

Energy Minimization and a Yang-Mills Gap Theorem

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Abstract

The theory of Yang-Mills theory possesses a notable vacuum sector consisting of flat connections, and the mass-gap conjecture asks whether the physical excitations retain a gap between the vacuum sector and the latter by an energy barrier. A naive classical reasoning just using non-vanishing curvature does not suffice, since non-flat configurations of arbitrary small energy can be attained before taking the field equations into account and gauge-fixing. In this paper, we construct an explicit energy-minimization setup which applies to gauge orbits, excludes pure-gauge dynamics, dissects out the vacuum directions in the field configuration space, and evaluates the actual orthogonal distance to the vacuum moduli space stratum. Our approach exploits Coulomb gauge fixing, Morse-Bott nature around the vacuum, elliptic coercivity of the Yang-Mills operator, critical-scale curvature bounds, and the Łojasiewicz-Simon gradient inequality. The central result shows that a smooth solution of the Yang-Mills equations on a closed Riemannian manifold with suitably small scale-critical curvature is flat. The analysis also shows the necessary Hamiltonian inequality for making the jump from the classical variational gap to a quantum mechanical spectral gap.

Keywords

Yang-Mills Theory, Mass Gap, Energy Minimization, Coulomb Gauge, Flat Connections

1. Introduction

The first formulation of Yang-Mills theory as a geometric field theory appeared in 1954, with the idea that internal symmetry gets promoted to local symmetry by way of a connection [1]. The connection itself represents the field, and its curva-

ture measures the strength of the field's gauge force. The classical Yang-Mills energy is the square size of this curvature. Its ground states are exactly flat connections, hence the vacuum geometry is the geometric moduli space of flat connections, not a chosen field. The quantum mass-gap problem, stated as one of the millennium problems by Jaffe-Witten, requires a quantum Yang-Mills theory on 4-dimensional space-time with a positive energy gap above its vacuum [2]. This includes two steps: 1) the construction of the quantum theory and 2) the proof of positivity in its physical Hamiltonian. In this paper, we consider the mechanism underlying the second step: we establish the existence of the necessary classical energy-gap structure.

The first difficulty arises from gauge symmetry. Connections contain redundant degrees of freedom because physically equivalent fields can have inequivalent connections. Uhlenbeck's compactness theorems and Coulomb gauge theory allow us to deal with such redundancies by proving curvature bounds imply controlled gauges [3]. Such an approach is crucial here since energy minimization must be performed on the quotient by gauge transformations, not on the unreduced connection space. The second difficulty is the geometry of the vacuum. A flat connection can have nontrivial deformations. There were early stability and gap results for Yang-Mills fields under certain geometric conditions [4] and more focused isolation results by Bourguignon-Lawson on the nature of curvature and topology [5]. There were also gaps in results by Xu for L^2 norm isolation of vacuum [6] and singularity removal theorem by Tao for higher dimensional Yang-Mills analysis [7]. All of the studies indicate that a true gap proof must be able to distinguish the normal directions of physics, gauging, and vacuums. The third difficulty is energy concentration. Price discovered the scale-sensitive structure of the Yang-Mills curvature [8]. Later, Naber and Valtorta gave the full energy identity for stationary Yang-Mills fields: weak limits could lose energy due to bubbling [9]. Any potential proof will have to account for this structure and cannot simply ignore it; there should either be a lower bound or an explanation for how much energy was lost to bubbles. The fourth difficulty is analyticity near the critical set. In 1983, Simon generalized it to infinite-dimensional geometric variational problems [10] and Łojasiewicz proved a finite-dimensional gradient inequality [11]. Råde brought the analytic-gradient method into the Yang-Mills heat-flow setting, proving continuous H^1 -flow for connections over closed two- and three-dimensional manifolds and polynomial convergence of the flow as time tends to infinity [12]. The Łojasiewicz-Simon technique was used by Feehan to prove an $L^{(d/2)}$ energy gap for Yang-Mills connections on arbitrary closed Riemannian manifold [13]. Such a global analytic step will play a key part in our argument in this paper. Other approaches include lattice constructions and constructive quantum field theories, each taking care of the physical question in a different way. Chatterjee provided an excellent demonstration of the continuum-limit problem by deriving the leading term in the Yang-Mills lattice free energy [14]. Quantum information theory takes a circuitry approach to mass-gap problem via entanglement and is based on an en-

tirely different analytical foundation from this paper's variational approach [15].

