to \theta_j = \theta_{ij} + \frac{\pi}{2} \to \cos\theta_j = \sin\theta_{ij}$$

Association angle $\tilde{\theta}_j$ is the angle between the direction of the mean velocity $\tilde{v}_j$ and the velocity $v_{j,1,2}$.

The exit angle $\tilde{\theta}_{ej}$ is defined as the angle between the refracted ray and the normal to the surface.

The relationship between the association angle $\tilde{\theta}_j$ and the exit angle $\tilde{\theta}_{ej}$ is given as

$$\tilde{\theta}_{ej} = \tilde{\theta}_j + \frac{\pi}{2} - \pi = \tilde{\theta}_j - \frac{\pi}{2} \to \tilde{\theta}_j = \theta_{ej} + \frac{\pi}{2} \to \cos\tilde{\theta}_j = \sin\theta_{ej}$$

From Part (I), the velocities $v_{j,1,2}$ of particle *j* is given by the relation [1]

$$v_{j,1,2} = \tilde{v}_j \pm d_j = \tilde{v}_j \left(1 \pm \frac{d_j}{\tilde{v}_j}\right) = \tilde{v}_j \left(1 \pm \frac{1}{\cos\theta_j}\right), \quad j = 1, \cdots, N \tag{1}$$

From Part (I), the potential energy V_j of particle *j* is given by the relation [1]

$$V_j = -m_j \tilde{v}_j v_j, \quad j = 1, \cdots, N$$

Substituting from the relation (1) into the potential energy relation gives

$$V_j = -m_j \tilde{v}_j v_j = -m_j \tilde{v}_j v_{j,1,2} = -m_j \tilde{v}_j^2 \left(1 \pm \frac{1}{\cos\theta_j}\right), \quad j = 1, \cdots, N \tag{2}$$

From Part (II), the complementary energy $\tilde{\beta}_j$ of particle *j* is given by the relation [2]

$$\tilde{\beta}_j = -m_j \tilde{v}_j v_j \cos\tilde{\theta}_j, \quad j = 1, \cdots, N \tag{3}$$

In Lagrange mechanics, the Lagrange function is written as the difference between the kinetic energy of a system of particles and the potential energy acting on the system [3]. The potential energy of the system of particles is a function of the generalized coordinates and generalized velocities. This is the general case. In some special cases, the function describing the potential energy of the system of particles is the sum of the potential energies of each individual particle, and it is a function of the generalized coordinates and generalized velocities of each particle. Therefore, the Lagrange function can be written as the sum of the Lagrange functions for each particle, and each of these functions is treated separately. Since we were able to write the potential energy of a system of particles as the sum of the potential energies of each individual particle, as demonstrated in Part (I) of this series, it became possible to write the Lagrange function for a single particle as the difference between the particle's kinetic energy and its potential energy.

The Lagrangian for particle *j* is the difference between the kinetic energy KE_j of the particle and its potential energy PE_j is written as follows [3]

$$L_j = KE_j - PE_j = \frac{1}{2} m_j v_j^2 - \left(V_j + \tilde{\beta}_j \right), \quad j = 1, \cdots, N$$

Substituting from Equations (2) and (3) into the Lagrangian, we get

$$L_j = \frac{1}{2} m_j v_j^2 + \left(m_j \tilde{v}_j v_j + m_j \tilde{v}_j v_j \cos \tilde{\theta}_j \right), \quad j = 1, \cdots, N$$

Substituting from relation (1) into the Lagrangian, we get

$$L_j = \frac{1}{2} m_j \tilde{v}_j^2 \left(1 \pm \frac{1}{\cos \theta_j} \right)^2 + m_j \tilde{v}_j^2 \left(1 \pm \frac{1}{\cos \theta_j} \right) + m_j \tilde{v}_j^2 \left(1 \pm \frac{1}{\cos \theta_j} \right) \cos \tilde{\theta}_j ,$$
$$j = 1, \cdots, N$$

Let generalized coordinates q_{j1} and q_{j2} be given as follows

$$q_{j1} = \left(1 \pm \frac{1}{\cos \theta_j} \right) \text{ and } q_{j2} = \tilde{\theta}_j , \quad j = 1, \cdots, N \tag{4}$$

The Lagrangian of particle *j* in terms of the generalized coordinates q_{j1} and q_{j2} is written

$$L_j \left(q_{j1}, q_{j2} \right) = \frac{1}{2} m_j \tilde{v}_j^2 {q_{j1}}^2 + m_j \tilde{v}_j^2 q_{j1} + m_j \tilde{v}_j^2 q_{j1} \cos q_{j2} \tag{5}$$

The Lagrangian of particle *j* satisfies the following differential equations [3]

$$\frac{\mathrm{d}}{\mathrm{d}t} \frac{\partial L_j}{\partial \dot{q}_{jk}} = \frac{\partial L_j}{\partial q_{jk}}, \quad k = 1, 2, \quad j = 1, \cdots, N \tag{6}$$

The Lagrangian of particle *j* is not function in $\dot{q}_{jk}$ so the left hand side of equations (6) is zero, *i.e.*

$$\frac{\partial L_j}{\partial q_{jk}} = 0, \quad k = 1, 2, \quad j = 1, \cdots, N \tag{7}$$

Differentiating the Lagrangian given by equations (7) with respect to q_{jk} and equating the result with zero gives

$$m_j\tilde{v}_j^2 q_{j1} + m_j\tilde{v}_j^2 + m_j\tilde{v}_j^2 \cos q_{j2} = 0, \quad j = 1,\cdots,N \tag{7.1}$$

$$-m_j\tilde{v}_j^2 q_{j1} \sin q_{j2} = 0, \quad j = 1,\cdots,N \tag{7.2}$$

Rewrite Equation (7.1) gives

$$q_{j1} + 1 + \cos q_{j2} = 0, \quad j = 1,\cdots,N \tag{8}$$

Using the relations (4) in Equation (8), we get

$$q_{j1} = -\left(1+\cos q_{j2}\right) \to \left(1 \pm \frac{1}{\cos\theta_j}\right) = -\left(1+\cos\tilde{\theta}_j\right), \quad j = 1,\cdots,N \tag{8.1}$$

Rearrange Equation (8.1) write

$$\pm\frac{1}{\cos\theta_j} = -\left(2+\cos\tilde{\theta}_j\right) \to \cos\theta_j = \mp\frac{1}{\left(2+\cos\tilde{\theta}_j\right)}, \quad j = 1,\cdots,N \tag{9}$$

$$\cos^2\theta_j = \frac{1}{\left(2+\cos\tilde{\theta}_j\right)^2} \to \frac{1}{\cos^2\theta_j} = \left(2+\cos\tilde{\theta}_j\right)^2, \quad j = 1,\cdots,N \tag{10}$$

From Equation (7.2), we get the following three cases

$$q_{j1} = 0, \quad j = 1,\cdots,N$$

$$\text{or} \quad \sin q_{j2} = 0, \quad j = 1,\cdots,N$$

$$\text{or both} \quad q_{j1} = 0 \quad \text{and} \quad \sin q_{j2} = 0, \quad j = 1,\cdots,N$$

Investigating the above cases with the validity relations (4) yields the following

Case 1 $q_{j1} = 0$

$$q_{j1} = \left(1 \pm \frac{1}{\cos\theta_j}\right) = 0 \to \frac{1}{\cos\theta_j} = -1 \to \cos\theta_j = -1, \quad j = 1,\cdots,N$$

$$\theta_j = \pm n\pi, \quad j = 1,\cdots,N$$

Substituting in Equation (7.1) with $q_{j1} = 0$ gives

$$m_j\tilde{v}_j^2 + m_j\tilde{v}_j^2 \cos q_{j2} = 0 \to \cos q_{j2} = -1, \quad j = 1,\cdots,N$$

From relations (4), we get

$$\cos q_{j2} = \cos\tilde{\theta}_j = -1 \to \tilde{\theta}_j = \pm(2n+1)\pi, \quad j = 1,\cdots,N$$

Case 2 $\sin q_{j2} = 0$

$$\sin q_{j2} = 0 \to \cos q_{j2} = \pm 1 \to \tilde{\theta}_j = \pm n\pi, \quad j = 1,\cdots,N$$

Substituting in Equation (7.1) with $\cos q_{j2} = \pm 1$ gives

$$m_j\tilde{v}_j^2 q_{j1} + m_j\tilde{v}_j^2 \pm m_j\tilde{v}_j^2 = 0 \to q_{j1} + 1 \pm 1 = 0, \quad j = 1,\cdots,N$$

This results in two choices for q_{j1}

The first choice

$$q_{j1} = 0 \to \theta_j = \pm n\pi, \quad j = 1,\cdots,N$$

The second choice

$$q_{j1} = -2 \quad \text{or} \quad q_{j1} = -2 \to \left(1 \pm \frac{1}{\cos\theta_j}\right) = -2 \to \frac{1}{\cos\theta_j} = -3\,, \quad j = 1,\cdots,N$$

$$\left|\theta_j\right| = 70.52877^\circ \quad \theta_j = 109.47122^\circ$$

<u>**Case 3**</u> $q_{j1} = 0$ <u>and</u> $\sin q_{j2} = 0$.

In this case

$$q_{j1} = \left(1 \pm \frac{1}{\cos\theta_j}\right) = 0 \to \frac{1}{\cos\theta_j} = -1 \to \cos\theta_j = -1 \to \theta_j = \pm n\pi\,, \quad j = 1,\cdots,N$$

$$\sin q_{j2} = 0 \to \cos q_{j2} = \pm 1 \to \tilde{\theta}_j = \pm n\pi\,, \quad j = 1,\cdots,N$$

Notice in this case values of θ_j and $\tilde{\theta}_j$ are equal *i.e.* $\theta_j = \tilde{\theta}_j\,, \quad j = 1,\cdots,N$.

2.2. Potential Energy - Radiation Conditions

From Part (II), the derivation of the general format of the complementary energy led to the result of defining the radiation conditions [2], where the adhesion force vanishes and the wave becomes no longer associated to the particle, the duality particle-wave ends, resulting in the wave being released from the substance and generating radiant wave outside the matter in the form of wave-particle duality. The particle then undergoes a new potential energy, an alternative to the previous before the fulfillment of the radiation condition and is appropriate to the changes resulting from the radiation in order to maintain dynamic equilibrium. In this section, the new potential energy influences the particle after the radiation occurred will be represented.

From Part (II), the radiation condition is given as follows [2]

$$\lim_{v_j \to c} \tilde{\beta}_j = \tilde{\beta}_{0j}\,, \quad j = 1,\cdots,N$$

$$\tilde{\theta}_j = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \cdots$$

From Equation (2), we have

$$V_j = -m_j \tilde{v}_j^2 \left(1 \pm \frac{1}{\cos\theta_j}\right) = -m_j \tilde{v}_j^2 q_{j1}\,, \quad j = 1,\cdots,N \tag{2.1}$$

Now we investigate the potential energies corresponding to the three cases for different values of q_{j1} as obtained in the previous section.

<u>**Case 1**</u>

$$\tilde{\theta}_j = \pm(2n+1)\pi\,, \quad q_{j1} = \left(1 \pm \frac{1}{\cos\theta_j}\right) = 0\,, \quad \theta_j = \pm n\pi\,, \quad j = 1,\cdots,N$$

Substituting in Equation (2.1), the resulting potential energy of particle *j* corresponding to the above value of q_{j1} is

$$V_j = 0\,, \quad j = 1,\cdots,N$$

This case where the potential energy vanishes, is investigated in details in Part (I) [1].

Case 2

In this case q_{j1} has two values for the same $\tilde{\theta}_j$

$$\tilde{\theta}_j = \pm n\pi \,, \quad q_{j1} = 0 \to \theta_j = \pm n\pi$$

Substituting in Equation (2.1), the resulting potential energy of particle *j* corresponding to the above value of q_{j1} is

$$V_j = 0 \,, \quad j = 1, \cdots, N$$

The case where the potential energy vanish investigated in details in Part (I) [1]

$$\tilde{\theta}_j = \pm n\pi \,, \quad q_{j1} = -2 \to \left(1 \pm \frac{1}{\cos\theta_j}\right) = -2 \to \frac{1}{\cos\theta_j} = -3 \,, \quad j = 1, \cdots, N$$

From Part (II), the restricted potential energy of particle *j* corresponding to constant $C_{1,2j} = q_{j1}$ is given as follows [2]

$$V_j = -\frac{m_j}{q_{j1}t^2} s_j^2 = -\frac{1}{q_{j1}} m_j 4\pi^2 \nu_j^2 s_j^2 \,, \quad j = 1, \cdots, N$$

Form Part (II) $\dfrac{1}{\cos\theta_j} = -3$ is corresponding to the potential energy of a linear oscillator which represents simple harmonic motion given [2]

$$V_j = -\frac{1}{2} K_{1,2j} s_j^2 \text{ where } K_j = 4\pi^2 m_j \nu_j^2 \,, \quad j = 1, \cdots, N$$

Or a linear oscillator with potential energy given as follows

$$V_j = \frac{1}{2} K_{1,2j} s_j^2 \text{ where } K_j = -4\pi^2 m_j \nu_j^2 \,, \quad j = 1, \cdots, N$$

Case 3

$$\tilde{\theta}_j = \pm n\pi \,, \quad q_{j1} = 0 \,, \quad \theta_j = \pm n\pi$$

Substituting in Equation (2.1), the resulting potential energy of particle *j* corresponding to the above value of q_{j1} is

$$V_j = 0 \,, \quad j = 1, \cdots, N$$

The case where the potential energy vanish investigated in details in Part (I) [1].

Now we investigate the values of the principle angle $\tilde{\theta}_j$ from different perspective.

The above derivations indicate that the new potential energy after the radiation condition is met is either zero or a linear oscillator potential energy. The radiation condition is met when particles collide and a particle jumps from energy level to another (changes its state). The results obtained in this section confirm those previously deduced in Part (II) using the triangle method for the geometric representation of the constant energy surface, review the cylinder, prism, and crystal shapes in Part (II) [2].

In Part (I), the rotational motion of the particle was explained when the potential energy vanishes, where the wave is confined between the origin, which is the center of rotation of the particle, and the position of the particle on the circumference. Likewise, in the linear motion limited between two points, the wave is confined between the two points, and the same equations of rotational motion are substituted with the distance between the two points of linear motion instead of the radius in rotational motion [1].

To study motion, a fixed point is necessary to which the change is referred, whether it is the motion of a particle or a wave. Representing the wave on the circumference of the path is incorrect mathematically and physically which violates the principle of frame of reference, an acceptable and agreed upon principle although it is common in many literatures.

Remark

The potential energy of particles vanishes near the surface of the matter.

2.3. Maxwell's Wave Equations in the External Space

This section will present an explanation of two formulations of Maxwell's wave equations.

The first formula, the original formula derived in Maxwell's work, is based on two assumptions: the first is the charge density $\rho = 0$ and the second is the current vector $J = 0$, these two assumptions represent the situation in the external space outside the matter. By substituting these two values into Maxwell's equations, Maxwell obtained the following wave equations.

$$\nabla^2 E = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \nabla^2 H = \mu_0 \varepsilon_0 \frac{\partial^2 H}{\partial t^2} \tag{11}$$

where

E is the electric field.

H is the magnetic field.

ε_0 is the Permittivity of free space.

μ_0 is the Permeability of free space.

And, then Maxwell compared equations (11) with the standard wave equation for a wave represented by function $\tilde{u}$ travels in the x direction with velocity $V_{\tilde{u}}$ is given as $\frac{\partial^2 \tilde{u}}{\partial x^2} = \frac{1}{V_{\tilde{u}}^2} \frac{\partial^2 \tilde{u}}{\partial t^2}$.

