

Deriving the Relativistic Compton Wavelength Once Again: Correcting the Moving-Electron Derivation and Defending Relativistic Mass

Espen Gaarder Haug

Tempus Gravitational Laboratory and Norwegian University of Life Sciences, Ås, Norway
Email: espenhaug@mac.com

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Abstract

A previous paper argued that the Compton wavelength of a moving particle should be extended by using relativistic mass. We argue that the central result of that paper was correct, but that the derivation used to reach it contained an important error: the moving electron's relativistic energy was included, while its initial relativistic momentum was omitted. This paper corrects the moving-electron Compton-scattering derivation and shows that the reduced relativistic Compton wavelength, $\lambda_{c,r} = \frac{h}{mc\gamma}$, appears naturally when the full relativistic energy-momentum structure is included. Finally, we discuss the view that the Compton wavelength is the more fundamental matter wavelength, while the de Broglie wavelength is a derivative quantity. In Haug's Planck-scale framework, the reduced relativistic Compton wavelength is bounded between the reduced rest-mass Compton wavelength and the Planck length, giving it a natural role in quantum-gravity considerations.

Keywords

Compton Wavelength, Relativistic, de Broglie Wavelength, Planck Scale

1. Introduction

The ordinary Compton [1] [2] wavelength of a particle of rest mass m is

$$\lambda_c = \frac{h}{mc}.$$

The reduced Compton wavelength is

$$\bar{\lambda}_c = \frac{\hbar}{mc}.$$

In conventional modern physics, the Compton wavelength is usually treated as a rest-mass quantity only. When a particle moves, standard presentations typically turn instead to the de Broglie wavelength. This historical habit is deeply connected to the modern resistance to relativistic mass.

This paper takes a different position. Relativistic mass is not a mathematical error. If it is used consistently, it is a legitimate and useful way of describing the mass-equivalent of total relativistic energy. If

$$m_r = \gamma m,$$

then the corresponding relativistic Compton wavelength is naturally

$$\lambda_{C,r} = \frac{h}{m_r c} = \frac{h}{\gamma m c} = \frac{\lambda_C}{\gamma}.$$

Likewise, the reduced relativistic Compton wavelength is

$$\bar{\lambda}_{C,r} = \frac{\hbar}{\gamma m c} = \frac{\bar{\lambda}_C}{\gamma}.$$

A previous paper by Haug [3] reached this same final expression for the relativistic Compton wavelength. We agree with the final result. The problem is not the endpoint; the problem is the derivation used to reach it. The earlier derivation modified the electron's rest energy mc^2 into γmc^2 , but did not include the electron's initial relativistic momentum $\gamma m v$. A moving electron has both relativistic energy and relativistic momentum. To use one while neglecting the other is inconsistent.

The purpose of the present paper is therefore twofold. First, we defend the final result

$$\lambda_{C,r} = \frac{h}{\gamma m c}.$$

Second, we correct the Compton-scattering derivation by including the initial momentum of the moving electron.

We also argue that the Compton wavelength is a stronger candidate for the true matter wavelength than the de Broglie [4] [5] wavelength. The de Broglie wavelength is

$$\lambda_B = \frac{h}{p}.$$

For a massive particle with relativistic momentum

$$p = \gamma m v,$$

one has

$$\lambda_B = \frac{h}{\gamma m v}.$$

Comparing this with

$$\lambda_{C,r} = \frac{h}{\gamma m c},$$

we obtain the exact relation

$$\lambda_B = \frac{c}{\nu} \lambda_{C,r}.$$

Thus, the de Broglie wavelength is always equal to c/ν multiplied by the relativistic Compton wavelength. At $\nu = 0$, the de Broglie wavelength is not mathematically defined, while the Compton wavelength is perfectly defined. This alone should make us cautious about treating the de Broglie wavelength as the primary matter wavelength.

