

Grassmannian Uncertainties and Rational Invariants

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Abstract

The joint image Grassmannian in epipolar geometry is iterated by base points of a pencil of conics. Rational points in image, singular two-dimensional minors, hyperbolic alternating forms and elliptic curve segments are discussed. Permutations in hyperbolic alternating are proposed for Grassmann variables leading to fermions. Unit values of binary forms are proposed for bosons. A fixed point Grassmannian partition function is discussed in conjunction with an application to physical fields. The amount of available world point information with respect to processable bits is estimated.

Keywords

Joint Image Grassmannian, Fermions, Grassmann Variables, Hyperbolic Alternating Forms, Partition Function

1. Introduction

Minkowski spacetime $\mathcal{M}$ seems to be a subset of complex projective geometry [1]. The question is about a relation between the joint image Grassmannian in epipolar geometry and Grassmann variables of fermions [2] [3]. This paper relates $\mathcal{M}$ to iterations of base points of a pencil of conics on complex plane by algebraic formulations. If iterations are chaotic, it is claimed that the simplest cycles constitute a hyperbolic border between the Julia and Fatou set even for a quadratic Newton method [4]. This border is used to explain Minkowski spacetime and the existence of a velocity of light. In projective lines in camera images, massive objects seem to be fixed points e_i (base points) of a pencil of conics. Being Weierstrass $\wp$ -functions base points e_i are roots of a cubic polynomial [5]. Rational substitutions of base points should generate modular invariants. As observable variables, it is claimed that its iterations are embeddable into a renormalization group

(RG) flow. The binary invariant should penetrate spacetime physical fields as a complex stability region. A quadratic map of binary curvature has been set responsible for quantum entanglement and entangled spacetime [6]. The measurement problem in physics is coupled to an imaging device. Devices with the highest information densities are up to four independent cameras. A measured information flow is supposed to be an optimal, fast algorithm of an input flow. Applications of elliptic curves in physics are limited by a number of nonlinear phenomena [7]. The present paper sets applications of elliptic curves in context to accessible approximations of bifurcation flows. It is claimed that one-dimensional chaotic maps of base points of a pencil of conics enable a linear theory. Also, hyperelliptic orthogonal substitutions of four complex points are exactly integrable mechanical systems [8]. Section 2 uses rational substitutions to epipolar geometry. Section 3 discusses reduction to elliptic curves by hyperbolic forms in conjunction with solutions of the fundamental matrix by vanishing minors. Section 4 links permutations in hyperbolic forms with a spinor group. Projective geometry is used to formulate a partition function in Section 5. Camera imaging of world points requires to solve singular determinants in terms of its vectorized matrices. The paper claims a relation between the joint image Grassmannian and Grassmann variables based on vanishing 2·2 minors. Further on, 2·2 minors of world variables and 2·2 alternating forms in hyperelliptic period 4·2 period matrices Π are compared. The common rational feature is a vanishing 2·2 form which is called alternating hyperbolic form. A fast-shuffling algorithm is proposed where a pair of found Grassmann variables indicates a zero determinant. Rationalized points enable to relate epipolar geometry to elliptic curves for base points of a pencil of conics. Here Grassmann variables result from permutation matrices of base points. This treatment is proposed to integrate fermions into projective space where the joint image Grassmannian and Grassmann variables are closely related. The paper claims that hits of world points in images allow to define partition functions in terms of base points. For a polynomial partition function the base point argument is set in context to a fugacity RG flow of points. The paper claims that massive world points are mappable to orbits on complex plane superimposed by a flow of modular invariants.

2. Epipolar Geometry and Rational Invariants

2.1. Existing Matter and Observable Reality

Image cameras are measuring devices of high-data transfer which require software for handling epipolar constraints. Introducing rectangular projection matrices $P^{(m,n)}$ in projective space from $\mathbb{P}^m$ to $\mathbb{P}^n$ the joint image for several cameras results from 2 - 4 reconstruction equations $X = P^{(4,3)}x$. The paper treats the map $\mathbb{P}^3$ to $\mathbb{P}^2$ by base points as observable reality (OR). Fixed points (base points) of pencils of conics in projective space are introduced as independent of images as existing matter. EM is claimed to be invariant rational base points. Fractional substitutions γ in two-dimensional minors as binary forms should be closely related

to matter. The alternating construct $(XY) = X_1Y_2 - X_2Y_1$ is a quadratic form $(\bar{\theta}_1\theta_2)\hat{\gamma}\begin{pmatrix}\theta_1\\\theta_2\end{pmatrix}$. The paper describes homogeneous variables by four variables θ_1, θ_2 and its conjugates $\bar{\theta}_1, \bar{\theta}_2$. If γ is a rational transformation the quadratic form is called rational invariant. This paper emphasizes $\gamma = \gamma^{(3)}$ as a quadratic map of a cubic polynomial. A joint image ξ in projective plane $\mathbb{P}^2$ are epipolar constraints for world points X, Y in projective space $\mathbb{P}^3$. Fundamental, trifocal and quadrifocal tensor $\hat{F}, \hat{T}, \hat{Q}$ yield constraints [2]

$$\mathbf{x}\hat{F}\mathbf{x} = \mathbf{x}\varepsilon^{(4)} \overbrace{\prod_{i=1,\dots,4} P_i^{(4,3)}} \mathbf{x} = 0 \quad (1)$$

$$\mathbf{x}\hat{T}\mathbf{x} = \mathbf{x}\varepsilon^{(4)} \overbrace{\prod_{i=1,\dots,4} P_i^{(4,3)}} \mathbf{x} = 0, \quad \mathbf{x}\hat{Q}\mathbf{x} = \mathbf{x}\varepsilon^{(4)} \overbrace{\prod_{i=1,\dots,4} P_i^{(4,3)}} \mathbf{x} = 0 \quad (2)$$

where $\varepsilon^{(n)}$ is the Levi-Civita symbol in n dimensions. Here anti-symmetrized three-component epipoles $\mathbf{e}^t \mathbf{x} = 0$ are denoted by a bracket. This 9(8), 27 and 81-point vector $\text{vec}\hat{F}$, $\text{vec}\hat{T}$, $\text{vec}\hat{Q}$ of a vectorized matrix denotes an over constrained system. An n^{th} order determinant $\hat{K}$ vanishes if a number of $(r+1)^2$, $(K = \hat{K}^t)$, $\frac{1}{2}(r+1)(r+2)$, $(K \neq \hat{K}^t)$ of its r^{th} minors is zero [9] [10]. This is used to propose a fast algorithm by finding three vanishing 2-2 minors to describe 9(6) vanishing 2-2 minors of $\hat{F}$, $\hat{T}$, $\hat{Q}$. Equations (1), (2) constitute a window by up to four linear independent camera images in $\hat{F}, \hat{T}, \hat{Q}$ in $\mathbb{P}^2$ which should hold for every measuring device for detecting world points.