Maxwell has concluded that the speed of the electromagnetic wave in space c is given by the relation

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad (12)\ [4]$$

The base units in the international system (SI) units for the right hand side of Equation (12) is a velocity units, so Maxwell derivation was correct mathematically and physically the numerical value of c calculated from the relation (12) is 3×10^8 meter/sec, the speed of light in space c obtained from experimentation is 299,792,458 meter/sec.

A solution to Maxwell's wave equations given by relations (11) becomes

$$H = H_0 \cos\left(c\sqrt{\varepsilon_0 \mu_0} x - ct + \tilde{\theta}_0\right) \tag{13}$$

Verifying that the solution of Maxwell's equation given above, we perform the following steps

$$\frac{\partial H}{\partial x} = -H_0 c\sqrt{\varepsilon_0 \mu_0} \sin\left(c\sqrt{\varepsilon_0 \mu_0} x - ct + \tilde{\theta}_0\right) \to \frac{\partial^2 H}{\partial x^2} = -c^2 \varepsilon_0 \mu_0 H$$

$$\to -H = \frac{1}{c^2 \varepsilon_0 \mu_0} \frac{\partial^2 H}{\partial x^2} \tag{14}$$

$$\frac{\partial H}{\partial t} = Hc \sin\left(\frac{\sqrt{\varepsilon_0 \mu_0}}{c} x - ct + \tilde{\theta}_0\right) \to \frac{\partial^2 H}{\partial t^2} = -c^2 H \to -H = \frac{1}{c^2} \frac{\partial^2 H}{\partial t^2} \tag{15}$$

Equating Equations (14) and (15) gives Maxwell's wave Equation (11).

The second formula, which other physicists have termed Maxwell's wave equations, is mentioned in numerous references. These equations are written as follows

$$\nabla^2 E = \frac{\mu_0 \varepsilon_0}{c^2} \frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \nabla^2 H = \frac{\mu_0 \varepsilon_0}{c^2} \frac{\partial^2 H}{\partial t^2} \tag{16}$$

Solutions for Equation (16) given above

$$H = H_0 \cos\left(\sqrt{\varepsilon_0 \mu_0} x - ct + \tilde{\theta}_0\right)$$

$$H = H_0 \cos\left(x - \frac{c}{\sqrt{\varepsilon_0 \mu_0}} t + \tilde{\theta}_0\right)$$

Equations (16) are not an equivalent to Maxwell's original equations; they are new equations indicating that $\frac{\mu_0 \varepsilon_0}{c^2}$ ratio is constant during the motion of the wave.

2.4. Photon's Kinetic Energy (KE_c) in the External Space

Reviewing the results obtained from the previous section, we find that the magnetic field H satisfies relations (13), which corresponds Maxwell's wave equation given in the previous section.

From Part (II), we also have the magnetic field H satisfies the following relation [2]

$$H = \frac{1}{e_j} \sqrt{\tilde{\delta}_{1,2j} \gamma_j} m_j \tilde{v}_j c \cos\left(\sqrt{\gamma_j} x_j - v_{2j} t + \tilde{\theta}_{0j}\right), \quad j = 1, \cdots, N \tag{17}$$

$$\text{where} \quad \sqrt{\gamma_j} = \frac{v_{2j}}{v_{1j}} \to v_{2j} = v_{1j} \sqrt{\gamma_j} \tag{18}$$

Compare this solution with Equation (13) to deduce the two particle velocities corresponding to the wave velocity derived from Maxwell's equation.

The arguments variables of both cos functions in relation (13) and relation (17) must be equal, we write

$$\left(\sqrt{\gamma_j} x_j - v_{2j} t + \tilde{\theta}_{0j}\right) = \left(c\sqrt{\varepsilon_0 \mu_0} x - ct + \tilde{\theta}_0\right), \quad j = 1, \cdots, N$$

From the above equality, we get the following two equalities

$$\sqrt{\gamma_j} = c\sqrt{\varepsilon_0 \mu_0} \quad \text{and} \quad v_{2j} = c, \quad j = 1, \cdots, N \tag{19}$$

Substituting from equalities (19) into relation (18), we get

$$v_{2j} = v_{1j}\sqrt{\gamma_j} \to c = v_{1j} c \sqrt{\varepsilon_0 \mu_0} \to v_{1j} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \tag{20}$$

We already know from Equation (12) that the speed of light $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ consequently, from equalities (20) we get

$$v_{1j} = c \quad \text{and} \quad v_{2j} = c, \quad j = 1, \cdots, N \tag{21}$$

From the relations (21), we can determine the value of the mean velocity $\tilde{v}_j$ as follows [1]

$$\tilde{v}_j = \frac{v_{1j} + v_{2j}}{2} = \frac{c + c}{2} = c, \quad j = 1, \cdots, N \tag{22}$$

Equation (22) given above indicates the mean velocity $\tilde{v}_j$ of particle *j* is equal of the speed of the wave *c*.

From Part (I), the relation between momentum of particle j p_j and wavelength λ_j of the associated wave is [1]

$$p_j = \frac{h}{\lambda_j}, \text{ where } \quad p_j = m_j \tilde{v}_j \tag{23}$$

From Equation (22) and the relation (23), we get

$$m_j \tilde{v}_j = m_j c = \frac{h}{\lambda_j} \tag{24}$$

Substituting in (24) by $\lambda_j = \frac{c}{\nu_j}$ we get

$$m_j c = \frac{h}{\lambda_j} = \frac{h\nu_j}{c} \to m_j c^2 = h\nu_j \tag{25}$$

The kinetic energy of the photon KE_c in the external space outside the matter is given as follows

$$KE_c = \frac{1}{2} m_j c^2 = \frac{1}{2} h\nu_j \tag{26}$$

2.5. Maxwell's Wave Equations in the Internal Space

Since the propagation of waves in the internal vacuum between particles differs from the propagation of waves in the vast external vacuum, there are constraints on the speed of electromagnetic waves. So, another solution is required for Maxwell's equation.

The external vacuum is vast enough to contain energy, and the energy contained in the internal vacuum is shared among molecules, atoms, and electrons. Therefore, propagation in this medium must be entirely different from the external vacuum. Effect of sharing energy in the internal vacuum is distributed among particles, altering wave propagation. Different behavior indeed, behavior in the internal vacuum differs from the external vacuum due to particle interactions.

The difference between vacuums will be considered in the new proposed solution. The external vacuum is vast and open, while the internal vacuum (between particles) contains interacting particles. Based on the preceding discussion, the results can be summarized and mathematically interpreted in a method that relies on a microscopic modification of the given solution to Maxwell's equation, illustrated in relation (13), to accommodate the propagation of electromagnetic waves in the internal vacuum of matter, rather than modifying Maxwell's equation itself. Modifying Maxwell's equation would require modifying classical electromagnetic theory, which is beyond the scope of this article.

:

The internal speed method

This method relies on the fact that the wave speed inside the material c_{int} is different from the wave speed c in the vacuum outside the material. This is achieved by setting a limit for the distance traveled by the wave in the proposed solution by multiplying the distance term by a factor and adjusting the value of the wave speed to the required speed within the material using the same factor, thus making the distance traveled by the wave as well as its speed proportional to dimensions of the matter The required solution is obtained by modifying Equation (13), so that the modified formula agrees with the previous arguments. The modified solution is written as follows:

$$H = H_0 \cos\left(\frac{c_{\text{int}}}{c}x - \frac{c_{\text{int}}}{c}ct + \tilde{\theta}_0\right) \tag{27}$$

Verifying that the solution of Maxwell's equation given above, we rewrite Maxwell's equations given by the relations (11) as follows

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \nabla^2 H = \frac{1}{c^2}\frac{\partial^2 H}{\partial t^2} \tag{28}$$

We derive the left-hand side of Maxwell's equation that is by differentiating the solution given by Equation (27) two times with respect to x as follows

$$\begin{aligned} &\frac{\partial H}{\partial x} = -H_0 \frac{c_{\text{int}}}{c}\sin\left(\frac{c_{\text{int}}}{c}x - \frac{c_{\text{int}}}{c}ct + \tilde{\theta}_0\right) \to \frac{\partial^2 H}{\partial x^2} = -\left(\frac{c_{\text{int}}}{c}\right)^2 H \\ &\to \frac{c^2}{c_{\text{int}}^2}\frac{\partial^2 H}{\partial x^2} = -H \end{aligned} \tag{29}$$

Now we derive the right hand side of Maxwell's equation by differentiating the solution given by Equation (27) two times with respect to t as follows

$$\begin{aligned} &\frac{\partial H}{\partial t} = H_0 \frac{c_{\text{int}}}{c}c\sin\left(\frac{c_{\text{int}}}{c}x - \frac{c_{\text{int}}}{c}ct + \tilde{\theta}_0\right) \to \frac{\partial^2 H}{\partial t^2} = -\left(\frac{c_{\text{int}}}{c}\right)^2 c^2 H \\ &\to \frac{c^2}{c_{\text{int}}^2}\frac{1}{c^2}\frac{\partial^2 H}{\partial x^2} = -H \end{aligned} \tag{30}$$

Equations (29) and (30) are equal, so the proposed solution given by Equation (27) is verifying Maxwell's wave Equation (28) representing wave propagation inside the matter, and the radiation ratio is $\tilde{\sigma} = \frac{c_{\text{int}}}{c}$.

Important Remark

In Part (II), a solution to Maxwell's equation was presented in the following form [2]

$$H = H_0 \cos\left(\frac{1}{\sqrt{\mu_0 \varepsilon_0}} x - ct + \tilde{\theta}_0\right)$$

This solution proposes a wave speed inside the matter of one meter per second, which is valid for some semiconductors and low temperature near the surface of the matter. There are materials in which the speed of electrons is less than one meter per second near the surface of the materials, so the proposed solution in Part (II) is a special solution. The author apologizes to the readers for not clarifying that solution given to Maxwell's equations in Par (II) is a special solution, and expects the readers to accept his apology.

2.6. Photon's Kinetic Energy ($KE_{c_{\text{int}}}$) in the Internal Space

In this section, the results obtained in the previous section, we will be used to determine the two velocities corresponding to the value of $\tilde{\sigma}$. Thus, we can calculate the mean velocity $\tilde{v}_j$ and, from this, determine the kinetic energy of the photons inside the matter.

The magnetic field H satisfies relation (27) corresponds Maxwell's wave equation given in the previous section.

From Part (II), we also have the magnetic field H satisfies the following relation [2]

$$H = \frac{1}{e_j}\sqrt{\tilde{\delta}_{1,2j}\gamma_j}\, m_j \tilde{v}_j c \cos\left(\sqrt{\gamma_j}\, x_j - v_{2j} t + \tilde{\theta}_{0j}\right), \quad j = 1, \cdots, N \tag{17}$$

$$\text{Where} \quad \sqrt{\gamma_j} = \frac{v_{2j}}{v_{1j}} \to v_{2j} = v_{1j}\sqrt{\gamma_j}, \quad j = 1, \cdots, N \tag{18}$$

Compare this relation (17) with the solutions of Maxwell's Equation (27) to deduce the two particle velocities corresponding to the wave velocity derived from Maxwell's equation.

The arguments variables of both cos functions in relations (27) and Equation (17) must be equal, we write

$$\left(\sqrt{\gamma_j}\, x_j - v_{2j} t + \tilde{\theta}_{0j}\right) = \tilde{\sigma} x - \tilde{\sigma} c t + \tilde{\theta}_0, \quad j = 1, \cdots, N \tag{31}$$

The above equality results two equalities determining the two speeds given as follows

$$\sqrt{\gamma_j} = \tilde{\sigma} \quad \text{and} \quad v_{2j} = \tilde{\sigma} c, \quad j = 1, \cdots, N \tag{32}$$

Substituting from equalities (32) into relation (18), we get

$$v_{2j} = v_{1j}\sqrt{\gamma_j} \to \tilde{\sigma} c = v_{1j}\tilde{\sigma} \to v_{1j} = c$$

$$v_{1j} = c \quad \text{and from equalities (32)} \quad v_{2j} = \tilde{\sigma} c, \quad j = 1, \cdots, N \tag{33}$$

From the relations (33), we can determine the value of the mean velocity $\tilde{v}_j$ as follows [1]

$$\tilde{v}_j = \frac{v_{1j} + v_{2j}}{2} = \frac{1}{2}(c + \tilde{\sigma} c) = \frac{c}{2}(1 + \tilde{\sigma}), \quad j = 1, \cdots, N \tag{34}$$

From Part (I), the relation between momentum of particle *j*, p_j and wavelength λ_j of the associated wave is [2]

$$p_j = \frac{h}{\lambda_j}, \text{ where } p_j = m_j \tilde{v}_j, \quad j = 1, \cdots, N \tag{23}$$

From Equation (34) and the relation (23), we get

$$m_j \tilde{v}_j = m_j \frac{c}{2}(1 + \tilde{\sigma}) = \frac{h}{\lambda_j}, \quad j = 1, \cdots, N \tag{35}$$

Substituting in (35) by $\lambda_j = \frac{c}{\nu_j}$ we get

$$m_j \frac{c}{2}(1 + \tilde{\sigma}) = \frac{h}{\lambda_j} = \frac{h\nu_j}{c} \to \frac{1}{2} m_j c^2 = \frac{h\nu_j}{(1 + \tilde{\sigma})}, \quad j = 1, \cdots, N \tag{36}$$

The kinetic energy of the photon $KE_{c_{\text{int}}}$ in the internal space inside the matter is given as follows

$$KE_{c_{\text{int}}} = \frac{1}{2} m_j c^2 = \frac{h\nu_j}{(\tilde{\sigma} + 1)}, \quad j = 1, \cdots, N \tag{37}$$

Note that the kinetic energy of the photon $KE_{c_{\text{int}}}$ in the internal space inside the matter is different than the kinetic energy of the photon KE in the external space outside the matter.

2.7. The Refraction Index of an Electromagnetic Wave

Equation (37) gives the value of the photon kinetic energy $KE_{c_{\text{int}}}$ in the internal space inside the matter, corresponds to a radiation ratio $\tilde{\sigma}$. Rearranging Equation (37), we get

$$\frac{1}{2} m_j c^2 (\tilde{\sigma} + 1) = h\nu_j \to \frac{1}{2} m_j c^2 \tilde{\sigma} + \frac{1}{2} m_j c^2 - h\nu_j = 0, \quad j = 1, \cdots, N \tag{38}$$

Now we will investigate the available solutions of Equation (38) for the value $\tilde{\sigma}$. Substitute in Equation (38) with $\tilde{\sigma} = \frac{c_{\text{int}}}{c}$ we get

$$\frac{1}{2} m_j c^2 \, \frac{c_{\text{int}}}{c} + \frac{1}{2} m_j c^2 - h\nu_j = 0 \to m_j c^2 + m_j c_{\text{int}} c - 2h\nu_j = 0, \quad j = 1, \cdots, N \tag{39}$$

Rewrite Equation (39), we get the following quadratic equation

$$c^2 + c_{\text{int}} c - \frac{2h\nu_j}{m_j} = 0, \quad j = 1, \cdots, N \tag{40}$$

The solutions to the quadratic Equation (40) represent the photon velocity *j*, given as follows:

$$c_{1,2j}=\frac{-c_{\text{int}}\pm\sqrt{c_{\text{int}}^2+4\dfrac{2h\nu_j}{m_j}}}{2}=\frac{-c_{\text{int}}\pm c_{\text{int}}\sqrt{1+4\dfrac{2h\nu_j}{m_jc_{\text{int}}^2}}}{2} \tag{41}$$

The refractive index $\tilde{n}$ defined as follows $\tilde{n}=\dfrac{c}{c_{\text{int}}}$. rewrite Equation (41) in order to get the refractive index $\tilde{n}$ by the formula

$$\tilde{n}=\frac{c}{c_{\text{int}}}=\frac{-1\pm\sqrt{1+4\dfrac{2h\nu_j}{m_jc_{\text{int}}^2}}}{2},\quad j=1,\cdots,N \tag{42}$$

2.8. Kinetic Theory - Kinetic Temperature - System's Temperature

From Part (I), we have the following relation [1]

$$\tilde{v}_j=\left(\frac{1}{2}v_j-\frac{\tilde{\alpha}_j\mathcal{E}+\tilde{\beta}_j}{m_jv_j}\right),\quad j=1,\cdots,N \tag{43}$$

where $\tilde{v}_j$ is the mean velocity of particle *j*.

