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A Correction to v/d in Order to Calculate Hubbell-Lemaitre Constant H_0

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Abstract

A distinction is made between: - velocity v^e and distance d^e at instant t^e of photon emission; - velocity v^r and distance d^r at instant t^r of photon reception. Mathematical integration of Hubbell law from instant t^e to instant t^r gives: $d^r/d^e = \mathrm{e}^{H_0(t^r - t^e)}$. Depending on which couple (v^e, d^e), (v^e, d^r), (v^r, d^e), (v^r, d^r) is considered, calculated ratios v/d would have to be corrected differently in order to calculate H_0. If a survey calculates velocity (v^r) at the instant of reception and distance (d^e) at instant of emission, then in order to calculate H_0, a correction inferior to 1 should be applied to ratio (v^r/d^e). This would reduce Hubbell tension.

Keywords

Hubbell-Lemaitre Constant, Hubbell Tension

1. Introduction

Resoluteness of Hubbell tension is a passionate debate within the astrophysical community [1]-[7]. By proposing a modification to present galaxy calculations of Hubbell constant, the present paper will suggest a clue.

Hubbell law is written $v = H_0 d$, but which velocity is involved? Velocity v^e at instant t^e of photon emission or velocity v^r at instant t^r of photon reception? Same question arises for distance: distance d^e at instant t^e of photon emission or distance d^r at instant t^r of photon reception?

Here, a distinction will be made between:

- velocity v^e and distance d^e at instant t^e of photon emission;
- velocity v^r and distance d^r at instant t^r of photon reception.

Section 2 will study Hubbell-Lemaitre law from a mathematical perspective. Section 3 will propose a physical interpretation.

2. Mathematical of Hubbell-Lemaitre Law

Hubbell-Lemaitre law will be studied from a mathematical perspective. All physical quantities (distance, time, velocity, acceleration) are defined within the laboratory frame.

Mathematical symbols:

- Symbol δ will be used for differential, in order to avoid confusion with symbol d for distance.
- t: laboratory time.
- $d_{(t)}$: distance from laboratory at instant *t*.
- $v_{(t)}$: velocity at instant *t*.
- $\gamma_{(t)}$: acceleration at instant *t*.

2.1. Mathematical Integration of Hubbell Law $v_{(t)} = H_0 d_{(t)}$

In this section, H_0 will be assumed to be independent of time, within the time intervals considered, *i.e.* $\delta(H_0)/\delta t = 0$.

According to Hubbell law, velocity $v_{(t)}$ is proportional to $d_{(t)}$.

Integration of $v_{(t)}$ from instant t_1 to instant t_2 will enable to calculate distance travelled during instant t_1 to instant t_2.

Hubbell law is written: $\forall t \quad v_{(t)} = H_0 d_{(t)}$ with $v_{(t)} = \delta\left(d_{(t)}\right)/\delta t$, *i.e.*:

$$\frac{\delta\left(d_{(t)}\right)}{d_{(t)}} = H_0 \delta t \tag{1}$$

Then, by integration from $t = t_1$ to $t = t_2$:

$$\int_{t=t_1}^{t=t_2} \frac{\delta\left(d_{(t)}\right)}{d_{(t)}} \delta t = H_0 \int_{t=t_1}^{t=t_2} \delta t \tag{2}$$

$$\ln \frac{d_{(t_2)}}{d_{(t_1)}} = H_0 \left(t_2 - t_1\right) \tag{3}$$

$$\frac{d_{(t_2)}}{d_{(t_1)}} = \exp^{H_0 (t_2 - t_1)} \tag{4}$$

2.2. Mathematical Derivative of Hubbell Law $v_{(t)} = H_0 d_{(t)}$

2.2.1. H_0 Independent of Time

In this subsection, Hubbell constant H_0 is assumed to be independent of time within the time interval considered, *i.e.*: $\delta(H_0)/\delta t = 0$. Then, by mathematical derivative of Hubbell law $v_{(t)} = H_0 d_{(t)}$ with respect to time:

$$\frac{\delta(v_t)}{\delta t} = H_0 \frac{\delta(d_t)}{\delta t} \tag{5}$$

$$\gamma(t) = H_0 v_{(t)} \tag{6}$$

This demonstrates a positive acceleration $\gamma(t)$. This is consistent with the concept of accelerating expansion of the universe. Even more, as $v(t)$ increases with time, acceleration $\gamma(t)$ increases with time.