The contribution of this paper is a gauge-reduced variational formulation that makes the mechanism behind such a gap transparent. The present work reorganizes the argument around four main points: the Morse-Bott geometry of the vacuum stratum, the Coulomb-sliced transverse Hessian, the scale-critical curvature threshold and the conditional Hamiltonian form inequality needed for a spectral gap. Our goal is therefore to isolate the classical gauge-reduced energy mechanism behind the Yang-Mills gap theorem and identify the Hamiltonian form estimate that a quantum theory would need to preserve.

2. Geometric Setting and Gauge-Invariant Energy

To begin, let X be a closed smooth Riemannian manifold of dimension $d \geq 2$. Let G be a compact Lie group with Lie algebra $\mathfrak{g}$, and let $P \rightarrow X$ be a smooth principal G -bundle. We denote the space of smooth connections on P by $\mathcal{A}(P)$, and the smooth gauge group by $\mathcal{G}(P)$. An invariant inner product on $\mathfrak{g}$, together with the Riemannian metric on X , gives pointwise norms on adjoint-valued forms.

For a connection A , the curvature is

$$F_A = dA + \frac{1}{2}[A \wedge A]. \quad (1)$$

And the Yang-Mills energy is

$$\mathcal{E}(A) = \frac{1}{2} \int_X |F_A|^2 \, d\text{vol}_g. \quad (2)$$

A gauge transformation $u \in \mathcal{G}(P)$ acts on A by

$$u(A) = uAu^{-1} - (du)u^{-1}. \quad (3)$$

The curvature transforms by conjugation,

$$F_{u(A)} = uF_Au^{-1}. \quad (4)$$

Equation (4) shows that the pointwise norm of curvature is gauge-invariant. Hence the functional in Equation (2) descends to the quotient space $\mathcal{A}(P)/\mathcal{G}(P)$. This quotient is the exact classical configuration space.

The first variation is obtained by differentiating the curvature along the path $A + ta$, where $a \in \Omega^1(X; \text{ad } P)$. The derivative at the origin is

$$\left. \frac{d}{dt} F_{A+ta} \right|_{t=0} = d_A a. \quad (5)$$

Using the closedness of X , integration by parts gives

$$D\mathcal{E}_A(a) = \int_X \langle d_A^* F_A, a \rangle \, d\text{vol}_g. \quad (6)$$

The Euler-Lagrange equation is therefore

$$d_A^* F_A = 0. \quad (7)$$

A connection satisfying Equation (7) is called Yang-Mills. A flat connection

satisfies $F_A = 0$, so it is an absolute minimizer of the energy. The flat moduli space is

$$\mathcal{M}_{\text{flat}}(P) = \{A \in \mathcal{A}(P) : F_A = 0\} / \mathcal{G}(P). \quad (8)$$

This point is essential for the rest of this work. A non-flat connection has positive Yang-Mills energy, but this positivity is not uniform on the full space of connections. A genuine gap appears only after the Yang-Mills critical equation is imposed, the gauge directions are removed, and the normal direction to the flat vacuum stratum is isolated.