2.2. Epipolar Geometry and Kummer Surface

The fundamental matrix is part of Kummer and Weddle surfaces $K(X)$ and $W(Y)$ as follows [9]. For a matrix $M_{n,m}$ of n rows and m columns $K(X)$ is a vanishing $M_{4,4}$ determinant

$$\det \hat{K} = \det(X_0 | X_1 | X_2 | X_3) = \det(Y_0 | Y_1 | Y_2 | Y_3) = 0 \quad (3)$$

Equation (3) is equivalent to a Grassmannian $G^{(4)}$ of four world points. The aim is to formulate $G^{(4)}$ invariant with respect to a fractional substitution γ of a given parameter θ . Four world points $X = (1, -\theta, \theta^2, 1)$ and $Y = (1, -\theta, \theta^2, -\theta^3)$ are rational on a twisted cubic curve parametrized by parameter θ . Then the γ -invariant condition $Y^t \hat{K} Y = 0$ reads

$$2(\theta_k \theta_{k+1})^2 (\theta_{k+2} \theta_k) (\theta_{k+2} \theta_{k+1}) - \phi^{(3)}(\theta_k) \phi^{(3)}(\theta_{k+1}) = 0. \quad (4)$$

which requires three parameters θ_k , θ_{k+1} and θ_{k+2} . Homogenous binary invariants $(XY) = X_1Y_2 - X_2Y_1$ and polynomials $\phi^{(n)}(z) = \sum_{i=0,\dots,n} \binom{k}{i} a_{n-i} z^i$ in (4) are added by a five-dimensional Grassmannian $G^{(5)} \simeq \varepsilon^{(5)}$ as a bordered determinant $\begin{pmatrix} l \\ l \end{pmatrix}$

$$G^{(5)} = \det M_{5,5} = \varepsilon^{(5)} \prod_{\mu=1,\dots,5} P_{\mu}^{(5,4)} X_{\mu} = \det \begin{bmatrix} K & l \\ l & o \end{bmatrix} = \binom{l}{l} = (l\wp_{+++})^2. \quad (5)$$

where $\varepsilon^{(n)}$ is the Levi-Civita symbol of size n . World points $X_{\mu} = (X, \lambda) = P_{\mu}^{(5,4)} X$ on world planes $\lambda \sum_i l_i X_i = 0$ of homogeneous $l_i = (l_0, -3l_1, 3l_2, l_3)$, $l_i = l_1^{3-i} l_2^i$ yield a γ -invariant quadric of derivatives $\wp_{+++}$ of the hyperelliptic $\wp$ -function on universal covering $u_{\pm}$. For $\det K = 0$, the Grassmannian $G^{(5)}$ is rationalizable due to the identity [9]

$$\binom{l}{l} \binom{m}{m} - \binom{l}{m}^2 = \binom{lm}{lm} \det K \rightarrow 0$$

The joint image Grassmannian is

$$\det K = \det M_{4,4} = G^{(4)} = \varepsilon^{(4)} \prod_{i=1,\dots,4} P_i^{(4,3)} \mathbf{x}_i. \quad (6)$$

Next the map $\mathbb{P}^3$ to $\mathbb{P}^2$ is discussed algebraically. World points on $K(X)$ are on hyperelliptic curves $\phi^{(6)}(\theta)$ generating six base points of a pencil of quadrics in $\mathbb{P}^3$. Image points of $\hat{F}$, $\hat{T}$, $\hat{Q}$ belong to a hyperelliptic curve $\phi^{(4)}(\theta)$ of four base points in a pencil of conics in $\mathbb{P}^2$ reducible to an elliptic curve $\phi^{(3)}(\theta)$. Pencils of quadrics obey symmetric world matrices and image matrices. The epipolar constraint $\mathbf{x}\hat{F}\mathbf{x} + \mathbf{x}\hat{F}'\mathbf{x} = C_1(\mathbf{x}) + \wp(u, \omega)C_2(\mathbf{x}) = 0$ is equivalent to a pencil of conics $C_1(\mathbf{x}), C_2(\mathbf{x})$ in $\mathbb{P}^2$. Base points $\mathbf{x}(\theta)$ depend on the Weierstrass function $\theta = \wp(u, \omega)$. For $\mathbf{x} = \mathbf{x}(\theta)$ epipolar geometry $\mathbf{x}\hat{F}\mathbf{x} + \mathbf{x}\hat{F}'\mathbf{x} = \phi^{(4)}(\theta)$ includes numerous invariant ways reducing a quartic $\phi^{(4)}(\theta)$ to Weierstrass form $\phi^{(3)}(\wp(u, \omega))$. Cubic modular invariant congruences mod $\phi^{(3)}(\theta)$ yield similar rationalized world points X and Y : $X = (\mathbf{x}, 1) = (1, -\theta, \theta^2, 1) = Y$. Binary invariance is adding points on elliptic curves where $\phi^{(3)}(\wp(u, \omega)) = \det(\hat{C}_1 + \wp(u, \omega)\hat{C}_2) \rightarrow \phi^{(4)}(\theta)$ [5]. Mean values of world points X in images $\mathbf{x}$ in pencils of quadrics in $\mathbb{P}^3$ and $\mathbb{P}^2$ allow a fast algorithm in terms of singular r^{th} minors. Therefore, three vanishing 2-2 minors in a fundamental matrix $\hat{F}$ entering the 3-way $\hat{T}$ and the 4-way $\hat{Q}$ are sufficient to describe 9(6) vanishing 2-2 minors in $\hat{K}$. This window to world points in $\mathbb{P}^3$ consists of the roots of $\phi^{(4)}(\theta)$ in the Grassmannian (5) and (6). This window is described by up to four linear independent camera images in $\hat{F}$, $\hat{T}$, $\hat{Q}$ in $\mathbb{P}^2$ which should hold for every measuring device. The present paper uses four fixed points (base points) of a pencil of camera images in order to describe regions around zeros of the partition function or zeta functions.