2. Relativistic Mass and the Historical Resistance to It

The rejection of relativistic mass is often presented as if it were a settled matter of physics. It is not. It is largely a matter of convention, pedagogy, and taste. Authors such as Adler [6], Okun [7], and Hecht [8] argued against the use of relativistic mass. However, other authors have defended it. Sandin's paper, *In Defense of Relativistic Mass*, is an explicit defense of the concept [9]. Rindler also allowed and defended the concept in relativistic discussions [10], see also Jammer [11] [12]. Oas [13] gives an extensive literature on relativistic mass and shows there is no full agreement on the validity of the concept, some physicists defend it, others think that relativistic mass should not be used.

The resistance to relativistic mass has had consequences. If one refuses to speak of

$$m_r = \gamma m,$$

then one naturally fails to ask whether the Compton wavelength should also possess a relativistic form. This is likely one reason the relativistic Compton wavelength has been neglected for so long.

The ordinary Compton wavelength is

$$\lambda_C = \frac{h}{mc}.$$

If the mass entering the matter-energy scale is instead the relativistic mass,

$$m_r = \gamma m,$$

then the corresponding wavelength is

$$\lambda_{C,r} = \frac{h}{m_r c} = \frac{h}{\gamma m c}.$$

This is not an exotic assumption. It is simply the same Compton formula applied to relativistic mass.

3. The Compton Wavelength as the Fundamental Matter Wavelength

The Compton wavelength has a direct connection to mass-energy. For a particle at rest,

$$E = mc^2.$$

Equating this to photon energy,

$$E = \frac{hc}{\lambda},$$

gives

$$\frac{hc}{\lambda} = mc^2.$$

Therefore,

$$\lambda = \frac{h}{mc} = \lambda_C.$$

Thus, the Compton wavelength is exactly the photon wavelength corresponding to the rest-mass energy of the particle.

For the reduced wavelength,

$$\bar{\lambda}_C = \frac{\hbar}{mc}.$$

This wavelength is defined at rest and is tied directly to the rest mass.

Haug has argued that this makes the Compton wavelength the true matter wavelength and that the de Broglie wavelength is derivative [14]. The argument is simple: the de Broglie wavelength can be written in terms of the relativistic Compton wavelength:

$$\lambda_B = \frac{h}{\gamma mv}.$$

Since

$$\lambda_{C,r} = \frac{h}{\gamma mc},$$

we have

$$\lambda_B = \frac{c}{v} \lambda_{C,r}.$$

Therefore,

$$\lambda_B = \frac{1}{\beta} \lambda_{C,r},$$

where

$$\beta = \frac{v}{c}.$$

The de Broglie wavelength is therefore a velocity-amplified form of the relativistic Compton wavelength. It is not primary in this relation. It is obtained from the relativistic Compton wavelength by multiplying by c/v . At $v = 0$, this expression is not mathematically defined:

$$\lambda_B = \frac{c}{0} \lambda_{C,r}.$$

By contrast,

$$\lambda_{C,r}(\nu = 0) = \lambda_C.$$

This is not a small technicality. A proposed fundamental matter wavelength that is not defined for a particle at rest is problematic, especially in theories where rest, Planck-scale localization, and mass-energy play central roles.

4. The Problem with the Previous Moving-Electron Derivation

The previous paper proposed the correct final result

$$\lambda_{C,r} = \frac{h}{\gamma mc}.$$

However, its derivation from Compton scattering was incomplete.

The derivation began by replacing the electron rest energy mc^2 with the moving-electron energy γmc^2 . In schematic form, it used

$$p_1 c + \gamma mc^2 = p_2 c + \gamma_a mc^2,$$

where p_1 and p_2 are the incident and scattered photon momenta, and γ_a is the Lorentz factor of the electron after scattering.