3. Coulomb Slicing and the Morse-Bott Vacuum Valley

The local analysis in this section assumes that the bundle P admits at least one flat connection. We fix one such connection and denote it by Γ . Thus the local discussion takes place near a chosen point $[\Gamma]$ of the flat moduli space. In this section, a flat connection Γ is called regular when the flat moduli space is locally represented near $[\Gamma]$ by a smooth finite-dimensional stratum whose tangent space is the harmonic space $\mathcal{H}_\Gamma^1$. The flat stratum is called Morse-Bott at Γ when the kernel of the Coulomb-sliced Yang-Mills Hessian is exactly $\mathcal{H}_\Gamma^1$, and the Hessian is strictly positive on the L^2 -orthogonal complement N_Γ . A Sobolev neighborhood of Γ means a sufficiently small $W^{1,2}$ -neighborhood inside the Coulomb slice $d_\Gamma^* a = 0$, after the local gauge freedom has been fixed. A nearby connection can be written in the form $\Gamma + a$, with $a \in \Omega^1(X; \text{ad } P)$. Infinitesimal gauge motion at Γ is generated by forms of the type $d_\Gamma \phi$, where $\phi \in \Omega^0(X; \text{ad } P)$. The Coulomb slice removes this motion through the condition

$$d_\Gamma^* a = 0. \quad (9)$$

The curvature expansion near the flat connection is

$$F_{\Gamma+a} = d_\Gamma a + \frac{1}{2}[a \wedge a]. \quad (10)$$

Since $F_\Gamma = 0$, the linearized curvature is simply $d_\Gamma a$. The second variation of the energy at Γ is

$$D^2 \mathcal{E}_\Gamma(a, a) = \|d_\Gamma a\|_{L^2(X)}^2. \quad (11)$$

Inside the Coulomb slice, the Hessian is the elliptic operator

$$H_\Gamma = d_\Gamma^* d_\Gamma. \quad (12)$$

The kernel of this operator contains infinitesimal flat deformations. We denote the harmonic space at Γ by

$$\mathcal{H}_\Gamma^1 = \{h \in \Omega^1(X; \text{ad } P) : d_\Gamma h = 0, d_\Gamma^* h = 0\}. \quad (13)$$

This finite-dimensional space is the tangent space to the flat moduli stratum at a regular flat point. Let N_Γ be the L^2 -orthogonal complement of $\mathcal{H}_\Gamma^1$ inside the Coulomb slice. The decomposition

$$\ker d_\Gamma^* = \mathcal{H}_\Gamma^1 \oplus N_\Gamma \quad (14)$$

separates the vacuum tangent directions from the physical normal directions.

The orange curve in **Figure 1** is the flat moduli stratum $\mathcal{M}_{\text{flat}}$. Along this curve, the Yang-Mills energy remains zero, so motion inside the vacuum costs no energy. The blue surface rises away from the orange valley in the normal direction. That rise represents Hessian coercivity after gauge directions and flat directions have been removed.