The kinetic theory assumes the existence of an ideal gas in which the molecules do not interact that is, the rate of speed of the molecules does not change when they collide with each other or with wall of the container, that is, they move freely, but they exert pressure on the wall of the container when they collide with it.

In this section, a mathematical proof will be deduced indicating that the speed given by Equation (43) is the speed of an un-reacting particle and satisfies the assumptions of the kinetic theory. Equation (43) was already derived in Part (I) [1] using the equation for the total energy of *N* moving particles. It was also established that each particle has a constant average velocity $\tilde{v}_j$, satisfying the assumptions of kinetic theory. This average velocity corresponds to the particle's actual moving velocity. Re-deriving Equation (43) based on a different approach will lead to the main equation of kinetic theory, and will provide additional information about the properties of the particle system in terms of energy distribution and how the particles interact with each other.

The conceptual aspect

Starting the deduction process by defining the set A is the set of all momentum values for all particles in the system as follows

$$A=\left\{m_1v_1,m_2v_2,m_3v_3,m_4v_4,\cdots,m_Nv_N\right\}$$

Consider the following two sets A_1, and A_2 subsets of set A where $A_1\subset A$, and $A_2\subset A$

$$A_1=\left\{m_1v_1,m_2v_2,m_3v_3,m_4v_4,\cdots,m_{N_1}v_{N_1}\right\},$$

$$A_2=\left\{m_1v_1,m_2v_2,m_3v_3,m_4v_4,\cdots,m_{N_2}v_{N_2}\right\}$$

where A is written as follows

$$A = A_1 \cup A_2 \quad \text{and} \quad A_1 \cap A_2 = \Phi$$

A_1 is the set of momentum values of the interacting particles with cardinality $\overline{A}_1 = N_1$.

A_2 is the set of momentum values of the non-interacting particles with cardinality $\overline{A}_2 = N_2$.

Let us define the sets X_1 and X_2 as follows

$$X_1 = \left\{ A_i \subset A : \overline{A}_i = \overline{A}_1 \right\} \quad \text{and} \quad X_2 = \left\{ A_i \subset A : \overline{A}_i = \overline{A}_2 \right\}$$

X_1 is the set of all subsets of A with cardinality N_1.

X_2 is the set of all subsets of A with cardinality N_2.

The cardinality of the sets X_1 and X_2 is given by the following relations

$$\overline{X}_1 = \overset{N}{\underset{N_1}{C}} = \frac{N!}{(N-N_1)!} \quad \text{and} \quad \overline{X}_2 = \overset{N}{\underset{N_2}{C}} = \frac{N!}{(N-N_2)!}$$

Let D be the set of all functions map $D: X_1 \to X_2$.

The number of functions map $X_1 \to X_2$ is the cardinality of D is given by the relation $\overline{D} = \left(\overline{X}_1\right)^{\overline{X}_2}$.

The number one-to-one (Injective) functions map $X_1 \to X_2$ is given by the relation

$$\overline{D}_{injective} = \frac{\overline{X}_2!}{\left(\overline{X}_2 - \overline{X}_1\right)!}$$

The derivation process acts to establish a relationship between the elements belonging to set A_2 specified above, and the elements belonging to set A_1 with the aim of deriving Equation (43). There will be $\overline{D}$ numbers of equations, the same like as Equation (43) by the end of the derivation process.

Implementation tools

The required necessary implementation tools in order to deduce a correlation between the elements of set A are as follows:

The set of adjusting coefficients, $\left\{ \tilde{f}_1, \tilde{f}_2, \tilde{f}_3, \cdots, \tilde{f}_N \right\} \subset \mathcal{R}$ is the set of real numbers.

$\tilde{f}_j$ to balances the velocities with the average speed.

The set of proportional factor matrix $\left\{ \sqrt{\gamma_{j,k}}, j = 1, \cdots, N; k = 1, \cdots, N_1 \right\} \subset \mathcal{R}$ is the set of real numbers to balance velocities of set A_1 with each other.

The set of proportional factor matrix $\left\{ \sqrt{\gamma_{j,k}}, j = 1, \cdots, N; k = 1, \cdots, N_2 \right\} \subset \mathcal{R}$ is the set of real numbers to balance velocities of set A_2 with each other.

b_0 and $\tilde{b}_0$ are algebraic transformation factors will be used in order to accomplish the derivation process.

Values belong to set A_1 will be in the right hand side of the initial equation during the derivation process, by using the implementation tools ultimately will become the right hand side of Equation (43) by the end of the derivation process.

Values belong to set A_2 will be in the left hand side of the initial equation during the derivation process, by using the implementation tools ultimately will become the left hand side of Equation (43) by the end of the derivation process. The initial equation is considered as a numerical measure for set A.

The implementation steps

Write the initial equation which it is a numerical measure of set A and it is the determination of the average of the elements of the set A and is given by the following summation

$$\overline{p} = \frac{1}{N}\sum_{j=1}^{N} m_j v_j \tag{44}$$

In order to distinguish the interacting particles between the non-interacting particles, split the summation in the right hand side into two summations, one for the interacting particles and another for the non interacting particles and write

$$\overline{p} = \frac{1}{N}\sum_{j=1}^{N_1} m_j v_j + \frac{1}{N}\sum_{j=1}^{N_2} m_j v_j, \quad N = N_1 + N_2 \tag{45}$$

where

N_1 is the number of interacting particles.

N_2 is the number of non-interacting particles.

Now separate the two summations, one for the interacting particles in the right hand side and another for the non interacting particles in the left hand side and rewrite Equation (45) as follows

$$\frac{1}{N}\sum_{j=1}^{N_2} m_j v_j = \overline{p} - \frac{1}{N}\sum_{j=1}^{N_1} m_j v_j \tag{46}$$

The left hand side of Equation (46) represents a numerical measure for the momentum of the non-interacting particles given by set A_2.

The right hand side of Equation (46) represents a numerical measure for the momentum of the interacting particles given by set A_1.

In order to distinguish the speed of the non-interacting particles, use the notation $v_j = \overline{v}_j$ in the left hand side of Equation (46), so Equation (46) is written

$$\frac{1}{N}\sum_{j=1}^{N_2} m_j \overline{v}_j = \overline{p} - \frac{1}{N}\sum_{j=1}^{N_1} m_j v_j \tag{47}$$

We now use the adjustment coefficients $\tilde{f}_j$ to generate N equations from Equation (47), with each equation corresponding to a single particle, by multiplying both sides of Equation (47) by the coefficients $\tilde{f}_j$ and $\frac{1}{m_j}$, yielding:

$$\frac{\tilde{f}_j}{m_j N}\sum_{k=1}^{N_2} m_k \overline{v}_k = \frac{\tilde{f}_j}{m_j}\overline{p} - \frac{\tilde{f}_j}{m_j N}\sum_{k=1}^{N_1} m_k v_k, \quad j = 1,\cdots,N \tag{48}$$

Consider the equality

Now let $\frac{1}{2}v_j = \frac{\tilde{f}_j}{m_j}\overline{p}, \quad j = 1,\cdots,N$ (49)

After substituting from equality (49) into Equation (47) the right hand side of Equation (47) becomes

$$\frac{1}{2}v_j - \frac{\tilde{f}_j}{m_j N}\sum_{k=1}^{N_1} m_k v_k, \quad j = 1,\cdots,N \tag{50}$$

Now we use the proportional factor matrix $\sqrt{\gamma_{j,k}}$ to balance velocities with each other by considering the following equality

$$v_k = \sqrt{\gamma_{j,k}}\,v_j \to v_k^2 = \sqrt{\gamma_{j,k}}\,v_j v_k \to v_k = \frac{v_k^2}{\sqrt{\gamma_{j,k}}\,v_j}, \quad j = 1,\cdots,N, \quad k = 1,\cdots,N_1 \tag{51}$$

$\sqrt{\gamma_{j,k}}$ is a proportional factor introduced in order to equate two different velocities to each other.

Substituting from equality (51) into the summation of the momentum term in Equation (50) we get

$$\sum_{k=1}^{N_1} m_k v_k = \sum_{k=1}^{N_1}\frac{m_k v_k^2}{\sqrt{\gamma_{j,k}}\,v_j} = \frac{b_{0j}}{v_j}\frac{\sum_{k=1}^{N_1} m_k v_k^2}{\sum_{k=1}^{N_1}\sqrt{\gamma_{j,k}}}, \quad j = 1,\cdots,N \tag{52}$$

b_{0j} is a proportional factor introduced to convert the sum of ratios into a single ratio between two sums which is required to continue the derivation process, and rewrite Equation (52) as follows

$$\sum_{k=1}^{N_1} m_k v_k = \frac{b_{0j}}{a_{0j}}\frac{\sum_{k=1}^{N_1} m_k v_k^2}{v_j}, \quad a_{0j} = \sum_{k=1}^{N_1}\sqrt{\gamma_{j,k}}, \quad j = 1,\cdots,N \tag{53}$$

Let $c_{0j} = \frac{b_{0j}}{a_{0j}}$.

We rewrite Equation (53), with considering the above relation as follows

$$\sum_{k=1}^{N_1} m_k v_k = 2c_{0j}\frac{\sum_{k=1}^{N_1}\frac{1}{2}m_k v_k^2}{v_j} = 2c_{0j}\frac{T_{N_1}}{v_j}, \quad j = 1,\cdots,N \tag{54}$$

where T_{N_1} is the kinetic energy of the N_1 interacting particles given by

$$T_{N_1} = \sum_{k=1}^{N_1}\frac{1}{2}m_k v_k^2$$

The kinetic energy of the N_1 interacting particles in terms of $\mathcal{E}_{N_1}$ the total energy and the potential energy V_{N_1} is written as follows

$$T_{N_1} = \mathcal{E}_{N_1} - V_{N_1} \tag{55}$$

Substituting from Equation (55) into Equation (54), we get

$$\sum_{k=1}^{N_1} m_k v_k = 2c_{0j}\frac{T_{N_1}}{v_j} = 2c_{0j}\frac{\mathcal{E}_{N_1} - V_{N_1}}{v_j}, \quad j = 1,\cdots,N \tag{56}$$

Substituting with $\sum_{k=1}^{N_1} m_k v_k$ given in Equation (56) into Equation (50), we get

$$R.H.S = \frac{1}{2}v_j - \frac{\tilde{f}_j}{m_j N}\sum_{k=1}^{N_1} m_k v_k = \frac{1}{2}v_j - \frac{2c_{0j}\tilde{f}_j}{N}\frac{\mathcal{E}_{N_1} - V_{N_1}}{m_j v_j}, \quad j = 1,\cdots,N \tag{57}$$

Let $\alpha_j = \dfrac{2c_{0j}\tilde{f}_j}{N}, \quad j = 1,\cdots,N$ (58)

Substituting with α_j given in Equation (58) into (57), we get

$$R.H.S = \frac{1}{2}v_j - \frac{\alpha_j \mathcal{E}_{N_1} - \alpha_j V_{N_1}}{m_j v_j}, \quad j = 1,\cdots,N \tag{59}$$

Now let $\tilde{\alpha}_j \mathcal{E} = \alpha_j \mathcal{E}_{N_1}$ and $\tilde{\beta}_j = -\alpha_j V_{N_1}, \quad j = 1,\cdots,N$ (60)

Equation (59) becomes

$$R.H.S = \frac{1}{2}v_j - \frac{\tilde{\alpha}_j \mathcal{E} + \tilde{\beta}_j}{m_j v_j}, \quad j = 1,\cdots,N \tag{61}$$

$R.H.S$ given in Equation (61) is the same $R.H.S$ of Equation (43), resulting in the $L.H.S$ of Equation (48) is equal the $L.H.S$ of Equation (43).

Written

$$\tilde{v}_j = \frac{\tilde{f}_j}{m_j N}\sum_{k=1}^{N_2} m_k \overline{v}_k, \quad j = 1,\cdots,N \tag{62}$$

Equating the $L.H.S$ given by Equation (62) with $R.H.S$ given by Equation (61), we get

$$\tilde{v}_j = \frac{1}{2}v_j - \frac{\tilde{\alpha}_j \mathcal{E} + \tilde{\beta}_j}{m_j v_j}, \quad j = 1,\cdots,N$$

<u>End of the derivation process</u>

The consequences of the derivation of Equation (43) are as follows:

-The law of ideal gas and kinetic temperature

The following steps will lead to the derivation of the law of ideal gas.

We go back to Equation (62) we have the term $\sum_{k=1}^{N_2} m_k \overline{v}_k$, Following the same procedure in deriving $\sum_{k=1}^{N_1} m_k v_k = \dfrac{2b_0}{a_0}\dfrac{T_{N_1}}{v_j}$ obtained in Equation (54) we can write $\sum_{k=1}^{N_2} m_k v_k$ as follows.

$$\sum_{k=1}^{N_2} m_k \overline{v}_k = \frac{2\tilde{b}_{0j}}{\tilde{a}_{0j}}\frac{T_{N_2}}{v_j} = 2\tilde{c}_{0j}\frac{T_{N_2}}{v_j}, \text{ where } \frac{\tilde{b}_{0j}}{\tilde{a}_{0j}} = \tilde{c}_{0j}, \quad j = 1,\cdots,N \tag{63}$$

And T_{N_2} is the kinetic energy of the N_2 non-interacting particles given by

$$T_{N_2} = \sum_{k=1}^{N_2} \frac{1}{2} m_k \overline{v}_k^2$$

Substituting from Equation (63) into Equation (62) we get

$$\tilde{v}_j = \frac{\tilde{f}_j}{m_j N}\sum_{k=1}^{N_2} m_k \overline{v}_k = \frac{2\tilde{c}_{0j}\tilde{f}_j}{m_j N}\frac{T_{N_2}}{v_j} \tag{64}$$

$$\frac{1}{N} = \frac{N_2}{N}\frac{1}{N_2} \tag{65}$$

Substituting from Equation (65) into Equation (64) we get.

$$\tilde{v}_j = \frac{f_j}{m_j N}\sum_{k=1}^{N_2} m_k v_k = \frac{2\tilde{c}_{0j}\tilde{f}_j}{m_j}\frac{N_2}{N}\frac{1}{v_j}\frac{T_{N_2}}{N_2} = \frac{2\tilde{c}_{0j}\tilde{f}_j}{m_j}\frac{N_2}{N}\frac{1}{v_j}\overline{T}_{N_2}, \quad j = 1,\cdots,N \tag{66}$$

where $\overline{T}_{N_2} = \dfrac{T_{N_2}}{N_2}$.