As an energy equation, this is not the main problem. The problem is that a moving electron also has initial momentum

$$\mathbf{p}_e = \gamma m \mathbf{v}.$$

The previous derivation then used a momentum-square expression appropriate for an initially stationary electron:

$$p_1^2 + p_2^2 - 2p_1 p_2 \cos \theta.$$

That expression corresponds to the square of

$$\mathbf{p}_1 - \mathbf{p}_2.$$

But if the initial electron is moving, the final electron momentum is not

$$\mathbf{p}_1 - \mathbf{p}_2.$$

It is

$$\mathbf{p}_f = \mathbf{p}_e + \mathbf{p}_1 - \mathbf{p}_2.$$

Thus,

$$p_f^2 = |\mathbf{p}_e + \mathbf{p}_1 - \mathbf{p}_2|^2.$$

Expanding,

$$p_f^2 = p_e^2 + p_1^2 + p_2^2 + 2\mathbf{p}_e \cdot \mathbf{p}_1 - 2\mathbf{p}_e \cdot \mathbf{p}_2 - 2\mathbf{p}_1 \cdot \mathbf{p}_2.$$

The omitted terms are

$$2\mathbf{p}_e \cdot \mathbf{p}_1$$

and

$$-2\mathbf{p}_e \cdot \mathbf{p}_2.$$

These are not optional corrections. They are required by momentum conservation.

There is also a sign issue. With λ_1 the incident photon wavelength and λ_2 the scattered photon wavelength, the standard rest-electron Compton shift is

$$\lambda_2 - \lambda_1 = \frac{h}{mc}(1 - \cos \theta),$$

not

$$\lambda_1 - \lambda_2 = \frac{h}{mc}(1 - \cos \theta).$$

Thus, the earlier paper had the right final wavelength scale, but the moving-electron scattering derivation was wrong.

5. Correct Moving-Electron Compton Derivation

Let the initial electron velocity be

$$\mathbf{v} = c\boldsymbol{\beta},$$

where

$$\boldsymbol{\beta} = \frac{\mathbf{v}}{c}, \quad \beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}.$$

The initial electron four-momentum is

$$P_i = (\gamma mc, \gamma mc\boldsymbol{\beta}).$$

Let the incident photon have energy

$$E_1 = \frac{hc}{\lambda_1}$$

and unit direction $\mathbf{n}_1$. Its four-momentum is

$$K_1 = \left(\frac{E_1}{c}, \frac{E_1}{c} \mathbf{n}_1 \right).$$

Let the scattered photon have energy

$$E_2 = \frac{hc}{\lambda_2}$$

and unit direction $\mathbf{n}_2$. Its four-momentum is

$$K_2 = \left(\frac{E_2}{c}, \frac{E_2}{c} \mathbf{n}_2 \right).$$

The scattering angle satisfies

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = \cos \theta.$$

Four-momentum conservation gives

$$P_i + K_1 = P_f + K_2.$$

Thus,

$$P_f = P_i + K_1 - K_2.$$

The electron rest mass is unchanged, so

$$P_f^2 = P_i^2 = m^2 c^2.$$

Therefore,

$$(P_i + K_1 - K_2)^2 = P_i^2.$$

Expanding,

$$P_i^2 + K_1^2 + K_2^2 + 2P_i \cdot K_1 - 2P_i \cdot K_2 - 2K_1 \cdot K_2 = P_i^2.$$

Since photons are massless,

$$K_1^2 = K_2^2 = 0.$$

Canceling P_i^2 , we obtain

$$2P_i \cdot K_1 - 2P_i \cdot K_2 - 2K_1 \cdot K_2 = 0.$$

Hence,

$$P_i \cdot K_1 - P_i \cdot K_2 = K_1 \cdot K_2.$$

Using the metric convention

$$A \cdot B = A^0 B^0 - \mathbf{A} \cdot \mathbf{B},$$

We have

$$P_i \cdot K_1 = (\gamma mc) \left(\frac{E_1}{c} \right) - (\gamma mc \boldsymbol{\beta}) \cdot \left(\frac{E_1}{c} \mathbf{n}_1 \right).$$

Therefore,

$$P_i \cdot K_1 = \gamma m E_1 (1 - \boldsymbol{\beta} \cdot \mathbf{n}_1).$$

Similarly,

$$P_i \cdot K_2 = \gamma m E_2 (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2).$$

For the photon-photon term,

$$K_1 \cdot K_2 = \left(\frac{E_1}{c} \right) \left(\frac{E_2}{c} \right) - \left(\frac{E_1}{c} \mathbf{n}_1 \right) \cdot \left(\frac{E_2}{c} \mathbf{n}_2 \right).$$