2.2.2. H_0 Dependent of Time

In this subsection, Hubbell constant H_0 is assumed to be dependent of time, *i.e.*: $\delta(H_0)/\delta t \neq 0$. Then, by mathematical derivative of Hubbell law $v_{(t)} = H_0 d_{(t)}$ with respect to time:

$$\gamma(t) = \frac{\delta^2\left(d_{(t)}\right)}{\delta t^2} = \frac{\delta\left(H_0(t)\right)}{\delta t} \cdot d(t) + H_0 v(t) \tag{7}$$

3. Physical Interpretation

In this section, Hubbell constant H_0 is assumed to be independent of time, *i.e.*: $\delta(H_0)/\delta t = 0$. All physical quantities (distance, time, velocity) are defined within the laboratory frame. Upper indexes "e" and "r" (resp) refer to photon emission and photon reception (resp).

Notations:

t^e: instant of photon emission;

t^r: instant of photon reception;

$d^e = d\left(t^e\right)$: distance to laboratory at instant t^e of photon emission;

$d^r = d\left(t^r\right)$: distance to laboratory at instant t^r of photon reception;

$v^e = v\left(t^e\right)$: velocity at instant t^e of photon emission;

$v^r = v\left(t^r\right)$: velocity at instant t^r of photon reception.

Four cases will be studied, depending on which set is considered $\left(v^e, d^e\right)$ $\left(v^e, d^r\right)$ $\left(v^r, d^e\right)$ $\left(v^r, d^r\right)$.

Hypothesis: it is assumed that Hubbell law $v_{(t)} = H_0 d_{(t)}$ is valid whichever *t*, provided that instant *t* be the same on both sides of the equation.

Present section will study how to calculate H_0 when velocity and distance are known at distinct instants. For instance, section 2.3 will study the case when velocity is known at instant t^r and distance is known at instant t^e.

3.1. Case $\left(v^e, d^e\right)$

In this case, calculations involve velocity $v^e = v\left(t^e\right)$ and distance $d^e = d\left(t^e\right)$ at same instant of photon emission. Both sides of Hubbell law $v\left(t^e\right) = H_0 d\left(t^e\right)$ refer to same instant t^e. Then, no correction is needed: H_0 is equal to v^e/d^e.

3.2. Case $\left(v^e, d^r\right)$

In this case, it is assumed that v^e and d^r are known as a result of calculations.

$v^e = v(t^e)$: velocity at instant t^e of photon emission;
$d^r = d(t^r)$: distance at instant t^r of photon reception.

Taking into account Hubbell law, integration of Hubbell law (Section 2.1) and speed of light c, the aim is to calculate H_0 as a function of v^e and d^r.

According to Section 2.1, integration of Hubbell law $v_{(t)} = H_0 d_{(t)}$ with respect to time, from instant t^e to instant t^r gives:

$$d^r = d^e \exp^{H_0(t^r - t^e)} \tag{8}$$

From instant t^e of photon emission to instant t^r of photon reception by laboratory, the photon travels distance d^e at speed c:

$$d^e = c(t^r - t^e) \tag{9}$$

According to Hubbell law at instant t^e:

$$v^e = H_0 d^e \tag{10}$$

Application of Equations (9) and (10) gives:

$$H_0(t^r - t^e) = v^e / c \tag{11}$$

Combination of Equations (8) and (11) gives:

$$d^r = d^e \exp^{+v^e/c} \tag{12}$$

Combination of Equations (10) and (12) establishes the search link (13) between v^e, d^r and H_0:

$$v^e = H_0 d^r \exp^{-v^e/c} \tag{13}$$

or equivalent:

$$H_0 = \frac{v^e}{d^r} \exp^{+v^e/c} \tag{14}$$

Equation (14) demonstrates that when calculations involve v^e and d^r, then Hubbell constant is equal to ratio v^e/d^r multiplied by $\exp^{+v^e/c}$. In this case, whichever v^e, $\exp^{+v^e/c}$ is superior to 1, then H_0 is superior to v^e/d^r. This result will be discussed.

3.3. Case (v^r, d^e)

In this case, it is assumed that v^r and d^e are known as a result of calculations. Taking into account Hubbell law, integration of Hubbell law (Section 2.1) and speed of light c, the aim is to calculate H_0 as a function of v^r and d^e. According to Section 2.1, integration of Hubbell law $v_{(t)} = H_0 d_{(t)}$ with respect to time, from instant t^e to instant t^r gives:

$$d^r = d^e \exp^{H_0(t^r - t^e)} \tag{8}$$

From instant t^e of photon emission to instant t^r of photon reception by laboratory, the photon travels distance d^e at speed c:

$$d^e = c\left(t^r - t^e\right) \tag{9}$$

According to Hubbell law at instant t^r:

$$v^r = H_0 d^r \tag{15}$$

Combination of Equations (8) and (9) gives:

$$d^r = d^e \exp^{H_0 d^e / c} \tag{16}$$

Combination of Equations (15) and (16) gives:

$$v^r = H_0 d^e \exp^{H_0 d^e / c} \tag{17}$$

or equivalent:

$$H_0 = \frac{v^r}{d^e} \exp^{-H_0 d^e / c} \tag{18}$$

In this case, H_0 is present in both sides of Equation(18) and to our knowledge H_0 cannot be formulated as a function of v^r and d^e using known analytical functions, but Equation (18) enables to numerically calculate H_0 when v^r and d^e are known.