$\overline{T}_{N_2}$ is the average kinetic energy of the N_2 non-interacting particles.

Now we express the average kinetic energy $\overline{T}_{N_2}$ in terms of the mean velocity $\tilde{v}_j$ by applying the proportional factor matrix $\sqrt{\gamma_{j,k}}$ to balance velocities of non-interacting particles with the mean velocity $\tilde{v}_j$. The average kinetic energy of the N_2 non-interacting particles is given by the following relation

$$\overline{T}_{N_2} = \frac{1}{N_2}\sum_{k=1}^{N_2}\frac{1}{2}m_k\overline{v}_k^2 \tag{67}$$

The proportional factor matrix $\sqrt{\gamma_{j,k}}$ correlates the velocities $\overline{v}_k$ with the velocity $\tilde{v}_j$ as follows

$$\overline{v}_k = \sqrt{\gamma_{j,k}}\tilde{v}_j \to \overline{v}_k^2 = \gamma_{j,k}\tilde{v}_j^2, \quad j = 1,\cdots,N, \quad k = 1,\cdots,N_2 \tag{68}$$

By substituting from the relations (68) in the relation (67), we obtain N equations for the average kinetic energy of the N_2 non-interacting particles written as follows

$$\overline{T}_{jN_2} = \frac{1}{N_2}\sum_{k=1}^{N_2}\frac{1}{2}m_k\overline{v}_k^2 = \frac{1}{m_j N_2}\frac{1}{2}m_j\tilde{v}_j^2\sum_{k=1}^{N_2}m_k\gamma_{j,k}, \quad j = 1,\cdots,N \tag{69}$$

Let $\tilde{a}_{2j} = \dfrac{1}{N_2}\sum_{k=1}^{N_2}m_k\gamma_{j,k}$ and substitute with $\tilde{a}_{2j}$ in Equation (69), we get

$$\overline{T}_{jN_2} = \frac{1}{N_2}\sum_{k=1}^{N_2}\frac{1}{2}m_k\overline{v}_k^2 = \frac{\tilde{a}_{2j}}{m_j}\frac{1}{2}m_j\tilde{v}_j^2, \quad j = 1,\cdots,N \tag{70}$$

From the arguments given previously the mean velocity $\tilde{v}_j$ is satisfying the assumptions of kinetic theory, so an enhanced ideal gas equation is written as

$$\frac{1}{2}m_j\tilde{v}_j^2 = \frac{3}{2}k_B\tilde{T}_j, \quad j = 1,\cdots,N \tag{71}$$

where k_B is Boltzmann constant and $\tilde{T}_j$ is kinetic temperature for particle *j*.

Substituting from Equation (71) into Equation (70) yield

$$\overline{T}_{jN_2} = \frac{\tilde{a}_{2j}}{m_j}\frac{3}{2}k_B\tilde{T}_j, \quad j = 1,\cdots,N \tag{72}$$

Substituting from Equation (70) into Equation (66) resulting in

$$\tilde{v}_j = \tilde{c}_{0j}\tilde{a}_{2j}\tilde{f}_j \frac{N_2}{N}\frac{1}{m_j v_j}\tilde{v}_j^2, \quad j = 1,\cdots,N \tag{73}$$

Substituting from Equation (72) into Equation (66) resulting in

$$\tilde{v}_j = 3\frac{\tilde{c}_{0j}\tilde{a}_{2j}\tilde{f}_j}{m_j}\frac{N_2}{N}\frac{1}{m_j v_j}k_B\tilde{T}_j, \quad j = 1,\cdots,N \tag{74}$$

Equation (71) is the ideal gas law in terms of the average kinetic energy and kinetic temperature.

The kinetic temperature $\tilde{T}_j$ is not the temperature of the system $\tilde{T}_S$. The following is a discussion explaining the nature of the kinetic temperature and its relation to the temperature of the system.

If the purpose of a particle's motion is to obtain its appropriate share of energy from the space occupied by the matter, according to the particle's mass and velocity, then the average velocity $\tilde{v}_j$, derived in the first part of this series of articles, indicates that when the particle's velocity reaches this value, its interaction with the surrounding space ceases. This interaction is the mechanism by which energy is exchanged between the space occupied by the matter and the particle. This is evident from the fact that the particle's energy is unaffected at this velocity and remains constant. In other words, when the particle's velocity reaches the average velocity value, it becomes independent of the velocities of other particles. The preceding explanation represents only one aspect of energy transfer: the transfer of energy from the space occupied by the matter in which the particles are moving to the particles themselves. The other aspect is the transfer of energy from the moving particles to the space occupied by the matter, resulting in the space occupied by the matter regaining the energy previously acquired by the particles. This occurs through collisions with other particles or friction, and in both cases, the energy transferred from the particle's motion to the space is thermal energy. From the preceding explanation and discussion, the ideal gas law given by Equation (71) can be interpreted as follows: the kinetic temperature on the right-hand side of Equation (71) is a criterion that measures the transfer of energy from the space occupied by the matter to a particle whose average kinetic energy is given on the left- hand side of the ideal gas equation. That is, at the kinetic temperature, the exchange of energy between the particles and the space occupied by the matter stops.

Continuing from what was mentioned earlier about kinetic temperature, we can conclude that there is another criterion for measuring energy transfer from particles to the space occupied by matter. This criterion must be temperature, just like kinetic temperature. We use the system's temperature, $\tilde{T}_S$, as a criterion for measuring energy transfer from particles to the space occupied by matter. The mathematical method of expressing the transfer of one value to another, or one quantity to another, is the mathematical operation represented by the ratio. This is done by placing the value from which the transfer is to be made in the denominator of the ratio and the value to which the transfer is to be made in the numerator of the ratio. The resulting numerical value expresses the transfer of the value of the denominator to the value of the numerator. Not only that, but it also indicates which of the two quantities is greater. If the value of the ratio is less than one, this means that the larger quantity becomes smaller as a result of the transfer. If the value of the ratio is greater than one, this means that the smaller quantity becomes larger as a result of the transfer. We apply this understanding to the kinetic temperature and the temperature of the system by determining the ratio Ti/Ts. According to the previous explanation, this ratio expresses the transfer of energy from the space occupied by the matter to the particle. As for the ratio Ts/Ti, it expresses the transfer of energy from the particle to the space occupied by the matter. The product of the two ratios represents the efficiency of energy transfer in both directions, from the space occupied by the matter and the particle and vice versa. This is another way of expressing the conservation of energy, which differs from the first law of thermodynamics.

The ideal gas equation provides a relationship between kinetic temperature and average velocity, where kinetic temperature is proportional to the square of the average velocity. To determine the relationship between kinetic temperature and system's temperature, the appropriate velocity related to the system's temperature is required, we use the relation (1) given in section 2.1 to define the relation between the mean velocity and the velocity of particle *j* and this will lead to the relation between kinetic temperature and system's temperature

$$v_{j,1,2} = \tilde{v}_j \left(1 \pm \frac{1}{\cos\theta_j}\right), \quad j = 1, \cdots, N \tag{1}$$

The kinetic energy of a particle *j* is given by the following relation

$$KE_j = \frac{1}{2} m_j v_j^2 = \frac{1}{2} m_j \tilde{v}_j^2 \left(1 \pm \frac{1}{\cos\theta_j}\right)^2, \quad j = 1, \cdots, N \tag{75}$$

By substituting from Equation (71) into the kinetic energy given by Equation (75), we get

$$KE_j = \frac{1}{2} m_j \tilde{v}_j^2 \left(1 \pm \frac{1}{\cos\theta_j}\right)^2 = \frac{3}{2} k_B \tilde{T}_j \left(1 \pm \frac{1}{\cos\theta_j}\right)^2, \quad j = 1, \cdots, N \tag{76}$$

Comparing Equation (71) with Equation (76), we find a similarity: the left side of both equations represents the kinetic energy of the particle, while the right side represents the product of Boltzmann's constant k_B and temperature. In Equation (71), the temperature is the kinetic temperature $\tilde{T}_j$, while in Equation (76), the kinetic temperature is multiplied by a factor given as $\left(1 \pm \frac{1}{\cos\theta_j}\right)^2$. Therefore, we conclude that $\tilde{T}_j \left(1 \pm \frac{1}{\cos\theta_j}\right)^2$ is also a temperature. From the preceding discussion, we know that temperatures are either kinetic or system's temperature;

thus, $\tilde{T}_j\left(1 \pm \frac{1}{\cos\theta_j}\right)^2$ is clearly the system temperature, consequently the relationship between the system's temperature and the kinetic temperature is given by the following equation.

$$\tilde{T}_S = \tilde{T}_j\left(1 \pm \frac{1}{\cos\theta_j}\right)^2, \quad j = 1, \cdots, N \tag{77}$$

Note that in Equation (77), for the system's temperature $\tilde{T}_S$ equal the kinetic temperature $\tilde{T}_j$ the factor $\left(1 \pm \frac{1}{\cos\theta_j}\right)^2$ must be equal one, *i.e.*, $\pm\frac{1}{\cos\theta_j} = 0$, and this condition is not possible.

Therefore, the temperature of the system $\tilde{T}_S$ will never be equal the kinetic temperature $\tilde{T}_j$., also the factor $\left(1 \pm \frac{1}{\cos\theta_j}\right)^2$ converts microscopic variable $\tilde{T}_j$ to macroscopic value $\tilde{T}_S$.

2.9. Wien Displacement Law

In this section, we present two different methods for deriving Wien's displacement Law.

The first method

In this method there are two different derivation techniques will be introduced to deduce Wien's displacement Law, based on the results obtained in the preceding Section 2.8, Section (2.6)

$$v_{j,1,2} = \tilde{v}_j\left(1 \pm \frac{1}{\cos\theta_j}\right) \to \frac{v_j}{\tilde{v}_j} = \left(1 \pm \frac{1}{\cos\theta_j}\right), \quad j = 1, \cdots, N \tag{1}$$

Now go back to Equation (73) in the previous Section 2.8 given as

$$\tilde{v}_j = \tilde{c}_{0j}\tilde{a}_{2j}\tilde{f}_j\frac{N_2}{N}\frac{1}{m_j v_j}\tilde{v}_j^2 \to \frac{1}{m_j}\tilde{c}_{0j}\tilde{a}_{2j}\tilde{f}_j\frac{N_2}{N}\frac{\tilde{v}_j}{v_j} = \pm 1, \quad j = 1, \cdots, N \tag{78}$$

Rearranging Equation (78) and substituting from Equation (1) above, we get two equations given as follows

$$\frac{1}{m_j}\tilde{c}_{0j}\tilde{a}_{2j}\tilde{f}_j\frac{N_2}{N} = \frac{v_j}{\tilde{v}_j}, \quad j = 1, \cdots, N \tag{79}$$

$$\frac{1}{m_j}\tilde{c}_{0j}\tilde{a}_{2j}\tilde{f}_j\frac{N_2}{N} = \pm\left(1 \pm \frac{1}{\cos\theta_j}\right), \quad j = 1, \cdots, N \tag{80}$$

Particle - Photon Interaction Technique

The following steps are a derivation for Wien displacement law.

From Equation (74) in Section 2.8 is given as follows

$$\tilde{v}_j = 3\frac{\tilde{c}_{0j}\tilde{a}_{2j}\tilde{f}_j}{m_j}\frac{N_2}{N}\frac{1}{m_j v_j}k_B\tilde{T}_j, \quad j = 1,\cdots,N \tag{74}$$

Substitute from Equation (79) into Equation (74) we get

$$\tilde{v}_j = 3\frac{v_j}{\tilde{v}_j}\frac{1}{m_j v_j}k_B\tilde{T}_j \to m_j\tilde{v}_j v_j = 3\frac{v_j}{\tilde{v}_j}k_B\tilde{T}_j, \quad j = 1,\cdots,N \tag{81}$$

In case of radiation from Equation (33) in Section 2.6

$$v_j = c, \quad j = 1,\cdots,N \tag{82}$$

And, from Equation (34) in Section 2.6, the mean velocity of photon moving inside the matter is given as follows

$$\tilde{v}_j = \frac{c}{2}\left(1+\tilde{\sigma}\right) = \frac{c}{2}\left(1+\frac{c_{\text{int}}}{c}\right), \quad j = 1,\cdots,N \tag{34}$$

By substituting in the right-hand side of Equation (81) from Equations (82) and (34) resulting in

$$R.H.S = 3\frac{v_j}{\tilde{v}_j}k_B\tilde{T}_j = 3\frac{c}{\frac{c}{2}\left(1+\frac{c_{\text{int}}}{c}\right)}k_B\tilde{T}_j = 3\frac{2}{\left(1+\frac{c_{\text{int}}}{c}\right)}k_B\tilde{T}_j \tag{83}$$

By substituting in the left-hand side of Equation (81) from Equations (82) and (23) resulting in

$$L.H.S = m_j\tilde{v}_j v_j = \frac{hc}{\lambda_j} \tag{84}$$

By equating the $R.H.S$ given by Equation (83) with the $L.H.S$ given by Equation (84) we get

$$\frac{hc}{\lambda_j} = 3\frac{2}{\left(1+\frac{c_{\text{int}}}{c}\right)}k_B\tilde{T}_j = b_w k_B\tilde{T}_j, \text{ where } \quad b_w = 3\frac{2}{\left(1+\frac{c_{\text{int}}}{c}\right)} \tag{85}$$

From Equation (85) we get Wien displacement law written as follows

$$\lambda_j = \frac{hc}{b_w k_B\tilde{T}} \tag{86}$$

The experimental value for $b_W = 4.965$, which corresponding to $\frac{c_{\text{int}}}{c} = 0.2084$

The critical angle $\theta_c = \arcsin\left(0.2084\right) \approx 12^\circ$.

Particle Interaction Technique

Substitute from Equation (80) into Equation (74) we get

$$\tilde{v}_j = -3\left(1-\frac{1}{\cos\theta_j}\right)\frac{1}{m_j v_j}k_B\tilde{T}_j \to m_j\tilde{v}_j v_j = -3\left(1-\frac{1}{\cos\theta_j}\right)k_B\tilde{T}_j \tag{87}$$

By substituting in the left-hand side of Equation (87) from Equations (82) and (23) resulting in

$$\frac{hc}{\lambda_j} = -3\left(1 - \frac{1}{\cos\theta_j}\right) k_B \tilde{T}_j = b_w k_B \tilde{T}_j \quad \text{where} \quad b_w = -3\left(1 - \frac{1}{\cos\theta_j}\right) \tag{88}$$

From Equation (88) we get Wien displacement law written as follows

$$\lambda_j = \frac{hc}{b_w k_B \tilde{T}_j} \tag{86}$$

The experimental value for $b_W = 4.965$.

Substitute by the experimental value $b_W = 4.965$, in Equation (88) and write

$$\frac{4.965}{3} = -\left(1 - \frac{1}{\cos\theta_j}\right) \rightarrow 1.655 = \frac{1}{\cos\theta_j} - 1 \rightarrow \frac{1}{\cos\theta_j} = 2.655 \tag{89}$$

which corresponds to $\theta_j = 67.8715^\circ$ from Equation (8.1) in Section 2.1 we calculate the value of $\tilde{\theta}_j$ given as $\tilde{\theta}_j \approx 49.0743^\circ$.