Thus,

$$K_1 \cdot K_2 = \frac{E_1 E_2}{c^2} (1 - \mathbf{n}_1 \cdot \mathbf{n}_2).$$

Since

$$\mathbf{n}_1 \cdot \mathbf{n}_2 = \cos \theta,$$

we have

$$K_1 \cdot K_2 = \frac{E_1 E_2}{c^2} (1 - \cos \theta).$$

Substitution into

$$P_i \cdot K_1 - P_i \cdot K_2 = K_1 \cdot K_2$$

gives

$$\gamma m E_1 (1 - \boldsymbol{\beta} \cdot \mathbf{n}_1) - \gamma m E_2 (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2) = \frac{E_1 E_2}{c^2} (1 - \cos \theta).$$

Now use

$$E_1 = \frac{hc}{\lambda_1}, \quad E_2 = \frac{hc}{\lambda_2}.$$

Then

$$\gamma m \left[\frac{hc}{\lambda_1} (1 - \boldsymbol{\beta} \cdot \mathbf{n}_1) - \frac{hc}{\lambda_2} (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2) \right] = \frac{h^2}{\lambda_1 \lambda_2} (1 - \cos \theta).$$

Divide by h :

$$\gamma mc \left[\frac{1 - \boldsymbol{\beta} \cdot \mathbf{n}_1}{\lambda_1} - \frac{1 - \boldsymbol{\beta} \cdot \mathbf{n}_2}{\lambda_2} \right] = \frac{h}{\lambda_1 \lambda_2} (1 - \cos \theta).$$

Multiply by $\lambda_1 \lambda_2$:

$$\gamma mc [\lambda_2 (1 - \boldsymbol{\beta} \cdot \mathbf{n}_1) - \lambda_1 (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2)] = h (1 - \cos \theta).$$

Therefore,

$$\lambda_2 (1 - \boldsymbol{\beta} \cdot \mathbf{n}_1) - \lambda_1 (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2) = \frac{h}{\gamma mc} (1 - \cos \theta).$$

Equivalently,

$$(1 - \boldsymbol{\beta} \cdot \mathbf{n}_1) \lambda_2 - (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2) \lambda_1 = \frac{h}{\gamma mc} (1 - \cos \theta).$$

Defining

$$\lambda_{C,r} = \frac{h}{\gamma mc},$$

we obtain

$$(1 - \boldsymbol{\beta} \cdot \mathbf{n}_1) \lambda_2 - (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2) \lambda_1 = \lambda_{C,r} (1 - \cos \theta).$$

This is the corrected moving-electron Compton formula.

6. Rest Limit

If the electron is initially at rest, then

$$\boldsymbol{\beta} = 0, \quad \gamma = 1.$$

Thus,

$$\boldsymbol{\beta} \cdot \mathbf{n}_1 = 0, \quad \boldsymbol{\beta} \cdot \mathbf{n}_2 = 0.$$

The corrected formula reduces to

$$\lambda_2 - \lambda_1 = \frac{h}{mc} (1 - \cos \theta).$$

This is precisely the standard Compton formula.

The relativistic Compton wavelength reduces to the ordinary Compton wave-

length:

$$\lambda_{C,r}(v=0) = \lambda_C.$$

Similarly,

$$\bar{\lambda}_{C,r}(v=0) = \bar{\lambda}_C.$$

This is a major advantage over the de Broglie wavelength. At $v=0$,

$$\lambda_B = \frac{h}{\gamma m v}$$

is not mathematically defined, while

$$\lambda_{C,r} = \frac{h}{mc}$$

is perfectly defined.

7. The De Broglie Wavelength Near Rest

The de Broglie wavelength is

$$\lambda_B = \frac{h}{\gamma m v}.$$

The relativistic Compton wavelength is

$$\lambda_{C,r} = \frac{h}{\gamma m c}.$$

Therefore,

$$\lambda_B = \frac{c}{v} \lambda_{C,r}.$$

Equivalently,

$$\lambda_B = \frac{1}{\beta} \lambda_{C,r}.$$

For small velocities, $\beta \ll 1$, the factor $1/\beta$ becomes enormous. Therefore, the de Broglie wavelength of an electron moving sufficiently close to rest can become larger than the diameter of the observable universe.