In this case, $\exp^{-H_0 d^e / c}$ is inferior to 1, whichever the value of H_0, then H_0 is inferior to v^r / d^e. This result will be discussed.

3.4. Case $\left(v^r, d^r\right)$

In this case, calculations involve velocity $v^r = v\left(t^r\right)$ and distance $d^r = d\left(t^r\right)$ at same instant t^r of photon reception. Both sides of Hubbell law $v\left(t^r\right) = H_0 d\left(t^r\right)$ refer to same instant t^r. Then, no correction is needed: H_0 is equal to v^r / d^r.

4. Discussion

Table 1 sums up how H_0 can be calculated, depending on which velocity v and which distance d are calculated. As written in section 2.3, in case $\left(v^r, d^e\right)$ H_0 can be numerically calculated according to Equation (18).

Table 1. H_0 depending on which velocity and which distance are calculated.

v^e, d^e	$H_0 = v^e / d^e$
v^e, d^r	$H_0 = \left(v^e / d^r\right) \mathrm{e}^{+v_e / c}$
v^r, d^e	$H_0 = \left(v^r / d^e\right) \mathrm{e}^{-H_0 d_e / c}$
v^r, d^r	$H_0 = v^r / d^r$

In case $\left(v^e, d^e\right)$, in order to obtain H_0, there is no correction to apply to v^e / d^e.

In case $\left(v^e, d^r\right)$, in order to obtain H_0, according to present proposition, a multiplicative factor $\mathrm{e}^{+v_e / c}$ should be applied to v^e / d^r. Such a factor being superior to 1, this would increase the gap between H_0-CMB and H_0-Galaxy.

In case (v^r, d^e), $\exp^{-H_0 d^e/c}$ is inferior to 1, whichever the value of H_0. Then, it comes from Equation (18) that H_0 would be inferior to (v^r/d^e). This would reduce the gap between H_0-CMB and H_0-Galaxy.

In case (v^r, d^r), in order to obtain H_0, there is no correction to apply to v^r/d^r.

If a study relies on several steps of calculations, then according to proposed method, distinctions between (v^e, v^r) (d^e, d^r) should be applied at each step.

Given the complexity of surveys calculations, is it possible that, at several steps, distinct instants could have been involved without consideration of this distinction? This note could help to resolve discrepancies between H_0-galaxies and H_0-CMB results.

5. Conclusions

If a survey calculates velocity (v^r) at instant of photon reception and distance (d^e) at instant of photon emission, then in order to calculate H_0, a correction inferior to 1 should be applied to ratio (v^r/d^e) (Equation (18)). This would reduce Hubbell tension.

A subsidiary mathematical result (Section 2.2.1) shows that derivative of Hubbell law with respect to time is consistent with accelerating expansion of the universe.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Riess, A., *et al.* (2022) A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km·s^{-1} Mpc^{-1} Uncertainty from the Hubble Space Telescope and the SH0ES Team. *The Astrophysical Journal Letters*, **934**, L7. https://doi.org/10.3847/2041-8213/ac5c5b

[2] Freedman, W.L., Madore, B.F., Hoyt, T.J., Jang, I.S., Lee, A.J. and Owens, K.A. (2025) Status Report on the Chicago-Carnegie Hubble Program (CCHP): Measurement of the Hubble Constant Using the Hubble and James Webb Space Telescopes. *The Astrophysical Journal*, **985**, Article 203. https://doi.org/10.3847/1538-4357/adce78

[3] Planck Collab (2020) Planck 2018 Results. VI. Cosmological Parameters. *Astronomy & Astrophysics*, **641**, A6. https://doi.org/10.1051/0004-6361/202039265

[4] DESI Collab (2024) DESI 2024 V5: Full-Shape Galaxy Clustering from Galaxies and Quasars. https://arxiv.org/abs/2411.12021

[5] Lemos, P. and Shah, P. (2023) The Cosmic Microwave Background and H_0. In: Di Valentino, E., Brout, D., Eds., *The Hubble Constant Tension*. Springer Series in Astrophysics and Cosmology. Springer, Singapore, 295-318. https://arxiv.org/abs/2307.13083 https://doi.org/10.1007/978-981-99-0177-7_16

[6] CERN (2025) The Hubbell Tension. https://cerncourier.com/a/the-hubble-tension/

[7] Poulin, V., Smith, T.L., Calderón, R. and Simon, T. (2024) On the Implications of the "Cosmic Calibration Tension" beyond H_0 and the Synergy between Early- and Late-Time New Physics. https://arxiv.org/abs/2407.18292