From Section 2.1, the relationship between θ_j, $\tilde{\theta}_j$ and the incidence angle θ_{ij}, exit angle $\tilde{\theta}_{ej}$ are given as follows

$$\theta_{ij} = \theta_j - \frac{\pi}{2} \rightarrow \theta_{ij} = 67.8715 - 90 = -22.1285^\circ \rightarrow \left|\theta_{ij}\right| = 22.1285^\circ$$

$$\text{and} \quad \tilde{\theta}_{ej} = \tilde{\theta}_j - \frac{\pi}{2} \rightarrow \tilde{\theta}_{ej} = 49.0743 - 90 = -40.9257^\circ$$

$\tilde{\theta}_{ej}$ is the exit angle for the wave, in order to determine the corresponding incidence angle, we use the relation

$$\frac{\text{Incidence Angel}}{\text{Exit Angel}} = \frac{\sin\tilde{\theta}_{ij}}{\sin\tilde{\theta}_{ej}} = \frac{c_{\text{int}}}{c} = 0.2084 \rightarrow \sin\tilde{\theta}_{ij} = 0.2084 \sin\tilde{\theta}_{ej}$$

$$\sin\tilde{\theta}_{ij} = 0.2084 \sin(-40.9257) = -0.2084 \times 0.65507 = -0.1365$$

$$\arcsin(-0.1365) = -7.8464^\circ$$

The incidence angle θ_{ij} is greater than the critical angle θ_c, *i.e.* $\theta_{ij} > \theta_c$, $22.1285^\circ > 12^\circ$.

The incidence angle of the wave $\tilde{\theta}_{ij}$ is less than the critical angle θ_c, *i.e.* $\tilde{\theta}_{ij} < \theta_c$, $7.8464^\circ < 12^\circ$.

This result means that the particle bounces back into the material at the surface, while the associated wave is able to escape from the material's surface in the form of radiation. The association persists because the termination of the association requires an association angle equal to π or multiples of π. See more details in Section 2.13.

$\tilde{T}_S$ is the temperature of the system, which is the temperature to be measured in the lab is given by relation (77) in the previous section is written as follows

$$\tilde{T}_S = \tilde{T}_j \left(1 - \frac{1}{\cos\theta_j}\right)^2, \quad j = 1, \cdots, N \tag{77}$$

Substituting from relation (89) with value of $\frac{1}{\cos\theta_j}$ in the above relation we get

$$\tilde{T}_S = \tilde{T}_j (1 - 2.655)^2 = 2.7390 \tilde{T}_j \tag{90}$$

Important Remark

The measured Cosmic Microwave Background (CMB) photon temperature is 2.725 K. In Equation (90), the system's temperature value $\tilde{T}_S = 2.7390\ \text{K}$ corresponding to particle's temperature $\tilde{T}_j = 1\ \text{K}$, the difference between the value of the system's temperature and the measured (CMB) temperature is given by the relation

$$\tilde{T}_S - \tilde{T}_C = 2.7390\ \text{K} - 2.725\ \text{K} = 0.014\ \text{K} \tag{91}$$

0.014 K, represents the difference of the performance of waves in the external space and the internal space of the matter. In order to assess the performance of the wave within and outside the material, we return to Section 2.8.

In Section 2.8, it was presented that temperature is a criterion for measuring energy transfer from the vacuum occupied by matter to the particles and vice versa. Here, the associated wave plays a crucial role as a medium through which energy and momentum are transferred. The difference between the vacuum temperature and the particle system temperature is due to expenditure of some of the transferred energy in associated wave's motion. Therefore, to calculate the wavelength of the associated wave, we follow these two steps:

Step 1: Determine the wave's kinetic energy corresponding to the temperature difference given by Equation (91). This represents the amount of energy the wave expended in transferring energy, as calculated by ideal equation given by Equation (71) in Section 2.8 as follows:

$$KE_w = \frac{3}{2} k_B \tilde{T}_w = \frac{3}{2} k_B 0.014$$

$$KE_w = \frac{1}{2} m_j \tilde{v}_j^2 = \frac{3}{2} k_B 0.014 \to m_j \tilde{v}_j^2 = 3 k_B 0.014 \tag{92}$$

Step 2: The quantum equation relating the energy of the associated wave to its wavelength, obtained in Part (I) as the particle-wave equation, its value is the value of the complementary energy at the speed of light [1], for particle *j* is written as

$$\tilde{\beta}_j = \nu_j h = \frac{ch}{\lambda_j}$$

Use equations (1) and (89), rewrite the particle-wave equation as follows

$$\tilde{\beta}_j = -m_j \tilde{v}_j v_j = -m_j \tilde{v}_j^2 \left(1 - \frac{1}{\cos\theta_j}\right) = -m_j \tilde{v}_j^2 (1 - 2.655)$$

$$\tilde{\beta}_j = 1.655 m_j \tilde{v}_j^2 = \frac{ch}{\lambda_j} \to m_j \tilde{v}_j^2 = \frac{1}{1.655} \frac{ch}{\lambda_j} \tag{93}$$

Equate the calculated wave's kinetic energy from Step 1 given by Equation (92) with Equation (93), this yields the following equation:

$$\lambda_j = \frac{ch}{3 \times 0.014 \times 1.655 \times k_B} \tag{94}$$

Substitute with values of h, c, and k_B given below in (94).

Plank's constant: $h = 6.62607015 \times 10^{-34}$ J·s.

Speed of light: $c = 2.99792458 \times 10^{8}$ m/s.

Boltzmann's constant $k_B = 1.380649 \times 10^{-23}$ J/K.

The calculations result in

$$\lambda_j = 0.207 \text{ m} = 20.7 \text{ cm} \tag{95}$$

Now we calculate the wavelength using Wien's law

$$\lambda_j = \frac{ch}{4.965 \times 0.014 \times k_B} = 0.207 \text{ m} = 20.7 \text{ cm} \tag{95.1}$$

The values of the wavelength calculated from duality equation and Wien's Law are the same.

The wavelength of 20.7 cm, calculated using Equation (95) above, corresponds to radio wavelength. This wavelength is also one that radio telescopes detect from distant galaxies. Furthermore, the calculated wavelength of 20.7 cm is based on the assumption that waves transfer energy from the vacuum to particles according to the speed and mass of those particles, as well as energy transfer from particles to the vacuum through collisions and friction. This occurs with air and water molecules in oceans, seas, rivers, and soil. This raises the question of the true source of the radio waves detected by radio telescopes: are they indeed coming from deep space, or are they waves from the atmosphere or reflected from the surface of the moon?

The second method- Free photons in external space

From Equation (26) in Section 2.7 we have the kinetic energy of a photon in the external space is given as

$$KE_{cj} = \frac{1}{2} m_j c^2 = \frac{1}{2} \nu_j h, \quad j = 1, \cdots, N \tag{26}$$

Substituting into (26) by $\lambda_j = \frac{c}{\nu_j}$ we get

$$KE_{cj} = \frac{ch}{2} \frac{1}{\lambda_j}, \quad j = 1, \cdots, N \tag{96}$$

The average photon kinetic energy is required in order to apply the kinetic theory formula, the kinetic theory formula is applicable on ideal gas where the particles are in free motion not under the influence of external factors, and in order to determine the average photon kinetic energy corresponds to free particle's motion, we define P_p The probability in the propagation direction (dependence probability), so the probability $\overline{P}_p = 1 - P_p$ is the probability that the photon's motion will not be influenced by any external factors (independence probability) [5]. $\overline{P}_p$ expresses the total probability of free motion required to determine the average kinetic energy of the photon.

By substituting by the value of $\overline{P}_p$ in Equation (96) we get, the average photon kinetic energy $\overline{KE}_{Rj}$ is given as follows:

$$\overline{KE}_{Rj} = \overline{P}_p \frac{ch}{2} \frac{1}{\lambda_j}, \quad j = 1, \cdots, N \tag{97}$$

Use the kinetic theory to determine the equivalent energy produced by the photon *j* for the radiant energy given by the following equation

$$\overline{KE} = \frac{3}{2} k_B \tilde{T}_j, \quad j = 1, \cdots, N \tag{98}$$

Now we equate Equation (97) and Equation (98) and this yields

$$\overline{KE}_{Rj} = \overline{P}_p \frac{ch}{2} \frac{1}{\lambda_j} = \frac{3}{2} k_B \tilde{T}_{N_2} \to \lambda_j = \frac{\overline{P}_p}{3} \frac{ch}{k_B \tilde{T}_{N_2}}, \quad j = 1, \cdots, N \tag{99}$$

Equation (99) is Wien's displacement law and b_W is Wien constant is given by the following relation

$$\frac{1}{b_W} = \frac{\overline{P}_p}{3} \tag{100}$$

The experimental value for Wien constant $b_W = 4.965$.

Comparing b_W given by Equation (100) and the experimental value indicates that, $\overline{P}_p = 0.6042$. The experimental value for $b_W = 4.965$ explains the relationship between electromagnetic radiation with short wave length and the emitting source temperature adequately, such as $b_W \approx 4.965$ for visible, UV, X-rays, and gamma rays. Wien's law failed to describe the electromagnetic radiation with long wave length, such as far infrared and law temperature this because Wien's law was derived with specific assumptions (thermal radiation from a blackbody). This research proposes that $\overline{b}_W$ a like Wien's constant should take different values corresponding to wavelengths of waves that the current Wien's constant cannot explain, particularly long-wavelength electromagnetic waves. This would broaden the range of practical applications of Wien's law, rather than limiting it to thermal radiation.

This proposal is implemented by assigning different and appropriate values to the ratio $\overline{P}_p$ and these examples of $\overline{P}_p$ values and the equivalent $\overline{b}_W$ obtained from Equation (100)

$\overline{P}_p = 0.625 \approx$ Equivalent $\overline{b}_W = 4.8$ for far infrared.

$\overline{P}_p = 0.6666 \approx$ Equivalent $\overline{b}_W = 4.5$ for far infrared.

$\overline{P}_p = 0.6042 \approx$ Equivalent $b_W = 4.965$ the current Wien's constant.

2.10. Universal Photon Gas Modulus (UPGM), κ_γ

Determining the Universal Photon Gas Modulus (UPGM), relies on a method linking two of the most famous laws of physics. The ideal Gas Law [6], sometimes called General Gas Law and the Law of Electrostatic Attraction [4] [7] [8].

The first is the ideal Gas Law given by the following equation,

$$PV = \frac{3}{2} N k_B \tilde{T}_j, \quad j = 1, \cdots, N \tag{101}$$

And the second is the Law of Electrostatic Attraction, given by the following equation.

$$F = \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{R^2} \tag{102}$$

To implement linking Equations (101) and (102), we begin with an observation: the Ideal Gas Law includes a value for volume, which implies the existence of an object. We will call this object a bubble. The volume of a bubble, of course, has a geometric shape that allows us to calculate its volume. There are several geometric shapes, but in deriving the Universal Photon Gas Modulus (UPGM), we will focus on the sphere. In a sphere, the ratio between the surface area A_B of a sphere with radius R_B and its volume V_B is given by the following equation.

$$\frac{A_B}{V_B} = \frac{4\pi R_B^2}{\frac{4}{3}\pi R_B^3} = \frac{3}{R_B} \rightarrow \frac{1}{3}\frac{A_B}{V_B} = \frac{1}{R_B} \tag{103}$$

Now, let's consider the law of electrostatic attraction and examine its relationship to a bubble, which, as mentioned earlier, obeys the ideal gas law. In the law of electrostatic attraction, R represents the distance between two charges, one positive and the other negative. In the spherical form of a bubble, R represents the distance between the center of the bubble and any point on its surface R_B. Therefore, we can write the law of electrostatic attraction for a spherical bubble using the equation for the ratio of the bubble's surface area A_B to its volume V_B, found in Equation (103), as follows:

$$F_B = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R_B^2} = \frac{1}{4\pi\varepsilon_0}\frac{1}{R_B}\frac{Ze^2}{R_B} = \frac{1}{4\pi\varepsilon_0}\frac{1}{3}\frac{A_B}{V_B}\frac{Ze^2}{R_B} \tag{104}$$

Rewrite Equation (104) as follows

$$\frac{F_B}{A_B} V_B = \frac{1}{3}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R_B} \tag{105}$$

From definition of pressure $P_B = \frac{F_B}{A_B}$ we substitute in Equation (105) to get

$$P_B V_B = \frac{1}{3}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R_B} \tag{106}$$

For a bubble with volume V_B and surface pressure P_B the law given by Equation (101) is written as follows

$$P_B V_B = \frac{3}{2} N_B k_B \tilde{T}_j \tag{107}$$

Now equating Equations (106) and (107) results

$$\frac{1}{3}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R_B}=\frac{3}{2}N_B k_B\tilde{T}_j \to \frac{1}{3\times 3\times 2}\frac{1}{\pi\varepsilon_0}\frac{Ze^2}{R_B}=N_B k_B\tilde{T}_j \tag{108}$$

The relation between the distance R_B and the wavelength λ_B is given as

$$2R_B = n_j\lambda_B \to R_B=\frac{n_B\lambda_B}{2} \tag{109}$$

Substituting from (109) into (108) and rewrite

$$\frac{1}{9\times 2}\frac{1}{\pi\varepsilon_0}\frac{Ze^2}{R_B}=\frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{Ze^2}{n_B\lambda_B}=N_B k_B\tilde{T}_j,\quad j=1,\cdots,N \tag{110}$$

Rearranging Equation (110) gives

$$\frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{Ze^2}{n_B\lambda_B}=N_B k_B\tilde{T}_j \to \lambda_B=\frac{1}{9}\frac{Ze^2}{\pi\varepsilon_0 n_B N_B}\frac{1}{k_B\tilde{T}_j},\quad j=1,\cdots,N \tag{111}$$

Wien's Displacement Law is

$$\lambda_j=\frac{hc}{b_w k_B\tilde{T}_j}=\frac{hc}{b_w}\frac{1}{k_B\tilde{T}_j}\quad \text{where}\quad b_w=4.965,\quad j=1,\cdots,N \tag{112}$$

For certain j $\lambda_j=\lambda_B$. *i.e.* the subscript j can be replaced by B or vice versa. Comparing equations (111) and (112) resulting in

$$\frac{hc}{b_w}=\frac{1}{9}\frac{Ze^2}{\pi\varepsilon_0 n_B N_B} \tag{113}$$

Rearranging the equality (113) gives

$$\frac{hc}{b_w}=\frac{1}{9}\frac{Ze^2}{\pi\varepsilon_0 n_B N_B}\to\frac{9\pi}{2b_w}\frac{n_B N_B}{Z}=\frac{e^2}{2\varepsilon_0 hc} \tag{114}$$

The right hand-side of Equation (114) is the Fine Structure Constant α (alpha) a fundamental physical constant describes interaction strength between electrons and photons. Origin of the name: "Fine structure" due to its effect on atomic spectral lines and it plays an important role in explaining quantum effects. Details in the discussion Section 3.

α is a dimensionless (a pure number) and its approximate value: $\alpha \approx$ 1/137.035999084.

Substituting by value of α in Equation (114), we get

$$\frac{9\pi}{2b_w}\frac{n_B N_B}{Z}=\frac{e^2}{2\varepsilon_0 hc}=\frac{1}{137.035999084}\to\frac{n_B N_B}{Z}=\frac{2b_w}{9\pi\times 137.035999084} \tag{115}$$

The right-hand side of Equation (115) is a constant and its reciprocal is

$$\frac{Z}{n_B N_B}=\frac{9\pi\times 137.035999084}{2b_w}=390.225 \tag{116}$$

The constant given by Equation (116) is named the **Universal Photon Gas Modulus (UPGM)** with symbol $\kappa_\gamma=390.225$.