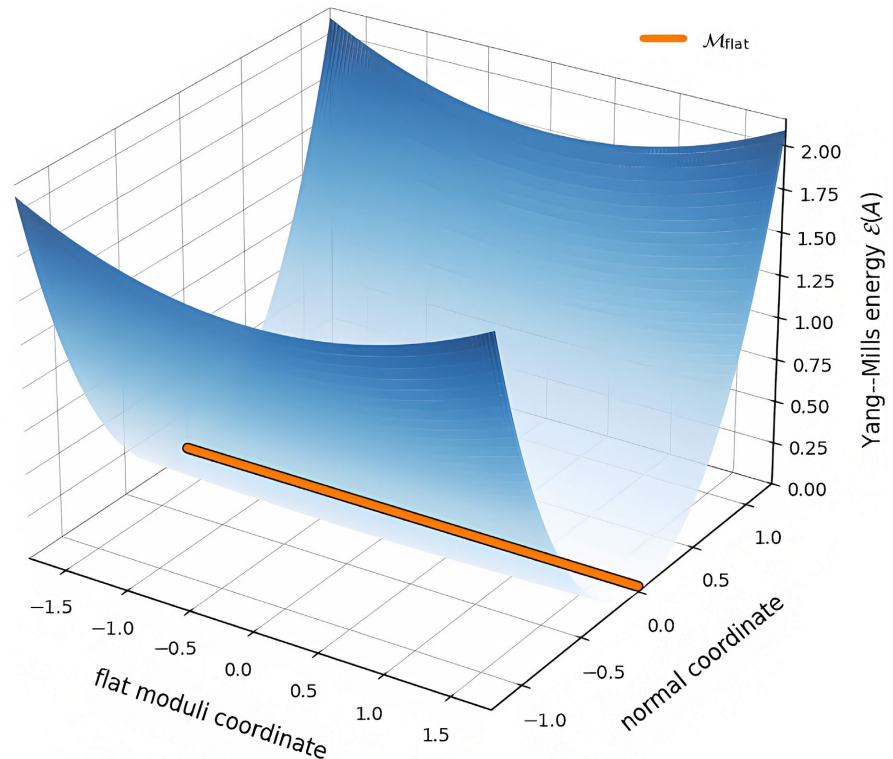


Figure 1. Morse-Bott vacuum valley for the Yang-Mills energy.

With the following theorem, we can record the local variational statement.

Theorem 1.

Let X be a closed smooth Riemannian manifold, let G be a compact Lie group, and let $P \rightarrow X$ be a smooth principal G -bundle. We assume that P admits a flat connection Γ and further that Γ is regular and that the flat moduli space is Morse-Bott at Γ . By fixing an exponent $p > d/2$, then there exist constants $r_\Gamma > 0$ and $c_\Gamma > 0$ such that every perturbation $a \in N_\Gamma$ satisfying

$$d_\Gamma^* a = 0, \|a\|_{W^{1,p}(X)} < r_\Gamma \quad (15)$$

obeys the transverse coercive estimate

$$\mathcal{E}(\Gamma + a) \geq c_\Gamma \|a\|_{W^{1,2}(X)}^2. \quad (16)$$

This estimate is local, gauge-reduced and transverse to the flat moduli stratum.

Proof of Theorem 1.

The Coulomb condition $d_\Gamma^* a = 0$ removes infinitesimal gauge directions. The Morse-Bott hypothesis states that the remaining zero directions of the Hessian are

exactly the infinitesimal flat deformations $\mathcal{H}_\Gamma^1$. Hence the restriction of $H_\Gamma = d_\Gamma^* d_\Gamma$ to the normal space N_Γ has no kernel. Since H_Γ is elliptic and self-adjoint on the closed manifold X , its first positive spectral value on N_Γ is strictly positive. Therefore, there exists $\lambda_\Gamma > 0$ such that

$$\langle H_\Gamma b, b \rangle_{L^2} \geq \lambda_\Gamma \|b\|_{W^{1,2}(X)}^2 \text{ for every } b \in N_\Gamma. \quad (17)$$

The Taylor expansion of $\mathcal{E}$ at Γ has the quadratic part in Equation (11), while the remaining terms are higher order in a . Sobolev multiplication controls these higher-order terms in a sufficiently small $W^{1,2}$ -neighborhood of Γ . Thus r_Γ can be chosen so that

$$\left| \mathcal{E}(\Gamma + a) - \frac{1}{2} \|d_\Gamma a\|_{L^2(X)}^2 \right| \leq \frac{1}{4} \|d_\Gamma a\|_{L^2(X)}^2 \quad (18)$$

for every $a \in N_\Gamma$ satisfying the stated smallness condition. Combining this estimate with the transverse Hessian bound gives the claimed lower bound.

4. Transverse Hessian Spectrum

The numerical source of the local gap is the first positive eigenvalue of the sliced Hessian. This eigenvalue appears only after two removals. The first removal is gauge motion. The second removal is infinitesimal motion inside the flat moduli space.

First, let the eigenvalues of H_Γ on the Coulomb slice be ordered as

$$0 = \mu_1 = \dots = \mu_k < \mu_{k+1} \leq \mu_{k+2} \leq \dots \quad (19)$$

The multiplicity k is the dimension of the harmonic space $\mathcal{H}_\Gamma^1$. The number

$$\lambda_\Gamma = \mu_{k+1} \quad (20)$$

is the first positive transverse eigenvalue. It controls the local normal energy cost.