3. Base Point Iterations and Permutations

3.1. Orthogonal Substitutions, and Elliptic-Curve Reduction by Alternating Forms

For vanishing binary forms four permuted variables constitute a group for both elliptic and hyperelliptic universal covering. A hyperelliptic Riemann surface splits into a product of elliptic curves if a hyperbolic alternating form $\alpha(\Pi_i, \Pi_j)$ van-

ishes for a complex base Π_i, Π_j of hyperelliptic tori of period $\Pi = M_{2,4}$ where Π_i is a column of Π . Hyperelliptic periods with $\alpha(\Pi_i, \Pi_j) = 0$ yield products of elliptic curves. Singular 2-2 minors are equivalent to binary invariants (XY) . 16 invariant alternating forms (XY) are hyperelliptic theta functions [11]-[15]. Singular 2-2 minors of matrices are equivalent to binary invariants (XY) . Further on, a hyperelliptic Riemann surface splits into a product of elliptic curves if hyperbolic alternating forms $\alpha(\Pi_i, \Pi_j)$ vanish for a base Π_i, Π_j of hyperelliptic tori of period $\Pi = M_{2,4}$ [15]. Determinant (6) can be written as a Plücker relation $\det \hat{K} = \prod_{ijkl} M_{ij} M_{kl}$ where 2-2 determinants M_{ij} are invariant in orthogonal substitution $g_{ij} = G_{i'ij'} g_{i'} g_{j'}$ [16] which is used to define anti-commuting matrices below [17]. Matrix $G_{i'ij'} = G_{i'li'} G_{ljj'}$ of $0, \pm 1$ entries is bilinear in the 3-way G_{ijk}

$$G_i = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_0 \end{pmatrix}, \begin{pmatrix} \sigma_1 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}, \begin{pmatrix} 0 & -\sigma_3 \\ \sigma_0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & \sigma_1 \\ i\sigma_2 & 0 \end{pmatrix}.$$

Bilinear composition g_{ij} in hyperelliptic theta functions $g_i = \mathcal{G}_{[gh]}(u_{\pm}, \Pi)$ with four-component indices $(ijkl)$ from 16 hyperelliptic characteristics $[gh]$ yield a quadrilinear invariance in the size 4 matrix K . The paper links (XY) to $\alpha(\Pi_i, \Pi_j)$ mediated by invariant Weierstrass relations on universal covering $m_{\pm}u$ and m_0u for matrices

$$m_0 = (DFT_2 \otimes I) (I \otimes DFT_2) \pi_2^4, m_+ = m_0 \eta_m, m_- = \eta_m m_0.$$

Matrices m_0 and $m_{\pm}$ are second and third roots of unity with

$$DFT_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \text{ stride-2 permutation matrix } \pi_2^4 \text{ of size 4 and}$$

$\eta_m = \text{diag}(-1, 1, 1, 1)$. The parameter θ parametrizes X and Y which are $\wp$ -functions and its derivatives. Theta functions $\mathcal{G}_{[gh]}(u_{\pm}, \Pi)$ remain unknown while iterating $\wp$. This algorithm assumes that base points $x(\wp)$ acquire a quadratic minimum iterating $m_{\pm}u$. Then $\wp$ and the Schwarzian derivative $F^{(3)}$, z are nearly constant whereas the twofold integral of $\mathcal{G}_{[gh]}(u_{\pm}, \Pi)$ over u is quadratic in u . Peculiar is self-similarity relations between four products of $\mathcal{G}_{[gh]}(u_{\pm}, \Pi)$ and universal covering $m_{\pm}u$ and m_0u . Self-similarity between universal covering $m_0u_{\pm}, m_{\pm}u_{\pm} \rightarrow u_{\pm}$ and alternating hyperbolic forms (XY) is realized by finding and shuffling all possible permutations which constitute a logarithmic Riemann surface $\mathbb{R}_L$. The paper claims that 2(3) vanishing forms (XY) or $\alpha(\Pi_i, \Pi_j)$ support rational points as a product of elliptic curves on surface $K(X)$.

3.2. Epilpar Constraints and Permutations

For arbitrary values of $\det \gamma$ the form (XY) is invariant if $(XY) = 0$. The paper uses a two-component form of homogenous coordinates

$$\theta \rightarrow \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \rightarrow \alpha_q^t \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \alpha_q$$

and defines each component $\theta_{1,2}$ four-component as a real permutation α_q of quartic roots x_i . For a complex cubic field x_i get complex which requires to formulate the permutation matrix α_q on complex plane. Accordingly, α_q can be regarded as a symbolic operator. The binary form expression (4) simplifies for a cubic field $\phi^3(\theta) = 0$. For a cubic field, the rational fractional transformation γ is a quadratic map. The resulting vanishing invariants $(\theta_q \theta_{q'})$ in (4) enable complex multiplication (CM) where variables θ and periods ω transform similarly. The present paper shows that the reduction processes