The following steps reveal the relationship between κ_γ and the photons gas pressure P_B given by Equation (106) given as follows

$$P_B V_B = \frac{1}{3}\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{R_B} \to P_B = \frac{1}{V_B 3\times 4\times\pi}\frac{Ze^2}{\varepsilon_0 R_B} \tag{106}$$

Substitute in Equation (106) $V_B = \frac{4}{3}\pi R_B^3$ and write

$$P_B = \frac{1}{4\times 4\times\pi^2}\frac{Ze^2}{\varepsilon_0 R_B^4}$$

From Equation (116) we get

$$\frac{Z}{n_B N_B} = \kappa_\gamma \to Z = n_B N_B \kappa_\gamma$$

The substitution from the above relation into the pressure equation yield

$$P_B = \frac{1}{4\times 4\times\pi^2}\frac{Ze^2}{\varepsilon_0 R_B^4} = \frac{1}{16\times\pi^2}\frac{n_B N_B}{R_B^4}\frac{e^2}{\varepsilon_0}\kappa_\gamma$$

The constant κ_γ appears in the photons gas pressure relation $P_B \propto \kappa_\gamma$, thus represents a dimensionless relative bulk modulus for the cosmic photon gas.

The relationships between Plank's constant *h* and the (UPGM) κ_γ

Substitute from Equation (116) into Equation (114) and write

$$\frac{1}{9}\frac{Ze^2}{\pi\varepsilon_0 n_B N_B} = \frac{1}{9}\frac{\kappa_\gamma e^2}{\pi\varepsilon_0} = \frac{hc}{b_w} \to h = \frac{1}{9}\frac{b_w e^2}{\pi\varepsilon_0 c}\kappa_\gamma \tag{117}$$

The following are the values of physical constants according to the Committee on Data for Science and Technology, 2018 adjustment (CODATA 2018)

(UPGM) constant $\kappa_\gamma = 390.225$ dimensionless

Planck constant $h = 6.62607015 \times 10^{-34}$ J·s

Elementary charge $e = 1.602176634 \times 10^{-19}$ C

Vacuum permittivity $\varepsilon_0 = 8.854187817 \times 10^{-12}$ F/m = $C^2/(N\cdot m^2)$

Speed of light: $c = 2.99792458 \times 10^8$ m/s

Boltzmann's constant $k_B = 1.380649 \times 10^{-23}$ J/K

Wien's constant $b = 4.965114231$ dimensionless

Substituting by the given constants above in Equation (117) to verify the results of the derivation, we obtain

$h = 6.62607015 \times 10^{-34}$ J·s = *h Perfect Match*

The relationships between Boltzmann's constant k_B **and the (UPGM)** κ_γ

From Equation (110), we have

$$\frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{Ze^2}{n_B\lambda_B} = N_B k_B \tilde{T}_j \to k_B \tilde{T}_j = \frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{Ze^2}{N_B n_B \lambda_B} \tag{118}$$

Substituting from Equation (116) into Equation (118), we get

$$k_B \tilde{T}_j = \frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{Ze^2}{N_B n_B \lambda_B} = \frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{e^2}{\lambda_B}\kappa_\gamma \tag{119}$$

For $\tilde{T}_j = 0.014 \quad \lambda_B = 0.207$ m

$$k_B = \frac{1}{9}\frac{1}{0.014\times 0.207}\frac{e^2}{\pi\varepsilon_0}\kappa_\gamma \quad (119.1)$$

Substituting by the given constants above in Equation (119.1) to verify the results of the derivation, we obtain

$k_B = 1.3806 \times 10^{-23}$ J/K = k_B 99.996% precision

The photon's thermal volume is defined as

$$\tilde{V}_\gamma = 9\times\lambda_B\times\Delta\tilde{T}_j \quad (120)$$

The relationships between independence probability $\bar{P}_p$ and the (UPGM) κ_γ.

Equation (23) in section 2.9 is written

$$p_j = \frac{h}{\lambda_j}, \text{ where } p_j = m_j\tilde{v}_j,\quad j = 1,\cdots,N \quad (23)$$

Substituting from Equation (23) above into Equation (119), we get

$$k_B\tilde{T}_j = \frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{e^2}{\lambda_B}\kappa_\gamma = \frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{e^2 m_j}{h}\tilde{v}_j\kappa_\gamma,\quad j = 1,\cdots,N \quad (121)$$

The enhanced ideal gas Equation (71) in Section 2.8 is written as

$$\frac{1}{2}m_j\tilde{v}_j^2 = \frac{3}{2}k_B\tilde{T}_j,\quad j = 1,\cdots,N \quad (71)$$

where k_B is Boltzmann constant and $\tilde{T}_j$ is kinetic temperature for particle *j*.

Equating equations (71) and (121) yield

$$\frac{3}{2}k_B\tilde{T}_j = \frac{3}{2}\frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{e^2 m_j\tilde{v}_j}{h}\kappa_\gamma = \frac{1}{2}m_j\tilde{v}_j^2 \rightarrow \tilde{v}_j = \frac{1}{3}\frac{e^2}{\pi\varepsilon_0 h}\kappa_\gamma,\quad j = 1,\cdots,N \quad (122)$$

Substituting by the constants given by CODATA 2018 above in Equation (122) to verify the results of the derivation, we obtain

$$\frac{e^2}{\pi\varepsilon_0 h} = \frac{2.5669699\times 10^{-38}}{1.8432237\times 10^{-44}} = 1.392638\times 10^6 \text{ m/s}$$

$$\tilde{v}_j = 1/3\times 1.392638\times 10^6\times 390.225 \text{ m/s}$$

$$\tilde{v}_j = 1.81189236\times 10^8 \text{ m/s}$$

To Compare the value of $\tilde{v}_j$ with respect the value of the speed of light *c* we calculate the ratio $\frac{\tilde{v}_j}{c}$ as follows:

$$\frac{\tilde{v}_j}{c} = \frac{1.81189236\times 10^8}{2.99792458\times 10^8} = 0.604394 \equiv \%60.4 \text{ of speed of light},\quad j = 1,\cdots,N \quad (123)$$

The independence probability $\bar{P}_p$ value obtained in section 2.9 $\bar{P}_p = 0.6042$ with a ***Perfect Match*** with the value calculated by Equation (123). The reciprocal of the photon's independent probability represents the **Photon Gas Refraction**

Index $E_{\theta\gamma}$, which is a broader physical understanding of the photon's independent probability. Details in Section 2.11.

Substituting from Equation (122) above into Equation (121) we get

$$k_B\tilde{T}_j = \frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{e^2 m_j}{h}\tilde{\upsilon}_j\kappa_\gamma = \frac{1}{9}\frac{1}{\pi\varepsilon_0}\frac{e^2 m_j}{h}\kappa_\gamma\frac{1}{3}\frac{e^2}{\pi\varepsilon_0 h}\kappa_\gamma, \quad j = 1,\cdots,N$$

$$k_B\tilde{T}_j = \frac{1}{9\times 3}\frac{1}{\pi^2\varepsilon_0^2}\frac{e^4\kappa_\gamma^2}{h^2}m_j \to \tilde{T}_j = \frac{1}{27k_B}\left(\frac{e^2\kappa_\gamma}{\pi\varepsilon_0 h}\right)^2 m_j, \quad j = 1,\cdots,N \tag{124}$$

Rewrite Equation (124) we get

$$m_j = 27\left(\frac{\pi\varepsilon_0 h}{e^2\kappa_\gamma}\right)^2 k_B\tilde{T}_j, \quad j = 1,\cdots,N \tag{125}$$

$$e^4 m_j = \tilde{T}_j\bar{k}_\gamma, \text{ where } \bar{k}_\gamma = 27\left(\frac{\pi\varepsilon_0 h}{\kappa_\gamma}\right)^2 k_B = 8.3165\times 10^{-115}\ \mathrm{J\cdot K} \tag{126}$$

i.e. $e^4 m_j$ is linearly proportional to $\tilde{T}_j$, $e^4 m_j \propto \tilde{T}_j$ and the proportional constant is $\bar{k}_\gamma = 8.3165\times 10^{-115}\ \mathrm{J\cdot K}$.

Substitute in Equation (126) $\tilde{T}_j = 1$ we get

$$\tilde{T}_j = 1 \to e^4 m_j = \bar{k}_\gamma = 8.3165\times 10^{-115}\ \mathrm{J\cdot K} \tag{127}$$

Equation (126) relates three values for three variables associated with particles: mass, charge, and temperature. If you have any two of these values, you can determine the value of the third variable. Additionally, this equation indicates that every object has a charge. This equation can be used to define the kinetic temperature of a particle.

Remark

The spherical shape of the bubble was assumed when determining the value of the Universal Photon Gas Modulus (UPGM) κ_γ. Of course, other geometric shapes of the bubble besides the spherical shape, such as the conical, prismatic, cubic, or crystalline shapes under study, can be assumed by following the same deductive steps used in the case of the spherical bubble shape.

2.11. Single-Photon Wavelength

In Section 2.9, Wien's law was derived using $\bar{P}_p$, the photon's independent probability in external vacuum. In the same section, the temperature difference between the material and the vacuum was used to calculate the wavelength of the wave responsible for this difference using equations (95) and (95.1) in section 2.9. In Section 2.10, from Equations (122) and (123), it was shown that the reciprocal of the photon's independent probability is the Photon Gas Refraction Index $E_{\theta\gamma}$. Therefore, the logical connection that summarizes and links all the results obtained in the previous two Sections 2.9 and 2.10, is the calculation of the photon's wavelength. In this section, we will derive the photon's wavelength using its thermal volume from Equation (121) and its volume in vacuum, given by the Heisenberg equation.

First, we will define the Photon Gas Refraction Index (PGRI) $E_{\theta\gamma}$ from Equations (122) and (123) as follows:

$$E_{\theta\gamma} = \frac{c}{\tilde{v}_j} \to \tilde{v}_j = \frac{c}{E_{\theta\gamma}}, \quad j = 1, \cdots, N \tag{128}$$

The relation between the Photon Gas Refraction Index (PGRI) $E_{\theta\gamma}$ and the photon's independent probability $\overline{P}_p$ is given as follows

$$E_{\theta\gamma} = \frac{1}{\overline{P}_p} \tag{129}$$

From Equation (129), the reciprocal of the photon's independent probability represents the value of the (PGRI), *i.e.* how much the photon is free corresponds to how much refraction.

From Equation (89) in Section 2.9 we get the relation between the (PGRI) $E_{\theta\gamma}$ and $\cos\theta_j$ as follows

$$E_{\theta\gamma} = -\left(1 - \frac{1}{\cos\theta_j}\right) \tag{130}$$

For $\theta_j = 67.8715^\circ$ we get

$$E_{\theta\gamma} = 2.655 - 1 = 1.655 \to \frac{1}{1.655} = 0.6042 = \overline{P}_p \tag{131}$$

From Equation (77) in Section 2.8 we get the relation between the (PGRI) $E_{\theta\gamma}$ and $\tilde{T}_S$, $\tilde{T}_j$ as follows

$$\tilde{T}_S = \tilde{T}_j E_{\theta\gamma}^2, \quad j = 1, \cdots, N \tag{132}$$

From Equation (132) $E_{\theta\gamma}$ can be considered a **Thermal Elasticity Factor.**

In order to obtain the relationship between $E_{\theta\gamma}$ and κ_γ substitute from Equation (128) into Equation (122) and write

$$\tilde{v}_j = \frac{1}{3}\frac{e^2}{\pi\varepsilon_0 h}\kappa_\gamma = \frac{c}{E_{\theta\gamma}} \to \frac{1}{E_{\theta\gamma}} = \frac{1}{3\pi}\frac{e^2}{\varepsilon_0 hc}\kappa_\gamma \tag{133}$$

$$\alpha = \frac{e^2}{2\varepsilon_0 hc} = \frac{3}{2}\frac{\pi}{E_{\theta\gamma}\kappa_\gamma} \tag{134}$$

Equation (134) gives the value of the Fine Structure Constant α (alpha).

From Equation (120) in Section 2.10 $\tilde{V}_\gamma$ the photon's thermal volume for photon j is given as follows

$$\tilde{V}_\gamma = 9 \times \lambda_j \times \Delta\tilde{T}_j \tag{120}$$

For photon j the photon's vacuum volume relation, given by the Heisenberg equation is [7]

$$\Delta x_j \cdot \Delta p_j = h, \quad j = 1, \cdots, N \tag{135}$$

The relation between λ_j and Δx_j is given as follows

$$2\Delta x_j = n_j \lambda_j \to \lambda_j = \frac{2\Delta x_j}{n_j}, \quad j = 1,\cdots,N \tag{136}$$

The enhanced ideal gas Equation (71) in Section 2.8 is written as

$$\frac{1}{2} m_j \tilde{v}_j^2 = \frac{3}{2} k_B \tilde{T}_j, \quad j = 1,\cdots,N \tag{71}$$

where k_B is Boltzmann constant and $\tilde{T}_j$ is kinetic temperature for particle *j*. Rewrite Equation (71) as follows:

$$\left(m_j \tilde{v}_j\right) \tilde{v}_j = p_j \tilde{v}_j = 3k_B \tilde{T}_j, \quad j = 1,\cdots,N \tag{137}$$

Rearranging Equation (137) we get

$$p_j \tilde{v}_j = 3k_B \tilde{T}_j \to p_j = \frac{3k_B}{\tilde{v}_j} \tilde{T}_j, \quad j = 1,\cdots,N \tag{138}$$

From Equation (128) we have $\tilde{v}_j = \frac{c}{E_{\theta\gamma}}$, substituting into Equation (138) result in

$$p_j = \frac{3k_B}{\tilde{v}_j} \tilde{T}_j = \frac{3k_B E_{\theta\gamma}}{c} \tilde{T}_j \to \tilde{T}_j = \frac{c}{3k_B E_{\theta\gamma}} p_j, \quad j = 1,\cdots,N \tag{139}$$

$$\Delta \tilde{T}_j = \frac{c}{3k_B E_{\theta\gamma}} \Delta p_j, \quad j = 1,\cdots,N \tag{140}$$

$\Delta \tilde{T}_j$ is the temperature difference before and after emission of the photon, so $\Delta \tilde{T}_j$ is the kinetic temperature of the photon *j*.

Substitute from Equations (136) and (140) into (120), we get

$$\tilde{V}_\gamma = 9 \times \lambda_j \times \Delta \tilde{T}_j = 9 \times \frac{2\Delta x_j}{n_j} \times \frac{c}{3k_B E_{\theta\gamma}} \Delta p_j, \quad j = 1,\cdots,N \tag{141}$$

Rearranging Equation (141), we get

$$\Delta p_j \Delta x_j = \lambda_j \times \Delta \tilde{T}_j \frac{3k_B E_{\theta\gamma}}{2c} n_j, \quad j = 1,\cdots,N \tag{142}$$

Substitute from Equation (135) "Heisenberg condition" into Equation (142), we get

$$h = \lambda_j \times \Delta \tilde{T}_j \frac{3k_B E_{\theta\gamma}}{2c} n_j, \quad j = 1,\cdots,N \tag{143}$$

The photon is a wave packet from the composition of two waves at least *i.e.* $n_j = 2$, consequently Equation (143) is written as follows

$$h = \lambda_j \times \Delta \tilde{T}_j \frac{3k_B E_{\theta\gamma}}{c} \to \lambda_j = \frac{hc}{3E_{\theta\gamma} k_B \Delta \tilde{T}_j}, \quad j = 1,\cdots,N \tag{144}$$

Substitute in Equation (144) with values $\Delta \tilde{T}_j = 0.014\ \text{K}$ and $E_{\theta\gamma} = 1.655$ corresponds to emission angle $\theta_j = 67.8715^\circ$ we get

$$\lambda_j = \frac{hc}{3E_{\theta\gamma} k_B \Delta \tilde{T}_j} = \frac{hc}{3 \times 1.655 \times 0.014 \times k_B} = 0.207\ \text{m} \tag{145}$$

The wavelength of the photon is equal to the wavelength of the wave calculated by Equations (95) and (95.1) in Section 2.9.