The initial zero eigenvalues (Figure 2) represent directions that don't produce physical energy growth: gauge motion and infinitesimal motion along the flat moduli space. The first positive eigenvalue, λ_Γ , marks the beginning of the

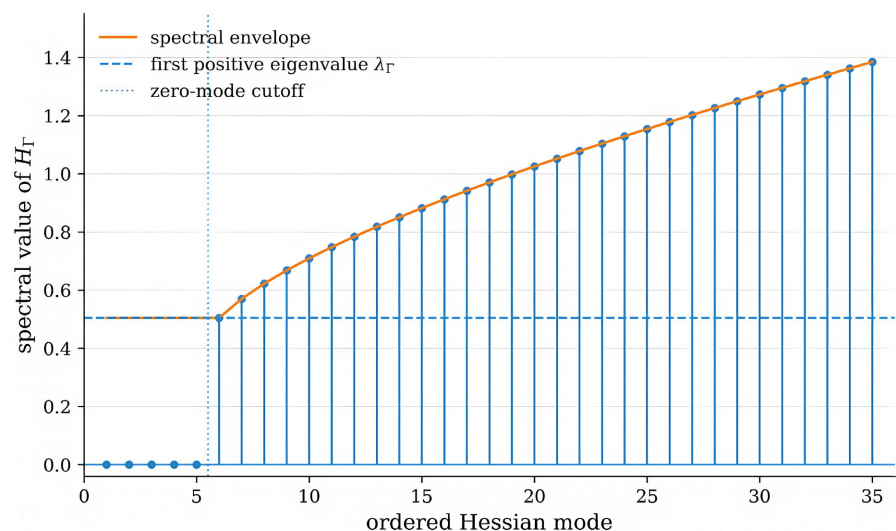


Figure 2. Transverse Hessian spectrum after Coulomb slicing.

genuine transverse spectrum. This is the quantitative source of coercivity. Once the zero modes are removed, every normal physical perturbation costs at least a fixed quadratic amount of energy.

Proposition 2.

The first positive transverse eigenvalue λ_{Γ} gives the quadratic lower bound for normal perturbations at the flat stratum.

Proof of Proposition 2.

Here, let $a \in N_{\Gamma}$. The spectral theorem applied to the self-adjoint elliptic operator H_{Γ} gives

$$\langle H_{\Gamma} a, a \rangle_{L^2} \geq \lambda_{\Gamma} \|a\|_{L^2}^2. \quad (21)$$

Elliptic regularity on the Coulomb slice strengthens this bound to the Sobolev estimate in Equation (16). The energy expansion in Equation (11) then converts Hessian positivity into energy positivity. No direction in N_{Γ} can approach the vacuum with nonzero Sobolev size and vanishing energy. The proposition gives the exact mathematical meaning of “energy minimization” in this problem. The energy is not uniformly coercive in all directions. It is coercive in the normal part of the gauge-reduced configuration space. A connection can be non-flat and have very small energy if it is not a Yang-Mills critical point. The gap theorem concerns critical points because the field equation removes arbitrary small fluctuations that don’t solve the variational problem. Proposition 2 gives the quantitative form of the local gap. The number λ_{Γ} is the spectral constant that turns normal displacement from the vacuum stratum into positive quadratic energy.

5. Scale-Critical Curvature and Hamiltonian Interpretation

The local theorem works inside a fixed neighborhood of the flat moduli space. A global statement requires a reason that a small Yang-Mills critical point enters that neighborhood. The correct control quantity is the scale-critical curvature norm. Under the natural Yang-Mills scaling, curvature has weight two. The L^p -norm of curvature scales with exponent

$$\alpha(d, p) = \frac{d}{p} - 2. \quad (22)$$

The scale-invariant choice is obtained by setting this exponent equal to zero. This gives

$$p = \frac{d}{2}. \quad (23)$$

This calculation for several dimensions is shown in **Figure 3**. The horizontal dashed line marks the scale-invariant level $\alpha = 0$. Each curve crosses this line exactly at $p = d/2$. That crossing is the reason the $L^{d/2}$ -curvature norm is the natural threshold in the gap theorem. Below that exponent, small-scale curvature concentration becomes stronger under rescaling. Above it, the norm loses sensitivity to the critical Yang-Mills scale.