$\phi^{(6)}(\theta) \rightarrow \phi^{(4)}(\theta) \rightarrow \phi^{(3)}(\theta)$ splits a hyperelliptic Riemann surface by matrices $\alpha_q \{x_i x_j x_k x_l\} \rightarrow \pm \infty \pm i\infty, e_j, e_k, e_l$ which permute four quartic roots x_i into cubic roots $e_{1,2,3}$. Numerous invariant transformations exist for the reduction $\phi^{(4)}(\theta) \rightarrow \phi^{(3)}(\theta)$. This paper uses a permutation method. Permutation matrix α_q forms a group of its products of order $G_{32}(\alpha_q)$ extending the Galois group S_4 of $\phi^{(4)}(\alpha_q \theta)$ of order $4!$ by the matrix -1 . Permutation is restricted to two cubic roots in

$$e_i = \sqrt{g_2/3} \left\{ \cos((\varphi_0 - \pi)/3), \cos((\varphi_0 + \pi)/3), \cos(\varphi_0/3) \right\} \quad (7)$$

with complex angle φ_0 defined by $\Delta = g_2^3 \sin^2 \varphi_0$ and $\Delta = g_2^3 - 27g_3^2$. In homogeneous variables $x_i \rightarrow \begin{pmatrix} x_{1i} \\ x_{2i} \end{pmatrix} \in \mathbb{C}^4 \otimes \mathbb{P}^1$ roots

$$\begin{pmatrix} i \sin(\varphi/3) \\ \cos(\varphi/3) \end{pmatrix} = \text{DFT}_2 \begin{pmatrix} e^{i\varphi/3} \\ e^{-i\varphi/3} \end{pmatrix} \quad (8)$$

are exponents. Roots for fundamental unit ε and regulator $R_\Delta = \ln \varepsilon$

$$e_i(\varepsilon, \varphi) = \left\{ \varepsilon^{-1}, \sqrt{\varepsilon} e^{i\varphi/3}, \sqrt{\varepsilon} e^{-i\varphi/3} \right\} \quad (9)$$

yield

$$i\sqrt{\Delta} = 2 \sin(2\varphi/3) - 4 \cosh\left(\frac{3}{2} R_\Delta\right) \sin(\varphi/3).$$

Permutations of x_i induce discrete dynamics from fractional substitutions $\theta_{k+N} = \gamma^{(3)} \circ \dots \circ \gamma^{(3)} \circ \theta_k$. Living in the Galois group S_3 the quadratic map γ is viewed as permutation of roots of a cubic polynomial

$\phi^{(3)}(x_i \simeq z) = \prod_{\beta=jkl} (x_i x_\beta)$. The Hermite-Tschirnhaus substitution in the Galois group S_3

$$\gamma = \gamma^{(3)} \circ z = F^{(3)}(w, z) = \begin{pmatrix} a & b \\ -1 & w \end{pmatrix} \circ z = \phi^{(3)}(w) / (w - z) - \frac{1}{3} \partial_w \phi^{(3)}(w) \quad (10)$$

is rational quadratic in z . The fractional substitution $\gamma^{(3)}$ creates a $z \wedge w$ transvectant [18]. This is equivalent to a Weierstrass form $\phi^{(3)}(z) \rightarrow 4z^3 - \gamma_2 z - \gamma_3$ with $\theta \rightarrow z = f(\omega)$ where $f(\omega)$ is a modular form of weight 0 for invariants $\gamma_{2,3}$. However, the $x_i - z$ dualism $\phi^{(3)}(x_i) = \prod_{\beta=jkl} (x_i x_\beta) \simeq \phi^{(3)}(z)$ yields for

permutations $(ijkl)$

$$z - e_i = \gamma^{(3)} \circ x = (x_i x_k)(x_i x_l) M_{j,i}(x) \quad (11)$$

with $M_{ij}(z) = \frac{z - z_i}{z - z_j}$. The paper claims that the ground state is a vacuum consisting of products of elliptic segments (11). Permutation (15) leads to

$$a = \frac{1}{3} \partial_w \phi^{(3)}(w) = 4w^2 - \frac{1}{3} g_2 \rightleftharpoons (x_i x_k)(x_i x_l) \quad (12)$$

and

$$b = \phi^{(3)} - \frac{w}{3} \partial_w \phi^{(3)} = \frac{2}{3} g_2 w - g_3 \rightleftharpoons x_j (x_i x_k)(x_i x_l) \quad (13)$$

which are different on a Riemann surface $\mathbb{R}_L$ of the complex logarithm and require a higher-dimensional space $\mathbb{C}^4 \otimes \mathbb{P}^1$ to be conjugate to a permutation α_q . The shuffle algorithm works with the symbolic derivative

$$\gamma^{(3)}(w) = \partial_z F^{(3)}(w, z). \quad (14)$$

$\gamma^{(3)}(w)$ is a fractional map, a quadratic map as well the permutation

$$\alpha_q \{x_i x_j x_k x_l \rightarrow \pm \infty \pm i\infty, e_j, e_k, e_l\} \quad (15)$$

Accordingly, $\phi^{(4)}(\alpha_q \theta)$ is determined mainly by permutations α_q on a hyperelliptic surface. The surprising result of this paper is that period-3 rotations or chaotic simplest cycles $\gamma^2 = \gamma$ obey a linearized theory based on interval $[0, 1]$. A scaling

$$z \rightarrow \gamma \circ z, e_i \rightarrow \gamma \circ e_i, u \rightarrow \sqrt{\gamma} \circ u$$

predicts for $\sqrt{\gamma} = \gamma$ the existence of a modular invariant of weight zero as well complex multiplication where $z \simeq u$.

4. Joint Image Grassmannian and Spinor Iteration

4.1. Permutations and Grassmann Variables

It is claimed that the integer permutation matrix α_q obeys also complex solutions which induces algebraic orbits around transcendental zeros e.g. of zeta functions [19]. Iterations on hyperelliptic surfaces $K(X)$ and $W(Y)$ are suspected to be related to a spinor concept [11] [12] [20]. The approach consists in an algorithm which shuffles singular 2·2 in size four matrices as hyperbolic alternating forms in a determinant of size four. The commutative group $G_{32}(K)$ of satellite points at $\pm u_{\pm}$ on $K(X)$ is claimed to be related to the fermion group $G_{32}(\alpha_q)$ which is introduced as follows. The paper links binary forms (XY) to alternating forms $\alpha(\Pi_i, \Pi_j) = \det(\Pi_i, \Pi_j)$ by invariant homogeneous variables $\gamma \circ \theta = \hat{\gamma} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \rightarrow \begin{pmatrix} c_1 \\ \bar{c}_2 \end{pmatrix}, \begin{pmatrix} \bar{c}_1 \\ c_2 \end{pmatrix}$. Earlier spinor formulations by means of alternating

forms (XY) describe quaternion dynamics [16]. The concept widens a scalar binary invariant as matrix form. The eigenstates of the matrix $\hat{\gamma}^{(3)}$ are taken 2-4 component $(\bar{c}_1 c_2)$ and $\begin{pmatrix} c_1 \\ \bar{c}_2 \end{pmatrix}$ which is capable to couple to a Green's function approach.