2.12. Emission of Accompanying Waves

The value of the Thermal Elasticity Factor (TEF) $E_{\theta\gamma}$, which is 1.655 at a photon gas temperature of 2.739 K. When the photon is transferred to external vacuum, the temperature of the particle's system becomes equal to the vacuum temperature of 2.725 K, and consequently, the value of the (TEF) changes. Using Equation (77), the (TEF) at that temperature can be calculated to be approximately 1.651. Thus, the (TEF) is at its highest value at 2.739 K and at its lowest value at 2.725 K. This variation in the (TEF) between the maximum and minimum values results in a fluctuation in the average velocity given by Equation (122). Since the average velocity is constant along the particle's path, a change in the average velocity corresponds to the particle shifting to a different path, leading to a fluctuation in the wavelength of 20.7 cm, which is the wavelength of the original wave and was calculated in Section 2.9. This fluctuation results in the emission of an accompanying wave with a new wavelength different from the original wavelength of 20.7 cm. It should be noted that the difference in the (TEF) corresponds to a difference in energy, representing the energy consumed by the duality wave for the purpose of transferring energy from the vacuum to the particle and vice versa. The following are the steps for calculating the wavelength of this wave.

Equation (144) gives the relationship between the wavelength, temperature, and the (TEF) $E_{\theta\gamma}$, which is another representation for Wien's Law as follows:

$$\lambda_j = \frac{hc}{3E_{\theta\gamma}k_B\tilde{T}_j}, \quad j = 1,\cdots,N \tag{144}$$

Now rewrite Equation (144) as follows

$$\lambda_j\left(E_{\theta\gamma},\tilde{T}_j\right) = \frac{\bar{\sigma}}{E_{\theta\gamma}\tilde{T}_j}, \quad \bar{\sigma} = \frac{hc}{3k_B} = 4.8\times10^{-3}\ \mathrm{m\cdot K}, \quad j = 1,\cdots,N \tag{146}$$

Equation (146) indicates that the wavelength is function in $E_{\theta\gamma}$ and $\tilde{T}_j$

Let us consider $\tilde{T}_j$ is constant, consequently $\bar{\sigma}_T = \dfrac{\bar{\sigma}}{\tilde{T}_j}$ is constant too, substitute in Equation (146) with $\bar{\sigma}_T$ and rewrite Equation (146) as follows

$$\lambda_j E_{\lambda\theta\gamma} = \bar{\sigma}_T, \quad j = 1,\cdots,N \tag{147}$$

Consider Equation (147) for $\Delta\lambda_j$ and $\Delta E_{\theta\gamma}$ and write

$$\Delta\lambda_j \cdot \Delta E_{\Delta\lambda\theta\gamma} = \bar{\sigma}_T, \quad j = 1,\cdots,N \tag{148}$$

The original wavelength λ_j obtained from Equations (95) and (95.1) in Section 2.9.

$\lambda_{ow} = 0.207\ \mathrm{m}$ and the (TEF), $E_{\lambda\theta\gamma}$ obtained from Equation (89) in section 2.9 $E_{\lambda\theta\gamma} = 1.655$ corresponding to system's temperature 2.739 K, for the same photon's kinetic temperature $\tilde{T}_j = 1$ and the measured (CMB) temperature 2.725

K, substitute in Equation (77) with these two values to determine the corresponding (TEF) $E_{\theta\gamma}$

$$\tilde{T}_S = \tilde{T}_j E_{\theta\gamma}^2 = 2.725 \to E_{\theta\gamma} = \sqrt{2.725} = 1.650757 \cong 1.651 \tag{149}$$

Now from Equations (131) and (149), we can determine $\Delta E_{\Delta\lambda\theta\gamma}$ the variation in (TEF) is given as follows

$$\Delta E_{\Delta\lambda\theta\gamma} = 1.655 - 1.651 = 0.004 \tag{150}$$

Equating Equations (147) and (148) gives

$$\lambda_j E_{\lambda\theta\gamma} = \Delta\lambda_j \cdot \Delta E_{\Delta\lambda\theta\gamma} \to \Delta\lambda_j = \frac{\lambda_{ow} E_{\lambda\theta\gamma}}{\Delta E_{\Delta\lambda\theta\gamma}}, \quad j = 1, \cdots, N \tag{151}$$

Now substitute into Equation (151) with values of λ_{ow}, $E_{\lambda\theta\gamma}$, and $\Delta E_{\Delta\lambda\theta\gamma} = \pm 0.004$ in order to determine the value of $\Delta\lambda_j$

$$\Delta\lambda_j = \frac{\lambda_{ow} E_{\lambda\theta\gamma}}{\Delta E_{\Delta\lambda\theta\gamma}} = \frac{0.207 \times 1.655}{\pm 0.004} = \pm 85.64625 \text{ m}$$

The relationship between the wavelength of the accompanying wave λ_{aw} and the wavelength of the original wave λ_{ow} is given as follows

$$\lambda_{aw} = \lambda_{ow} \pm \Delta\lambda_j = 0.207 \pm 85.646 \text{ m}$$

$$\lambda_{aw} = 85.853 \text{ m} \quad \text{or} \quad \lambda_{aw} = 85.439 \text{ m} \tag{152}$$

Now we consider the case where the (TEF) $E_{\lambda\theta\gamma}$ is constant and the kinetic temperature of the photon $\tilde{T}_j$ is changed due to collision or friction (review the discussion in Section 2.8 regarding $\tilde{T}_j$ and $\tilde{T}_S$), then a system's temperature difference.014 K occurs.

Now rewrite Equation (146) as follows

$$\lambda_j = \frac{\bar{\sigma}_\delta}{\tilde{T}_j E_{\theta\gamma}^2}, \quad \bar{\sigma}_\delta = \bar{\sigma} E_{\theta\gamma}, \quad j = 1, \cdots, N \tag{153}$$

where $\bar{\sigma}_\delta$ is constant.

From Equation (132) $\tilde{T}_S = \tilde{T}_j E_{\theta\gamma}^2$, we rewrite Equation (153) as follows

$$\lambda_j = \frac{\bar{\sigma}_\delta}{\tilde{T}_{\lambda S}}, \quad j = 1, \cdots, N \tag{154}$$

Arranging Equation (154) and write

$$\lambda_j \tilde{T}_{\lambda S} = \bar{\sigma}_\delta \tag{155}$$

Consider the Equation (155) for $\Delta\lambda_j$ and $\Delta\tilde{T}_S$ and write

$$\Delta\lambda_j \cdot \Delta\tilde{T}_{\Delta\lambda S} = \bar{\sigma}_\delta, \quad j = 1, \cdots, N \tag{156}$$

Equating Equations (155) and (156) gives

$$\lambda_j \tilde{T}_{\lambda S} = \Delta\lambda_j \cdot \Delta\tilde{T}_{\Delta\lambda S} \to \Delta\lambda_j = \frac{\lambda_{ow} \tilde{T}_{\lambda S}}{\Delta\tilde{T}_{\Delta\lambda S}}, \quad j = 1, \cdots, N \tag{157}$$

Now substitute into Equation (157) with values of λ_{ow}, $\tilde{T}_{\lambda S}$, and $\Delta\tilde{T}_{\Delta\lambda S}$ in order to determine the corresponding value of $\Delta\lambda_j$ the values are

$$\lambda_{ow} = 0.207 \text{ m}, \quad \tilde{T}_{\lambda S} = 2.739 \text{ K}, \quad \Delta\tilde{T}_{\Delta\lambda S} = \pm 0.014 \text{ K}$$

$$\Delta\lambda_j = \frac{\lambda_{ow}\tilde{T}_{\lambda S}}{\Delta\tilde{T}_{\Delta\lambda S}} = \frac{0.207 \times 2.739}{\pm 0.014} = \pm 40.4980 \text{ m}$$

The relationship between the wavelength of the accompanying wave λ_{aw} and the wavelength of the original wave λ_{ow} is given as follows

$$\lambda_{aw} = \lambda_{ow} \pm \Delta\lambda_j = 0.207 \pm 40.4980 \text{ m}$$

$$\lambda_{aw} = 40.70507 \text{ m} \quad \text{or} \quad \lambda_{aw} = 40.2918 \text{ m} \tag{158}$$

The wavelengths of the accompanying wave 40+ m and 85+ m, calculated using equations (152) and (158) above, corresponds to a standard radio wavelength with a long wavelength. These wavelengths are also one that ordinary radio set detect from distant broadcasting stations. It's worth noting that radio waves with a wavelength greater than 10 m are transmitted to locations far from Earth's transmission stations through reflection and repeated bouncing off the ionosphere and Earth's surface during the night. This is one of the characteristics of all the waves with wavelength greater than 10 m. Furthermore, the calculated wavelengths of 40+ m and 85+ m is based on the assumption that waves transfer energy from the vacuum to particles according to the speed and mass of those particles, as well as energy transfer from particles to the vacuum through collisions and friction. This occurs with air and water molecules in oceans, seas, rivers, and soil. This raises the question of the true source of the radio hiss, detected by regular radio sets, Is the hiss you hear when tune radio frequencies to 3.8 - 3.502 MHz or 7.0 - 7.45 MHz, while these frequencies are not occupied by man-made electromagnetic waves, are they indeed useless noise or un-modulated natural carries reflected from the ionosphere?

2.13. Microscopic Dynamics of the Radiation-Emission Process

In Section 2.2, it was established that the condition for radiation is that the angle θ be 70.52 degrees, and that when this condition is met, the wave-particle duality is broken, and the wave is released and exits from the surface of the material. In Sections 2.9, 2.10, and 2.11, the value of θ at which wave emission or photon emission occurs was 67.78 degrees. Therefore, is there a discrepancy between the results of Section 2.2 and Sections 2.9, 2.10, and 2.11? In this section, we provide a mathematical explanation for this difference in the value of θ. We begin by stating that radiation and emission occur temporally separate but spatially related. This is because the surface of the material consists of a layer with two surfaces. Radiation is an electromagnetic wave, associated with a theta angle of 67.78 degrees, while emission is associated with the breaking of the wave-particle duality, resulting in the emission of a photon. This occurs when the theta angle is 70.52 degrees.

The goal is simple: to prove that 67.87380˚ and 70.52877˚ are not random numbers, but are related to an internal wave. The mathematical representation of such wave is a first-order linear relationship, which is an acceptable approximation of that wave.

The proposed mathematical form of the wave is a sin function, and the reason of this choice is that the solution to Schrödinger equation for the well problem is a sin function [2].

The Taylor expansion for $\sin\theta$ is given

$$\sin(\theta) = \sin(\theta_0) + (\theta - \theta_0)\cos(\theta_0) + R_{\sin} \tag{159}$$

$R_{\sin}$ is the remainder of the sin function given by Lagrange form of remainder as follows

$$R_{\sin} = -\frac{(\theta - \theta_0)^2}{2}\sin(\xi), \quad \xi \in (\theta_0, \theta) \tag{160}$$

$A = \dfrac{1}{R_{\sin}}$, can be regarded as the amplitude of the wave given by Equation (159).

Now, we examine the sin function given by Equation (159) for the values of the radiation condition and photon's emission.

Start angle $\theta_0 = 67.87380^{\circ}$, Final angle $\theta = 70.52877^{\circ}$.

1) Compute the difference $\Delta\theta_{\mathrm{r}}$

$$\Delta\theta_{\mathrm{d}} = 70.52877^{\circ} - 67.87380^{\circ} = 2.65497^{\circ}$$

$$\Delta\theta_{\mathrm{r}} = \Delta\theta_{\mathrm{d}} \times \frac{\pi}{180} = 2.65497^{\circ} \times \frac{\pi}{180} = 0.046337 \text{ rad}$$

2) Substitute with values of θ_0 and θ:

$$\sin(\theta_0) = \sin(67.87380^{\circ}) = 0.9263565$$

$$\cos(\theta_0) = \cos(67.87380^{\circ}) = 0.376647$$

$$\text{L.H.S } \sin(\theta) = \sin(70.52877^{\circ}) = 0.9428089$$

From the right-hand side, we calculate the summation of the first and second term as follows

$$0.9263565 + 0.376647 \times 0.046337 = 0.9263565 + 0.017452692$$

In order to determine $R_{\sin}$ rewrite Equation (159) as follows

$$\begin{aligned} R_{\sin} &= (\sin(\theta) - \sin(\theta_0)) - \Delta\theta_{\mathrm{r}}\cos(\theta_0) \\ &= (0.9428089 - 0.9263565) - 0.017450878 \end{aligned}$$

$$R_{\sin} = 0.0164524 - 0.017450878 = -0.000998478$$

Difference = −0.000998478 ~0.1%.

3. Result

The difference of 0.1% means that the term $R_{\sin}$ represents 0.1% loss of the wave's strength in its journey between the initial angle and the final angle, due to the presence of an obstacle, which is the physical surface of the matter. So the angles 67.87380˚ and 70.52877˚ are related by a sin wave with linear approximation `$\sin(\theta) \approx \sin(\theta_0) + (\theta - \theta_0)\cos(\theta_0)$ with less than 0.1% loss of the wave strength due to the physical surface obstruction of the binding wave, consequently linear approximation is practically valid.

The physical surface of the matter is located at the loss of the strength of the wave occurs at angle ξ, in order to determine the angle ξ rearrange Equation (160) and write as follows

$$R_{\sin} = -\frac{(\theta-\theta_0)^2}{2}\sin(\xi) \to \sin(\xi) = -\frac{2\times R_{\sin}}{(\Delta\theta_r)^2} = \frac{0.001996956}{0.002147117} \cong 0.9300$$

$$\xi = \arcsin(0.9300) \cong 68.44^\circ$$

The barrier that impedes the propagation of the binding wave at a 68.44˚ angle is the atoms on the material's physical surface, which absorb less than 1% of the binding wave's energy and then re-emit this amount of energy as electromagnetic radiation. Before calculating the binding wavelength, it is necessary to clarify the concept of a bifacial surface layer. The concept of a two-sided surface layer is based on the classical concept of free electrons on the surfaces of materials, which act as conductors of electricity and heat. This concept has evolved to suggest that the free electrons are trapped within this layer between its two faces. This is where the similarity between the surface layer and a quantum well lies. The relationship between total energy of the particle and the mean velocity will be used to determine the internal binding wavelength of the surface layer.