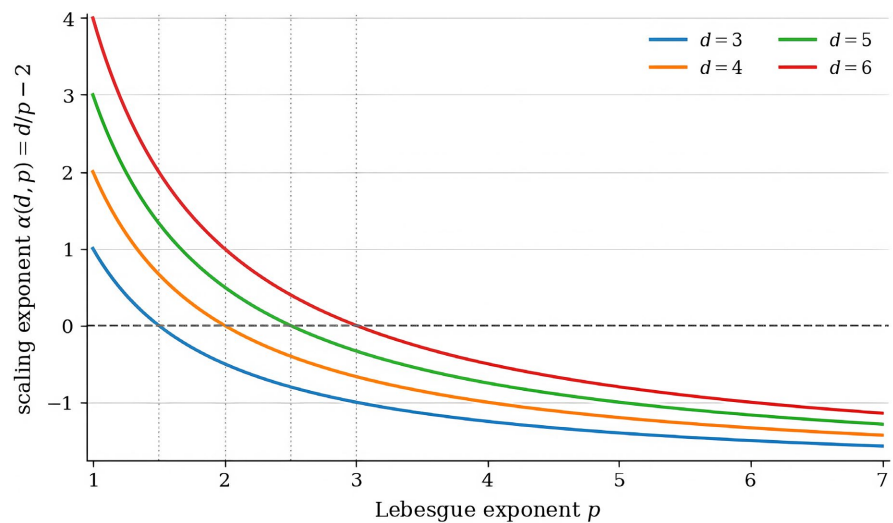


Figure 3. Scale-critical curvature exponent map.

The global theorem has two readings. If P admits flat connections, the theorem says that every sufficiently small Yang-Mills critical connection is flat. And if P admits no flat connection, the same statement means that no smooth Yang-Mills critical connection on P can satisfy the small $L^{d/2}$ -curvature bound. Thus the theorem does not assume in advance that the flat moduli space is nonempty; rather, small Yang-Mills curvature forces flatness, and flatness is impossible on bundles that admit no flat connection.

Theorem 3.

Let X be a closed smooth Riemannian manifold of dimension $d \geq 2$, then G be a compact Lie group and $P \rightarrow X$ be a smooth principal G -bundle. There exists a constant

$$\varepsilon = \varepsilon(X, g, G, P) > 0 \quad (24)$$

such that every smooth Yang-Mills connection A on P satisfying

$$\|F_A\|_{L^{\frac{d}{2}}(X)} \leq \varepsilon. \quad (25)$$

is flat. In particular, if P admits no flat connection, then no smooth Yang-Mills connection on P satisfies this small-curvature bound.

Proof of Theorem 3.

We choose $\varepsilon > 0$ below the small-curvature threshold in Uhlenbeck gauge fixing and below the threshold in Feehan's $L^{d/2}$ -energy gap theorem. Let A be a smooth Yang-Mills connection satisfying $\|F_A\|_{L^{\frac{d}{2}}(X)} \leq \varepsilon$. The smallness of the scale-critical curvature gives local Coulomb gauges on a finite cover of X , with uniform Sobolev control on the local connection forms. Feehan's global distance-to-flat argument promotes these local estimates to a global statement: after a gauge transformation, A lies in a sufficiently small Sobolev neighborhood of a flat connection Γ . More precisely, for some fixed exponent $p > d/2$, the gauge representative of A is close to Γ in a $W^{1,p}$ -connection chart.

This is the analytic neighborhood where the Yang-Mills functional is real analytic and the Łojasiewicz-Simon gradient inequality applies. Hence there exist constants $C > 0$ and $\theta \in [1/2, 1)$ such that

$$\|\mathcal{E}(B) - \mathcal{E}(\Gamma)\|^{1-\theta} \leq C \|d_B^* F_B\|_{\mathcal{X}^*} \quad (26)$$

for every connection B in this chart. Here $\mathcal{X}^*$ is the dual Banach space in which the Yang-Mills gradient is measured.