4.2. Rational Invariants with Vanishing Minors and Grassmann Variables

Four roots in $\phi^{(4)}(\theta)$ yield an eight-component homogeneous vector to describe an eight-point algorithm of $\text{vec} \hat{F}$. A permutation of two roots $\alpha_q x_i$ yields two matrices c_1, c_2 and its conjugates $\bar{c}_1, \bar{c}_2$ discussed in [17].

$$\begin{aligned}\alpha_{1,2} &= \bar{c}_{1,2} + c_{1,2} \\ \alpha_{3,4} &= i(\bar{c}_{1,2} - c_{1,2})\end{aligned}\quad (16)$$

where Grassmann variables $c_1, c_2, \bar{c}_1, \bar{c}_2 : c_i^2 = 0, \bar{c}_i^2 = 0$ constitute four permutation matrices α_q where $c_1, c_2, \bar{c}_1, \bar{c}_2, \alpha_q \in M_{4,4}$. Roots $x_i \rightarrow \infty$ shifted to infinity require homogeneous variables for 16 alternating forms

$$\alpha(\Pi_i, \Pi_j) \rightarrow \alpha(\alpha_q \Pi_i, \alpha_q \Pi_j) \in \mathbb{C}^4 \otimes \mathbb{P}^1. \text{ Grassmann variable } c = z - e_i = a^+$$

describes an annihilation process a^+ in an empty state. Grassmann variable

$$c \simeq \frac{d}{d(z - e_i)} \simeq a \text{ describes a creation process } a \text{ in an occupied state. The per-}$$

mutation process $\alpha_q x_i \in M_{4,4}$ yields a square $(z - e_i)^2$ which is a product of 8

Grassmann variables on the r.h.s of (11) which leads to $(z - e_i)^2 \rightarrow 0$. Grassmann

variables $c_1, c_2, \bar{c}_1, \bar{c}_2$ are defined by sparse matrices permuting two-variables as a rotation by $\pm\pi$ in interval $[0, 1]$. Homogeneous roots are defined by rows

$$(\bar{c}_1 c_2), \text{ columns } \begin{pmatrix} c_1 \\ \bar{c}_2 \end{pmatrix} \text{ and matrices } \begin{pmatrix} b & 0 \\ 0 & \bar{b} \end{pmatrix} \text{ in unit binary forms}$$

$$\alpha(b, \bar{b}) = b\bar{b} - \bar{b}b = 1. \text{ The general permutation}$$

$$x_i \rightarrow \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix} \rightarrow \alpha_q^t \begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix} \alpha_q \rightarrow (\bar{c}_1 c_2) \begin{pmatrix} b & 0 \\ 0 & \bar{b} \end{pmatrix} \begin{pmatrix} c_1 \\ \bar{c}_2 \end{pmatrix} \quad (17)$$

seems to be related to the concept of a paired state. The polynomial of the fractional substitution γ

$$\det \gamma^{(3)} = \phi^{(3)} = \int \mathcal{D}c_1 \mathcal{D}c_2 \mathcal{D}\bar{c}_1 \mathcal{D}\bar{c}_2 e^{(\bar{c}_1 c_2) \gamma^{(3)} \begin{pmatrix} c_1 \\ \bar{c}_2 \end{pmatrix}} = \prod_{\sigma=1,2} \int \mathcal{D}c_\sigma \mathcal{D}\bar{c}_\sigma e^{\bar{c}_\sigma \gamma_{\sigma\sigma'}^{(3)} c_{\sigma'}} \quad (18)$$

is defined by Grassmann variables as its determinant [3]. Homogeneous

$$z \rightarrow \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \simeq x_i \rightarrow \alpha_q x_i \rightarrow \begin{pmatrix} c_1 \\ \bar{c}_2 \end{pmatrix} z \quad (19)$$

have nilpotent components. Grassmann variable c linearizes both polynomial

$$\phi^{(3)}(cz) \rightarrow -g_2 cz - g_3 \text{ and substitution}$$

$$\gamma^{(3)}(cw) = \begin{vmatrix} -\frac{1}{3}g_2 & -\frac{2}{3}g_2w - g_3 \\ -1 & cw \end{vmatrix} = -\begin{vmatrix} \frac{1}{3}g_2 & g_3 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -\frac{2}{3}g_2w \\ 0 & w \end{vmatrix} c \quad (20)$$

5. Entropy in Reconstructing World Lines

5.1. Information Units

An information bit can be set finding a zero determinant in Section 2.1 which splits into the number of represented possible linear independent objects

$N_o = 2^n$ of an n -dimensional vector. Vectorizations in Section 2.1 yield

$N_o = 2^{3^4}$ linear independent computer visioned images of $3^2, 3^3, 3^4$ elements in epipolar geometry $\hat{F}, \hat{T}, \hat{Q}$ which is about the third root of $N_{edd} = 2^{2^8}$. The Eddington number N_{edd} is accepted as the number of fermions in the universe.

World line variations $X_{k+1} - X_k$ which live in the Kronecker product of vectorizations $\text{vec} \delta \hat{K} = \text{vec} \hat{K} \otimes \text{vec} \hat{K}$ are 2^8 -component which yields N_{edd} linear independent objects. For $n \rightarrow \infty$ a vanishing Grassmannian $G^{(n)}$ of EM should correspond to transcendental roots. OR is estimated by a vectorized Grassmannian $G^{(5)}$ of dimension

$(\dim \text{vec} \hat{K} + \dim \hat{l}) \cdot (\dim \text{vec} P_i^{(5,4)})^5 \cdot (\dim \text{vec} P_i^{(4,3)})^4 = 2^{20} \cdot 5^6 \cdot 3^4$. This would yield to $2^{2^{20} \cdot 5^6 \cdot 3^4}$ linear independent objects as compared to 2^{3^4} objects in OR.