Let the diameter of the observable universe be denoted by D_U . The condition

$$\lambda_B > D_U$$

becomes

$$\frac{h}{\gamma m_e v} > D_U.$$

For very small v , $\gamma \approx 1$, so this is approximately

$$v < \frac{h}{m_e D_U}.$$

There is no mathematical obstruction to choosing such a small velocity in ordinary theory. Hence the de Broglie wavelength can be made arbitrarily large as

$v \rightarrow 0$.

At $v = 0$ itself,

$$\lambda_B = \frac{h}{0},$$

which is not mathematically defined. Standard theory often ignores this issue because, according to the usual Heisenberg uncertainty principle [15], one does not normally demand exact rest and exact localization simultaneously. However, this attitude becomes much less satisfactory in Planck-scale physics and quantum gravity, where rest, mass, and limiting length scales may play central roles.

8. Haug's Maximum Velocity and the Planck-Length Lower Bound

In standard special relativity, matter is usually said to satisfy

$$v < c.$$

If that were the only restriction, then

$$\gamma \rightarrow \infty$$

as

$$v \rightarrow c,$$

and therefore

$$\lambda_{c,r} = \frac{\bar{\lambda}_c}{\gamma}$$

would tend toward zero.

However, in Haug's [16] [17] framework, matter has a maximum velocity below c , given by

$$v_{\max} = c \sqrt{1 - \frac{l_p^2}{\bar{\lambda}_c^2}},$$

where l_p is the Planck length [18] [19] and

$$\bar{\lambda}_c = \frac{\hbar}{mc}$$

is the reduced rest-mass Compton wavelength. The maximum velocity formula can simply be found by setting the maximum relativistic energy for an elementary particle to the Planck energy or the maximum length contraction to the Planck length:

$$\begin{aligned} mc^2 \gamma &\leq m_p c^2 \\ \frac{\hbar}{\bar{\lambda}_c} \frac{1}{c} c^2 \gamma &\leq \frac{\hbar}{l_p} \frac{1}{c} c^2 \\ v &\leq c \sqrt{1 - \frac{l_p^2}{\bar{\lambda}_c^2}}, \end{aligned} \tag{1}$$

From this formula,

$$\frac{v_{\max}^2}{c^2} = 1 - \frac{l_p^2}{\bar{\lambda}_C^2}.$$

Thus,

$$1 - \frac{v_{\max}^2}{c^2} = \frac{l_p^2}{\bar{\lambda}_C^2}.$$

The Lorentz factor at maximum velocity is

$$\gamma_{\max} = \frac{1}{\sqrt{1 - v_{\max}^2/c^2}}.$$

Substituting the above expression gives

$$\gamma_{\max} = \frac{1}{\sqrt{l_p^2/\bar{\lambda}_C^2}}.$$

Hence,

$$\gamma_{\max} = \frac{\bar{\lambda}_C}{l_p}.$$

The reduced relativistic Compton wavelength is

$$\lambda_{C,r} = \frac{\bar{\lambda}_C}{\gamma}.$$

At maximum velocity,

$$\lambda_{C,r,\min} = \frac{\bar{\lambda}_C}{\gamma_{\max}}.$$

Using

$$\gamma_{\max} = \frac{\bar{\lambda}_C}{l_p},$$

we get

$$\lambda_{C,r,\min} = \frac{\bar{\lambda}_C}{\bar{\lambda}_C/l_p}.$$

Therefore,

$$\lambda_{C,r,\min} = l_p.$$

Thus, in Haug's framework,

$$l_p \leq \lambda_{C,r} \leq \bar{\lambda}_C.$$

The reduced relativistic Compton wavelength is limited between the reduced rest-mass Compton wavelength and the Planck length.