Apply the inequality to the gauge representative of A . Since A is Yang-Mills,

$$d_A^* F_A = 0. \quad (27)$$

The right-hand side vanishes. Therefore

$$\mathcal{E}(A) = \mathcal{E}(\Gamma). \quad (28)$$

The connection Γ is flat, so $\mathcal{E}(\Gamma) = 0$. Hence $\mathcal{E}(A) = 0$. Since

$$\mathcal{E}(A) = \frac{1}{2} \int_X |F_A|^2 \, d\text{vol}_g \quad (29)$$

and the integrand is nonnegative, $F_A = 0$. Thus A is flat.

Theorem 3 is a classical Yang-Mills gap statement which concerns smooth Yang-Mills critical connections on closed Riemannian manifolds under a small scale-critical curvature hypothesis. The role of Theorem 3 is to identify the classical variational mechanism that such a quantum construction would have to retain.

The passage from the classical gap theorem to a quantum mass gap is conditional. One must first construct a gauge-invariant quantum Yang-Mills theory with a physical Hilbert space and a nonnegative self-adjoint Hamiltonian.

Theorem 4.

Assume that a gauge-invariant quantum Yang-Mills theory has been constructed with physical Hilbert space $\mathcal{H}_{\text{phys}}$, nonnegative self-adjoint Hamiltonian H , normalized vacuum vector Ω , and closed Hamiltonian quadratic form $\mathfrak{h}$. Assume that $H\Omega = 0$ and that there exists $m_0 > 0$ such that

$$\mathfrak{h}[\Psi] \geq m_0 \|\Psi\|_{\mathcal{H}_{\text{phys}}}^2 \quad \text{for every } \Psi \perp \Omega. \quad (30)$$

Then H has a spectral gap above the vacuum, and

$$\Delta \geq m_0. \quad (31)$$

Proof of Theorem 4.

Let E_H be the spectral resolution of H . The closed quadratic form satisfies

$$\mathfrak{h}[\Psi] = \int_{\text{Spec}(H)} \lambda \, d\|E_H(\lambda)\Psi\|^2. \quad (32)$$

Suppose, for contradiction, that H has nonzero spectrum in the interval $(0, m_0)$. Then there exists a nonzero vector

$$\Psi \in \text{Ran } E_H((0, m_0)) \quad (33)$$

with $\Psi \perp \Omega$. The spectral integral gives

$$\mathfrak{h}[\Psi] < m_0 \|\Psi\|_{\mathcal{H}_{\text{phys}}}^2, \quad (34)$$

which contradicts the assumed form inequality. Hence the spectrum has no non-zero part in $(0, m_0)$, and the gap satisfies $\Delta \geq m_0$.

6. Conclusions

The frame developed in this paper separates the energy-gap mechanism into its essential geometric parts. The vacuum is the flat moduli stratum. Gauge directions are removed by Coulomb slicing. Tangent directions to the flat stratum are separated by the Morse-Bott decomposition. The Hessian becomes coercive only on the normal complement. The local theorem proves that the Yang-Mills energy rises quadratically in those normal directions. The transverse Hessian spectrum gives the quantitative constant behind this rise. The global theorem then uses the scale-critical $L^{d/2}$ -curvature norm to bring a small Yang-Mills critical connection into the analytic neighborhood of the flat stratum. The Łojasiewicz-Simon inequality turns vanishing Yang-Mills gradient into equality of energy with a flat connection. Since flat connections have zero energy, the curvature must vanish.

This study therefore proves a classical variational gap theorem for Yang-Mills critical points. It also gives a precise conditional spectral criterion: a quantum mass gap follows once an independently constructed Yang-Mills Hamiltonian has a closed quadratic form that dominates the norm on the vacuum-orthogonal sector. Arbitrary non-flat fields can have very small classical energy. Non-flat Yang-Mills critical points below the scale-critical curvature threshold cannot occur.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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