The paper links topological entropy of world lines to base point iterations in projective reconstruction. World point hits of a 16-component vectorized form $\text{vec} \hat{K}$ are approximated by zeros of Grassmannian $G^{(4)}$ or $G^{(5)}$. The vectorized matrix $\text{vec} \delta \hat{K}$ of world line variations $X_{k+1} - X_k$ is 2^8 -component which yields a number of bits 2^{2^8} comparable to the Eddington number. Zeros of a general Grassmannian $G^{(\infty)}$ are assumed to be transcendental. This paper discusses algorithmically accessible zeros of $G^{(\infty)}$ by $\rightarrow G^{(4)}$ of a reduced partition function $\zeta(z, G^{(4)})$.

5.2. Partition Functions of Base Points

The paper approximates a Grassmannian $G^{(n)}$ for arbitrary world point shifts $\text{vec} \hat{K} \otimes \dots \otimes \text{vec} \hat{K}$ with $n \rightarrow \infty$ and transcendent roots by $n = 4$. Quartic roots correspond to epipolar geometry and the algorithmically accessible group S_4 . Zeros for $G^{(\infty)} \rightarrow G^{(4)}$ in a reduced partition function $\zeta(z, G^{(4)})$ orbit four conjugates of transcendental roots. A transcendental function $\zeta(z, G^{(\infty)}) \rightarrow \zeta(z)$ e.g. Riemann zeta and xi functions

$$\zeta(z) = \prod_{\rho_i} (1 - z/\rho_i) = \prod_{\rho_i} (1 + e^{E_i}) \quad (21)$$

with transcendental roots ρ_i is formally equivalent to a fermion partition function of particles with energies

$$E_i = \ln z - \ln \rho_i + i\pi$$

which proves equivalence found in [19]. Computationally roots of $\zeta(z)$ are

suspected by secants between zero values $0+$ and $0-$. The present algorithm claims that a fast-shuffling algorithm of rational binary invariants creates orbits $\theta_{k+N} = \gamma^{(3)} \circ \dots \circ \gamma^{(3)} \circ \theta_k \in S_3, S_4$ of high lap number N [20]. These orbits are connected with finite values in (4)-(6) which are claimed to be statistical energies E_i . A partition function

$$\zeta(z, G^{(4)}) \simeq \Pi e^{G_i + \ln G^{(4)}(z)} \simeq \Pi e^{G_i + \ln \phi^{(4)}(z)} \simeq q(z, z_1) q(z, z_2) \quad (22)$$

encircles zeros which are base points by orbits quadrilinear on complex plane. This emphasizes the role of Newtonian root finding for a complex quadric $q(z, z_1)$. The quantity G_i is defined below by the condition that near a suspected zero an information bit can be set. Approach (22) remembers Yang-Lee zeros and phase transitions [21] [22]. Orbits around base points (11) enable an algebraic definition of topological entropy

$$h_t(N) = \lim_{N \rightarrow \infty} \frac{C(N, g_2, g_3)}{N} \quad (23)$$

by cardinality of orbits $C(N, u, \omega_k)$ on universal covering [23]-[26]

$$C(N, u, \omega_k) = \ln \frac{\prod_{i,k} \sigma(u - u_i, \omega_k)}{\prod_{i,k} \sigma(u - v_i, \omega_k)} = \ln \prod_k \det \wp^{(i)}(u_j, \omega_k) \quad (24)$$

with n^{th} order Weierstrass sigma functions and i^{th} derivatives of the Weierstrass $\wp$ -function. The $c, \bar{c}$ algorithm consists in finding two finite entries in an otherwise zero 4×4 matrix with $c^2 = 0$, $\bar{c}^2 = 0$ which leaves open absolute values of iterated periods ω_k and therefore, also of the Dedekind eta function $\eta(\omega)$. Branches $\mathbb{R}_L$ contain a network of branching lines generated by $\gamma^{(3)} \circ z$. On definite points on complex plane $\mathbb{C}$ hyperbolic forms $\alpha(\alpha_q u, \alpha_q v)$ exist with rational points on elliptic curves. Orthogonal substitutions $g_{ij \in [gh]}$ are self-similar to $\alpha(\alpha_q \Pi_i, \alpha_q \Pi)$ [27]. The doubly-periodic motion is visualized by spheres in spheres on $\mathbb{C}$ which is a self-similar precession of a spinning top which recovers astronomical motions. In this case, the partition function (22) gets polynomial $\zeta(z, G^{(4)}) \simeq \phi^{(N)}(z)$ which corresponds to Yang-Lee zeros near impenetrable spheres [21]. Driven by permutations (15), the vacuum state

$$\phi_{vac}(z) = \prod_i (z - e_i) \quad (25)$$

is a product of lines on elliptic curves with fluctuating zeros $(z - e_i)[c, \bar{c}]$. This approach concerns algorithmic orbits where transcendental zeros in (21) cannot be reached [20]. Valid periods ω_k are found by Grassmann variables $c, \bar{c}$ for CM of periods ω of elliptic tori $z \simeq \omega$. Self-similar universal covering $m_0 u_{\pm}, m_{\pm} u_{\pm} \rightarrow u_{\pm}$ and alternating hyperbolic forms (XY) shuffles all possible permutations on $\mathbb{R}_L$. From (11) one gets $\phi^{(3)}(\theta) \simeq \phi^{(4)}(\theta)$ up to a quadratic pre-factor. Orthogonal substitutions $g_{ij \in [gh]}$ are bilinear in $\mathcal{G}(u_{\pm}(i), \Pi)$. For $q = k, k+1, k+2, k+3$ zeros in (22) transform as

$$\phi^{(4)}(z) = \prod_{i=1,2,3,4} \mathcal{G}(u_{\pm}(i), \Pi) \quad (26)$$

Formal zeros in

$$Y' \bmod \phi^{(3)}(\theta) K(X) Y \bmod \phi^{(3)}(\theta) = \mathbf{x} \hat{\mathbf{F}} \mathbf{x} + \mathbf{x} \hat{\mathbf{F}}' \mathbf{x} = \phi^{(4)}(\theta)$$