For the non-reduced Compton wavelength,

$$\lambda_{C,r} = 2\pi\lambda_{C,r}.$$

Therefore,

$$2\pi l_p \leq \lambda_{C,r} \leq \bar{\lambda}_C.$$

This is an essential correction to the naive statement that the relativistic Compton wavelength tends to zero. Under Haug's maximum-velocity condition, the reduced relativistic Compton wavelength reaches the Planck length, not zero. This is supported by a new theory of quantum gravity; see Haug [14] [20]. That said, this new maximum velocity is not needed for the derivation of the relativistic Compton wavelength, but it provides a new perspective on the minimum length of the Compton wavelength.

9. Rest, Planck-Scale Physics, and the Certainty-Uncertainty Principle

The standard Heisenberg uncertainty principle is often invoked to argue that exact rest is not physically meaningful, because exact momentum and exact position cannot both be known. In conventional quantum mechanics, this makes the undefined rest-limit of the de Broglie wavelength less troubling. If a particle is never treated as exactly at rest in a fully localized sense, then the divergence or undefined character of

$$\lambda_B = \frac{h}{p}$$

at $p = 0$ is usually ignored.

However, this standard attitude may fail at the Planck scale. In Haug's [14] quantum-gravity program, rest and the Planck scale play central roles. Haug has argued for replacing the ordinary uncertainty principle, at the deepest level, with a certainty-uncertainty principle in which there is certainty at the Planck scale and in the rest-state limit. In such a framework, the fact that the de Broglie wavelength is not mathematically defined at rest becomes a serious weakness, not a harmless curiosity.

The relativistic Compton wavelength behaves very differently. It is defined at rest:

$$\lambda_{C,r}(v=0) = \bar{\lambda}_C.$$

It remains defined for every physically allowed velocity:

$$0 \leq v \leq v_{\max}.$$

And in Haug's framework, it has a Planck-length lower bound:

$$\lambda_{C,r,\min} = l_p.$$

Thus, the reduced relativistic Compton wavelength is mathematically and physically well behaved over the full allowed domain:

$$l_p \leq \lambda_{C,r} \leq \bar{\lambda}_C.$$

The de Broglie wavelength, by contrast, becomes arbitrarily large near rest and is not defined at exact rest:

$$\lambda_B = \frac{c}{v} \lambda_{C,r}.$$

This strongly supports the view that the de Broglie wavelength is derivative, while the Compton wavelength is more fundamental.

10. Physical Interpretation

The corrected derivation supports four central claims. First, the previous paper's final expression,

$$\lambda_{C,r} = \frac{h}{\gamma mc},$$

is correct under the relativistic-mass convention.

Second, the previous scattering derivation was incomplete because it omitted the moving electron's initial relativistic momentum. The corrected derivation includes this momentum and yields

$$(1 - \boldsymbol{\beta} \cdot \mathbf{n}_1) \lambda_2 - (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2) \lambda_1 = \lambda_{C,r} (1 - \cos \theta).$$

Third, the de Broglie wavelength is related to the relativistic Compton wavelength by

$$\lambda_B = \frac{c}{v} \lambda_{C,r}.$$

Therefore, the de Broglie wavelength is a velocity-amplified derivative of the relativistic Compton wavelength.

Fourth, in Haug's maximum-velocity framework, the reduced relativistic Compton wavelength is bounded by

$$l_p \leq \lambda_{C,r} \leq \bar{\lambda}_C.$$

It does not go to zero. It reaches the Planck length. This makes the relativistic Compton wavelength a natural bridge between special relativity, matter waves, and Planck-scale physics.

Comparison with Standard Covariant Treatments

It is useful to compare the present result with the standard covariant treatment of Compton scattering from a moving electron, or equivalently inverse Compton scattering. In the conventional approach one normally does not introduce a "relativistic Compton wavelength" as a separate named wavelength. Instead, the calculation is written entirely in terms of four-momentum invariants. The electron four-momentum is

$$P_i = (\gamma mc, \gamma m \mathbf{v}),$$

and the scattering condition follows from

$$(P_i + K_1 - K_2)^2 = m^2 c^2.$$

This gives the standard covariant moving-electron relation

$$(1 - \boldsymbol{\beta} \cdot \mathbf{n}_1) \lambda_2 - (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2) \lambda_1 = \frac{h}{\gamma mc} (1 - \cos \theta).$$

Thus, the factor $h/(\gamma mc)$ appears directly once the initial electron momentum is included consistently.