We have the relation

$$\sin(\theta) = \sin(\theta_0) + (\theta-\theta_0)\cos(\theta_0) + R_{\sin} \tag{159}$$

The reminder $R_{\sin}$ can be expressed as follows

$$\begin{aligned}&(\theta-\theta_0)\cos(\theta_0) + R_{\sin}\\ &= (\theta-\theta_0)\cos(\theta_0) - \frac{1}{2!}(\theta-\theta_0)^2\sin(\theta_0) + \frac{1}{3!}(\theta-\theta_0)^3\cos(\theta_0)\\ &-\frac{1}{4!}(\theta-\theta_0)^4\sin(\theta_0) + \cdots\end{aligned}$$

Now we group the odd and the even terms and separate them into two terms x_1, x_2 and rewrite the reminder $R_{\sin}$ as follows

$$\begin{aligned}&(\theta-\theta_0)\cos(\theta_0) + R_{\sin}\\ &= (\theta-\theta_0)\cos(\theta_0) + \cos(\theta_0)\sum_1^\infty \frac{1}{2i+1!}(\theta-\theta_0)^{2i+1}\\ &-\sin(\theta_0)\sum_1^\infty \frac{1}{2i!}(\theta-\theta_0)^{2i}\end{aligned} \tag{161}$$

$$x_1 = (\theta-\theta_0) + \sum_1^\infty \frac{1}{2i+1!}(\theta-\theta_0)^{2i+1}, \quad x_2 = \sum_1^\infty \frac{1}{2i!}(\theta-\theta_0)^{2i} \tag{162}$$

Substitute from Equations (162) into Equation (161) and write

$$(\theta-\theta_0)\cos(\theta_0) + R_{\sin} = \cos(\theta_0)x_1 - \sin(\theta_0)x_2 \tag{163}$$

Determine the left-hand side of Equation (163) using the relation $R_{\sin} = 0.0164524 - 0.017450878$ as follows

$$\begin{aligned}&0.376647\times 0.046337+R_{\sin}\\&=0.017452692+0.0164524-0.017450878=0.016454214\end{aligned}\tag{164}$$

Now the expansion of the function $e^{(\theta-\theta_0)}$ in power series is given as follows:

$$\begin{aligned}e^{(\theta-\theta_0)}&=1+(\theta-\theta_0)+\frac{(\theta-\theta_0)^2}{2!}+\frac{(\theta-\theta_0)^3}{3!}+\frac{(\theta-\theta_0)^4}{4!}\\&+\frac{(\theta-\theta_0)^5}{5!}+\cdots+\frac{(\theta-\theta_0)^n}{n!}+\cdots,\end{aligned}$$

Now we group the odd and the even terms and separate them into two terms x_1, x_2 and rewrite the power $e^{(\theta-\theta_0)}$ series as follows

$$e^{(\theta-\theta_0)}=1+(\theta-\theta_0)+\sum_1^\infty\frac{1}{2i+1!}(\theta-\theta_0)^{2i+1}+\sum_1^\infty\frac{1}{2i!}(\theta-\theta_0)^{2i}\tag{165}$$

Substitute from Equations (162) into Equation (165) and write the $e^{(\theta-\theta_0)}$ function as follows

$$e^{(\theta-\theta_0)}=1+x_1+x_2\to e^{(\theta-\theta_0)}-1=x_1+x_2\tag{166}$$

$$\sin(\theta_0)=\sin(67.87380^\circ)=0.9263565\tag{167}$$

$$\cos(\theta_0)=\cos(67.87380^\circ)=0.376647\tag{168}$$

$$\Delta\theta_{\mathrm{r}}=\Delta\theta_{\mathrm{d}}\times\frac{\pi}{180}=2.65497^\circ\times\frac{\pi}{180}=0.046337\text{ rad}\tag{169}$$

Substitute from Equations (164), (167), (168), and (169) into (163) and write

$$0.376647x_1-0.9263565x_2=0.016454214\tag{170}$$

Now we determine the left-hand side of Equation (166) as follows

$$e^{(\theta-\theta_0)}-1=1.047427334-1=0.047427334$$

Rewrite Equation (166) as follows

$$x_1+x_2=0.047427334\tag{171}$$

Equation (170) and Equation (171) are both linear equations into variables x_1, x_2 we solve the two linear equations simultaneously using Cramer's method [9] to determine the variables x_1, x_2.

$$\Delta=\begin{vmatrix}0.376647 & -0.9263565\\ 1 & 1\end{vmatrix}=0.376647+0.9263565=1.3030035$$

$$\Delta_1=\begin{vmatrix}0.016454214 & -0.9263565\\ 0.047427334 & 1\end{vmatrix}=0.016454214+0.9263565\times 0.047427334$$

$$\Delta_1=0.016454214+0.043934619=0.06038833$$

$$\Delta_2=\begin{vmatrix}0.376647 & 0.016454214\\ 1 & 0.047427334\end{vmatrix}=0.376647\times 0.047427334-0.016454214$$

$$\Delta_2=0.017863363-0.016454214=0.001409149069$$

$$x_1=\frac{\Delta_1}{\Delta}=\frac{0.0603883}{1.3030035}=0.046345462\tag{172}$$

$$x_2 = \frac{\Delta_2}{\Delta} = \frac{0.001409149069}{1.3030035} = 0.001018146217 \tag{173}$$

Substitute from equations (172) and (173) into Equation (163) and write

$$(\theta - \theta_0)\cos(\theta_0) + R_{\sin} = 0.046345462 \times \cos(\theta_0) - 0.001018146217 \times \sin(\theta_0) \tag{174}$$

Substitute from equations (174) into Equation (159) and write

$$\sin(\theta) = \sin(\theta_0) + 0.046345462 \times \cos(\theta_0) - 0.001018146217 \times \sin(\theta_0) \tag{175}$$

Rearrange Equation (175) and write

$$\sin(\theta) = (1 - 0.001018146217) \times \sin(\theta_0) + 0.046345462 \times \cos(\theta_0)$$

$$\sin(\theta) = (0.998981854) \times \sin(\theta_0) + 0.046345462 \times \cos(\theta_0) \tag{176}$$

From Part (I) $\sin\theta$ and follows [1]are given as $\cos\theta$

$$\sin\theta_j = \frac{\sqrt{\frac{2\mathcal{E}_j}{m_j}}}{d_j}, \quad \cos\theta_j = \frac{\tilde{\upsilon}_j}{d_j}, \quad j = 1, \cdots, N \tag{177}$$

Substituting from (177) into (176) results the general format of Equation (176)

$$\frac{\sqrt{\frac{2\mathcal{E}_j}{m_j}}}{d_j} = (0.998981854) \times \frac{\sqrt{\frac{2\mathcal{E}_{0j}}{m_j}}}{d_j} + 0.046345462 \times \frac{\tilde{\upsilon}_j}{d_j}, \quad j = 1, \cdots, N \tag{178}$$

Rearrange Equation (178) and write

$$\sqrt{\mathcal{E}_j}\left(1 - (0.998981854) \times \frac{\sqrt{\mathcal{E}_{0j}}}{\sqrt{\mathcal{E}_j}}\right) = 0.046345462 \times \sqrt{\frac{m_j}{2}}\tilde{\upsilon}_j, \quad j = 1, \cdots, N \tag{179}$$

$$\frac{\sqrt{\mathcal{E}_{0j}}}{\sqrt{\mathcal{E}_{1j}}} = \frac{\sin(\theta_0)}{\sin(\theta)} = \frac{0.9263565}{0.9428089} = 0.982549592 \tag{180}$$

By substituting in Equation (179) from relation (180) to define the wave propagation region $(67.87380^\circ \to 70.52877^\circ)$ we get

$$\sqrt{\mathcal{E}_j}\,(1 - 0.998981854 \times 0.982549592) = 0.046345462 \times \sqrt{\frac{m_j}{2}}\tilde{\upsilon}_j$$

$$(1 - 0.981549213) \times \sqrt{\mathcal{E}_j} = 0.046345462 \times \sqrt{\frac{m_j}{2}}\tilde{\upsilon}_j$$

$$0.018450786 \times \sqrt{\mathcal{E}_j} = 0.046345462\sqrt{\frac{m_j}{2}}\tilde{\upsilon}_j \to \sqrt{\mathcal{E}_j} = \frac{0.046345462}{0.018450786} \times \sqrt{\frac{m_j}{2}}\tilde{\upsilon}$$

$$\sqrt{\mathcal{E}_j} = 2.511842151 \times \sqrt{\frac{m_j}{2}}\tilde{\upsilon} \to \mathcal{E}_j = \frac{6.309350993}{2} m_j \tilde{\upsilon}^2 = 3.15467549 m_j \tilde{\upsilon}^2 \tag{181}$$

Rewrite Equation (181) as follows

$$\mathcal{E}_j = 3.15467549 m_j \tilde{\upsilon}^2 = 3.15467549 \frac{p_j^2}{m_j} \tag{182}$$

Equation (23) in Section 2.9 is given as

$$p_j^2 = \frac{h^2}{\lambda_b^2} \tag{23}$$

Substitute from Equation (23) into Equation (182) and write

$$\mathcal{E}_j = 3.15467549 \frac{p_j^2}{m_j} = 3.15467549 \frac{h^2}{m_j \lambda_b^2} \tag{183}$$

The enhanced ideal gas energy given by Equation (71) in Section 2.8 is written as

$$KE = \frac{1}{2} m_j \tilde{v}_j^2 = \frac{3}{2} k_B \tilde{T}_j \to \frac{p_j^2}{m_j} = 3k_B \tilde{T}_j , \tag{184}$$

Substitute from Equation (184) into Equation (183) and write

$$3.15467549 \times 3k_B \tilde{T} = 3.15467549 \frac{h^2}{m_j \lambda_b^2} \to k_B \tilde{T}_j = \frac{h^2}{3m_j \lambda_b^2} = \frac{ch}{\lambda_b} \frac{h}{3m_j \lambda_b c} \tag{185}$$

Rearrange Equation (185) we get

$$\lambda_b = \frac{ch}{k_B \tilde{T}_j} \frac{h}{3m_j \lambda_b c} \tag{186}$$

Rewrite Equation (144) given in Section 2.12 as follows

$$\lambda_j = \frac{hc}{3E_{\theta\gamma} k_B \tilde{T}_j} = \frac{ch}{k_B \tilde{T}_j} \frac{1}{3E_{\theta\gamma}} \tag{187}$$

Comparing equations (186) and (187) result in

$$\frac{1}{3E_{\theta\gamma}} = \frac{h}{3m_j \lambda_b c} \to \lambda_b = \frac{h}{m_j c} E_{\theta\gamma} \tag{188}$$

The ratio $\frac{h}{m_j c}$ is Compton wavelength of the electron is given as

$$\frac{h}{m_e c} = 2.426 \times 10^{-12} \text{ m} \tag{189}$$

From Section 2.12 $E_{\theta\gamma} = 1.655$.

Substitute into Equation (188) with values of $E_{\theta\gamma}$ and $\frac{h}{m_e c}$ we get

$$\lambda_b = 1.655 \times 2.426 \times 10^{-12} \text{ m} \approx 4.02 \times 10^{-12} \text{ m} = 0.00402 \text{ nm} = 4.02 \text{ pm} \tag{190}$$

The value λ_b of the wavelength of the biding wave in the surface layer, calculated in Equation (190) is closer to the order of wavelengths of hard X-rays, whose wavelength ranges between 0.1 nm to 0.001 nm (100 pm to 1 pm), which is the same band used in hospitals to examine bones and detect tumors, in addition to its use in crystallography.

Important Remarks

1) Although there are no assumptions related to the special theory of relativity

in deriving the wavelength of the binding wave, the Compton wavelength of the electron appeared in the equation for calculating the wavelength of the binding wave [Equations (188), (189), and (190)]. It is known that the Compton effect studies the collision of an electron with a photon, considering equations from the special theory of relativity.

2) The calculated wavelength of the binding wave is in the hard X-ray range, and the equations used to determine the wavelength are completely independent of the jumps of atomic electrons between energy levels, which confirms the nature of the X-rays and the existence of another source of electromagnetic radiation other than the jumps of electrons between energy levels.

4. Discussion

In section 2.10, while relating the general gas law to the law of electrostatic attraction, the fine-structure constant emerged, and the following are details concerning this constant.

First introduced in physics by Arnold Sommerfeld in 1916, in the context of Bohr's atomic model (explaining relativistic effects), Sommerfeld added relativistic corrections to the Bohr model found that:

- α appears in equations for electron energy in the atom.
- Describes relativistic and electromagnetic interaction effects.
- Value of α measured with high precision using various experiments (hydrogen spectrum, quantum Hall effect ...).

α is a fundamental constant of nature represents:

- Strength of electromagnetic interaction.
- Ratio of electron velocity in the first Bohr orbit to the speed of light.
- Electromagnetic interaction is relatively weak.
- Electron in atom moves with velocity $\ll c$.
- Appears in Quantum Electrodynamics (QED) equations.

Uses of the Constant:

- Precise calculations of atomic and molecular energies in atomic and molecular physics.
- Interpretation of spectral lines.
- Calculations of electronic transition probabilities.
- In particle physics describes electromagnetic interactions between particles.
- Its precise measurement tests QED validity.

In Section 2.13

The difference between the Two-Dimensional Electron Gas (2DEG) model and the bifacial surface layer concept is that the 2DEG model is derived from the physical properties of semiconductors, while the surface layer concept is based on the condition that angles are related to radiation and emission from materials.

5. Conclusions

- It has been established that the connection between Maxwell's concept and

Schrödinger's concept is through the Lagrangrian, and this is an understanding that was not known in the relationship between the solution of Maxwell's equation and the solution of Schrödinger's equation, and both of them represent a wave, and this new understanding opens up new horizons in theoretical and analytical research to link concepts to each other.

- The kinetic energy of a photon, whether in the vacuum within matter or in external vacuum, could be deduced from a solution to Maxwell's equation. The kinetic energy of the photon deduced in this way served as a means of deriving Wien's displacement law.
- A novel mathematical method was introduced for deriving the fundamental equation of the kinetic theory, from which Wien's law of displacement was deduced. The use of this mathematical method raises new questions about the origin of classical mechanics and its relationship to statistical methods, an area of inquiry unknown to researchers.
- An old problem in physics, namely the relationship between kinetic temperature and system temperature, was solved by deriving a mathematical relationship linking them. An understanding of the meaning of temperature and the transfer of thermal energy from a vacuum to particles and vice versa was presented.
- The detection of waves with a wavelength very close to the wavelength of the hydrogen line is similar to waves received by radio telescopes, while the deduction of the existence of these waves is based on the idea of energy transfer from the vacuum to particles and vice versa. This is a process that takes place continuously in the air, water and surrounding soil, which requires a more accurate re-examination of the source of the waves received by radio telescopes.
- A new view of the atom was introduced after linking the general gas law with the law of electrostatic attraction, leading to the concept of the photon gas. As a result, a new physical constant was discovered named ***Universal Photon Gas Modulus (UPGM)*** κ_γ, related to the fine-structure constant, and new relationships emerged between constants such as Planck's constant, Boltzmann's constant, Wien's constant, vacuum permeability to the electric field, and electron charge. This necessitates a revision to define independent and related constants in physics.
- We conclude from the equations presented in this article that thermal radiation necessitates electromagnetic radiation and the emission of photons.
- The existence of a wave within the hard X-ray wavelength was deduced without resorting to an explanation involving electron jumps between energy levels, which supports the idea of a wave trapped until conditions allow it to radiate.
- The boundary of matter can be modeled as a potential layer confined between two surfaces. This layer performs two concurrent functions. First, it acts as a confining well that prevents the escape of constituent particles into the exter-

nal vacuum by establishing bound states with energies less than the vacuum level, which defines the work function and explains the photoelectric effect. Second, the same layer acts as a potential barrier that permits radiation-like transmission through quantum tunneling, wherein the particle's wave function extends into the classically forbidden region and yields a finite probability of emergence beyond the layer. Thus, the layer simultaneously prohibits classical escape while allowing quantum-mechanical transmission.

It was deduced that there are two accompanying waves to the original wave, with wavelengths of 85.439 m - 85.853 m and 40.2918 m - 40.70507 m corresponding to 3.5087 MHz - 3.4918 MHz and 7.4389 MHz - 7.365502 MHz. These wavelengths are similar to normal radio waves and are natural, not man-made. These waves can be used to establish a low-cost, 24/7, non-stop, and energy-free international communication system covering the entire planet.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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