are expected on products of elliptic tori for Grassmann variables (18) where

$$\phi^{(4)}(z) = \phi^{(3)} = \det \gamma^{(3)} \quad (27)$$

A Grassmann variable enters orthogonal substitutions $\mathcal{G}_{ij \in [gh]}$ via the permutation matrix α_q in distinction to [16] [27]. Shifts by $\pm \pi/3$ in angle

$$\frac{1}{3}(\varphi_0 - \pm \pi) \text{ in (7)-(9) permute hyperbolic forms}$$

$\alpha(\alpha_q \Pi_i, \alpha_q \Pi_j) = \alpha(c \Pi_i, c \Pi_j)$. Permutations as $\pm \pi$ shifts can be mapped to rotations in interval $[0, 1]$. Accordingly, a high statistical weight of simplest cycles

of changes $\pm \frac{\pi}{3}$ in interval $[0, 1]$ relates to the concept of spin. Then periods

ω get statistical products of bases Π_i, Π_j and of Grassmann variables $c, \bar{c}$. The partition function (22) gets polynomial as a product of sigma functions

$\sigma(u, \omega) = \prod_{\rho} (1 - \rho) e^{\rho + \frac{1}{2} \rho^2}$. Over lattice sites ω one gets for an orthogonal substitution $\text{DFT}_4 u_i$ in $\rho_i = u_i / \omega$ in

$$\prod_{i=1,2,3,4} \sigma(u_i, \omega) = \prod_{\rho_i} (1 - \rho_i) \quad (28)$$

if $\sum_{i=1,2,3,4} \left(\rho_i + \frac{1}{2} \rho_i^2 \right) = 1$. Theta and sigma functions

$$\ln \sigma(u, \omega) = \ln \mathcal{A}(u/\omega_2, \omega) + \eta_2 u^2 / 2\omega_2 - \frac{1}{8} \ln \Delta + \frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln \omega_2$$

of periods $\eta_i = \zeta(\omega_i/2, \omega)$ of Weierstrass zeta functions for discriminant Δ suggest an exponential

$$G_i = \sum \eta_2 u^2 / 2\omega_2 - \frac{1}{8} \ln \Delta + \frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln \omega_2 \quad (29)$$

This algorithm determines periods ω via Grassmann variables $c, \bar{c}$. Orbits in (22) and (25) yield a product permutations in

$$\zeta(z, G^{(4)}) \rightarrow \prod_{N, \rho_i} (1 - z_{k+N} / \rho_i) \quad (30)$$

where $z_{k+N} = \gamma^{(3)} \circ \dots \circ \gamma^{(3)} \circ z_k$. However, Dedekind eta functions $\eta(\omega)$ left undetermined. Certain powers $\eta^N(\omega)$ ($N = 2, 3, 8, 24$) are defined due to symmetries, e.g. $\eta^2(\omega) = \omega$ in CM. The statistical weight in (22) is estimated by the Grassmannian $G^{(5)}$ of maximal elements. Statistically a differential structure should emerge. First the logarithmic derivative of (11) yields [18].

$$\frac{(dz)^2}{\phi^{(3)}(z)} = \frac{(dx)^2}{\phi^{(4)}(x)} \quad (31)$$

independent on permutation α_q . An introduced RG beta function

$$\beta(z) = \frac{dz_k}{d \ln z_{k+N}} = \phi^{(3)}(z) \quad (32)$$

is discussed in context of differentiability of a flow of coupling constants $\gamma^{(3)} \circ \dots \circ \gamma^{(3)} \circ z$ [28]. It is assumed that four-component decompositions z_q create stability regions of constant $\beta(z)$ which allow to introduce a vacuum energy density [28]. For constant $\beta(z)$ invariant relations (31) and (32)

$$dz_{k+N} d \ln z_k = inv. \quad (33)$$

are compatible if $z_{k+N} = \ln z_{k+1} + \ln z_{k+2}$. Conformal invariance demands three $\gamma^{(3)}$ -steps $k, k+1, k+2$ or $z_{k+3} \in \{z_k \rightarrow z_{k+1} \rightarrow z_{k+2}\}$. Then simplest cycles are compatible with (32) for invariant half-order differential combinations [29] [20]

$$d_z \ln(\phi(z) - \phi(w)) d_w \ln(\phi(z) - \phi(w)) - d_z \ln(z - w) d_w \ln(z - w) \quad (34)$$

which allow to define a metric line element

$$ds^2 = y^{-2} dz d\bar{z} = -d_z \ln(z - \bar{z}) d_{\bar{z}} \ln(z - \bar{z}) \text{ for complex conjugates } w = \bar{z}.$$

A constant $\beta(z) = g_w^2$ function is treated as a coupling constant g_w for universal covering $du = g_w^{-1} dz$. A differential form of Grassmann variables $c, \bar{c}$ is

$$\bar{c} = z - e_i \text{ and } c = \frac{d}{d(z - e_i)}. \text{ Variable (11) permuted by } x_i \rightarrow \alpha_q x_i \text{ yields}$$

$c, \bar{c} \in G_{32}(\alpha_q)$ which would correspond to $(z - e_i)^2 \rightarrow 0$. Grassmann variables

$c, \bar{c}$ and the γ -eigenstates $\begin{pmatrix} c_1 \\ \bar{c}_2 \end{pmatrix}, \begin{pmatrix} \bar{c}_1 \\ c_2 \end{pmatrix}$ are understood as sparse $M_{4,4}$ -matrices

permuting two x_i components. Accordingly, permutations $c, \bar{c}$ are one-periodic base point variations in epipolar geometry $\mathbf{x} \hat{\mathbf{F}} \mathbf{x}$ on the border to elliptic curves. Projective space of fixed points of conics consists of segments of elliptic curves. Transcendent bifurcation lines z of base points are algorithmically accessible by cubic number fields. A regulator derivative $N_z = \bar{c}c = \frac{d}{dR_\Delta}$ confirms

an optimal algorithm for a number operator with eigenvalues 0 and 1. The vacuum state (25) gets an eigenstate of partition function (20) written as

$$\prod_{\rho_i} e^{-n E_i} \phi_{vac}(z) = \prod_{\rho_i} (1 + e^{-E_i}) \phi_{vac}(z) \quad (35)$$