In standard inverse-Compton language, the factors

$$1 - \boldsymbol{\beta} \cdot \mathbf{n}_1 \quad \text{and} \quad 1 - \boldsymbol{\beta} \cdot \mathbf{n}_2$$

are usually interpreted as Doppler or aberration factors associated with transforming photon energies between the laboratory frame and the instantaneous rest frame of the electron. The ordinary Compton wavelength $h/(mc)$ is then retained as a rest-frame invariant, while the laboratory-frame wavelength relation contains explicit Lorentz and angular factors. The present interpretation rewrites the same covariant structure by identifying

$$\lambda_{C,r} = \frac{h}{\gamma mc}$$

as the relativistic Compton wavelength associated with the electron's total relativistic mass-energy. If one instead defines a relativistic wavelength as $\lambda_\gamma = \gamma \lambda$, then this corresponds to a Doppler-stretched or frame-dependent wavelength scale rather than to the mass-energy Compton scale $h/(\gamma mc)$. The distinction is important: standard covariant Compton and inverse-Compton scattering preserve the invariant rest mass m , but the laboratory-frame equation naturally contains the combination $h/(\gamma mc)$ after the full energy-momentum balance is written out.

Therefore, the present result is not in conflict with standard covariant treatments. It is a reinterpretation of the same Lorentz-covariant equation. The conventional formulation emphasizes invariants and Doppler factors; the present formulation emphasizes that the coefficient multiplying $1 - \cos \theta$ can be read as a relativistic Compton wavelength when relativistic mass $m_r = \gamma m$ is used consistently.

11. Conclusions

The previous paper [3] reached the correct final expression for the relativistic Compton wavelength:

$$\lambda_{C,r} = \frac{h}{\gamma mc}.$$

However, the derivation used to obtain this result from moving-electron Compton scattering was incomplete. It included the electron's relativistic energy but omitted the electron's initial relativistic momentum.

The corrected derivation gives

$$(1 - \boldsymbol{\beta} \cdot \mathbf{n}_1) \lambda_2 - (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2) \lambda_1 = \frac{h}{\gamma mc} (1 - \cos \theta).$$

Equivalently,

$$(1 - \boldsymbol{\beta} \cdot \mathbf{n}_1) \lambda_2 - (1 - \boldsymbol{\beta} \cdot \mathbf{n}_2) \lambda_1 = \lambda_{C,r} (1 - \cos \theta).$$

The relativistic Compton wavelength is therefore not weakened by correcting

the earlier derivation. It is strengthened. The correction shows that $h/(\gamma mc)$ appears naturally when the moving electron's full relativistic energy-momentum structure is included.

Furthermore, Haug's maximum-velocity framework implies

$$v_{\max} = c \sqrt{1 - \frac{l_p^2}{\bar{\lambda}_C^2}},$$

and therefore

$$\gamma_{\max} = \frac{\bar{\lambda}_C}{l_p}.$$

Consequently,

$$\lambda_{C,r,\min} = l_p.$$

Thus, the reduced relativistic Compton wavelength is limited between the reduced rest-mass Compton wavelength and the Planck length:

$$l_p \leq \lambda_{C,r} \leq \bar{\lambda}_C.$$

The de Broglie wavelength behaves very differently:

$$\lambda_B = \frac{c}{v} \lambda_{C,r}.$$

It becomes arbitrarily large near rest and is not mathematically defined at $v = 0$. Standard theory often avoids concern about exact rest through the Heisenberg uncertainty principle, but in Haug's Planck-scale quantum-gravity program, rest and Planck-scale certainty play a central role. In that context, the Compton wavelength is not merely an alternative matter wavelength. It is the more fundamental one.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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