Generally, the regulator index R_Δ is a quotient $R_\Delta \rightarrow R_\Delta / \det \ln \varepsilon_r$ with circulant units ε_r [30] [31]. The fundamental unit of a cubic number field ε_r is estimated from the class number formula [32]

$$h_\Delta \ln \varepsilon = \ln \Delta(\chi_0) - \ln \Delta(\chi_1) \quad (36)$$

of character $\chi_i \simeq w = w_0, w_1, w_2$ of representations of the Galois group. Via $e_i \rightarrow \alpha_q e_i \rightarrow c e_i$ the logarithm $R_\Delta = \ln \varepsilon$ is given by a linear c -term in $\ln(z - c e_i)$. Unit forms $\alpha(b, \bar{b})$ in (17) yield a zero contribution in (36). The pulsating vacuum state (24) depends on whether the hyperelliptic surface of base points splits into a regulator index $R_\Delta \det \ln \varepsilon_r$ reducible to

$R_\Delta = \ln \varepsilon = \ln(z - e_i)$ or not. Permutation (17) suggests the importance of three-

step simplest cycles where $\phi_{vac}(z) = \prod \phi^{(3)}(z)$ gets a product of modular invariants. As a product of vertices $\bar{c}_1 b c_2, c_1 \bar{b} \bar{c}_2$ the base point vacuum $\phi_{vac}(z)$ is stable and noncritical for constant beta function (32). From (25), one gets

$$N_z \phi_{vac}(z) \approx \begin{cases} \phi_{vac}(z) & \Delta = \Delta' \\ 0 & \Delta \neq \Delta' \end{cases} \quad (37)$$

Ⓒ whether pencils of quadrics in projective space are dependent or not. The compression algorithm $\mathbb{R}_L \mathbb{P}^3 \rightarrow \mathbb{P}^2 \rightarrow \mathbb{C}$ sets an information bit if the texture of elliptic line segments (25) encircles a transcendent zero of partition function (22).

5.3. Permutations of Roots as a Paired Fermion State

Searching complex solutions in permutations (11) relates quartic roots to a product of four Grassmann variables and two commuting permutations $b, \bar{b}$ in unit binary forms $\alpha(b, \bar{b})$. This approach works if the fractional substitution $\gamma^{(3)}$ creates a paired fermion state. For an imaginary cubic field $\phi_{vac}(z) = \prod \phi^{(3)}(z)$ one has $z - e_{1,2} \approx c_{1,2}$ whereas a third real root $z - e_3$ would not be influenced by the permutation α_q . Per R_Δ -definition the number operator N_z is idempotent $N_z^2 = N_z$ and is zero or 1 whenever z is on an elliptic curve or not. A regulator index $\ln \varepsilon_r$ is a determinant of a circulant matrix

$$\det \ln \varepsilon_r = \Pi \phi^{(N)}(\ln(z - e_i)) = \Pi \phi^{(N)}(\ln(\gamma^{(3)}(cw) \circ x)) = \alpha + \beta c$$

Linear in c from (22) two terms remain as a peculiar realization of the Kronecker-Weber theorem. The quadratic fractional substitution $\gamma^{(3)}(cw)$ gets linear.

6. Conclusions

A fast algorithm which shuffles vanishing binary forms by permutations of quartic roots in complex space is proposed. Base points of pencils of conics in epipolar geometry are relatable to a partition function comparable to that of a paired state describing OR and EM. It is conjectured that OR and EM physical fields carry generally a binary invariance as rational entities. A two-component matrix form is introduced which relates binary forms parametric to a fugacity RG flow of point-like objects. The next conclusion is to conjecture a relation between EM and a cubic number field by identifying the rationalizing parameter with a fugacity RG flow of orbits of point-like quadruples and pairs. Accordingly, OR and EM is related to quartic partition function zeros as a paired state related to a quartic polynomial of epipolar geometry. It is claimed that the algorithm captures accessible partition function zeros as OR and EM by iterated orbits of paired states travelling around transcendent zeros. The iterated partition function (22) represents orbits around of algorithmically inaccessible transcendent zeros of, e.g., the Riemann zeta function from the mathematical point of view. The flow variable is taken as the rationalizing parameter of a pencil of conics in projective plane and pencil of

quadrics in projective space as the variable of a quartic polynomial. Relation (14) opens an algorithmic theory of fugacity and chemical potential via a Lyapunov exponent. The paper claims a bridge between iterated algebraic base (fixed) points and theoretical physics by invariant binary forms. Regular γ -orbits constrain iterates to a cubic modular invariant. An iterated partition function (22) limits the number of regular non-stochastic orbits by class number one cubic number fields. Therefore, the present approach is of interest for phase transitions in EM which will be the topic of a forthcoming paper. Minkowski spacetime $\mathcal{M}$ would be a subset of imaged base points $\mathcal{M} \subset \mathbb{R}_L \{ \mathbb{P}^3 \rightarrow \mathbb{P}^2 \rightarrow \mathbb{C} \}$ in (22). The base point partition function (22) links zeros of the fundamental matrix in camera imaging to a logarithmic Riemann surface. The base point parameter is a symbolic derivative (14) which splits into a product over iterates as a fugacity RG flow where the Lyapunov exponent acts a chemical potential. As a first prediction partition, function zeros (22) are encircled by orbits in a quartic and quadratic polynomial. Base points linkable to a fugacity RG flow are those fixed points in images which are accessible through a fast algorithm. This fast algorithm links the joint image Grassmannian to Grassmann variables by permutations of quartic base points. Grassmann variables are defined through vanishing alternating two-forms as rational substitutions on elliptic curves. It is claimed that a flow of base points is controlled by an invariant polynomial $\phi^{(3)}(z)$ where a modular function of weight zero is equivalent to a one- and two-loop beta function (32).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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