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Advancements in Time Modeling: Relationalism, Divisional Structures, and Geometry

Steven D. P. Moore

Independent Researcher, Milton, Canada
Email: steven.moore86400@protonmail.com

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Abstract

This article broadens terminology and approaches that continue to advance time modelling within a relationalist framework. Time is modeled as a single dimension, flowing continuously through independent privileged points. Introduced as absolute point-time, abstract continuous time is a backdrop for concrete relational-based time that is finite and discrete, bound to the limits of a real-world system. We discuss how discrete signals at a point are used to temporally anchor zero-temporal points $[t = 0]$ in linear time. Object-oriented temporal line elements, flanked by temporal point elements, have a proportional geometric identity quantifiable by a standard unit system and can be mapped on a natural number line. Durations, line elements, are divisible into ordered unit ratio elements using ancient timekeeping formulas. The divisional structure provides temporal classes for rotational (Rt_{24t}) and orbital (Rt_{18}) sample periods, as well as a more general temporal class (Rt_{12}) applicable to either sample or frame periods. We introduce notation for additive cyclic counts of sample periods, including divisional units, for calendar-like formatting. For system modeling, unit structures with dihedral symmetry, group order, and numerical order are shown to be applicable to Euclidean modelling. We introduce new functions for bijective and non-bijective mapping, modular arithmetic for cyclic-based time counts, and a novel formula relating to a subgroup of Pythagorean triples, preserving dihedral n-polygon symmetries. This article presents a new approach to model time in a relationalistic framework.

Keywords

Relationalism, Mohist Geometry, Euclidean Geometry, Relational-Time, Discrete-Time, Continuous-Time, Planck Time, Zero-Time

1. Introduction

Time modeling has been a fundamental aspect of various scientific disciplines, with significant implications for both theoretical and applied research. Leibniz' various discussions of relationalism were largely metaphysical and philosophical [1], leaving gaps in a physical system discussed to this day. [2]-[5] This study discusses a model for time within the context of relationalism yet uses an approach that is not focused on alignment to Eurocentric mathematics. Complementing and building from previous works [6] [7], this article focuses on definitions and clarity as well as geometric forms and mapping functions that combine independent elements of space with independent elements of time.

With respect to time, Leibniz had three key theses ([8], p 263). The first is that time is relational, meaning it is not a physical entity, but rather a relation or ordering of entities. This work was before Maxwell's work, which was an effective way to model dimensional quantities. [9] Second, time is ideal, with no existence apart from the things it relates to. Lastly, time is a continuous quantity.

Leibniz was a student of ancient Chinese works [10] [11], which included geometry and discrete mathematics. [12]-[15] Instead, Leibniz primarily utilized geometry as a way to express his metaphysical theories while applying continuous mathematics to discrete Chinese mathematical approaches. This was during a time that was referred to as "bringing about world harmony and Christian-European rule." ([10], p 436) On the other hand, translations of Mohist works from ancient China highlight a form of geometry that emphasizes practical and utilitarian applications, particularly techniques for precise building and construction. [12] [14] [15]

The rishta (rt) system was introduced as an overarching framework for modeling matter, space, time, and motion using object-relativity from real-world systems, an alternative tool for relationalistic modeling. [6] [7] The system aligns and builds applications consistent with dimensional analytics, signal systems, set theory, and advancements in relationalism. The rishta system incorporates definitions from Euclidean geometry [16], but also follows a similar utilitarian or corporeal geometry interpreted from translations of the Mohist canon. [14] [15] To model intervals of time bound by the limits of a real-world system, applications included the use of ancient divisional structures of time. [6] [7]

Modern physics is largely harmonized with Greek-centric philosophies, which include works by Aristotle. [17]-[19] Aristotle considered "…no continuum can be made up out of indivisibles, as for instance a line out of points, granting that the line is continuous and the point indivisible". ([20], Book 6, Chap. 1) However, debates on whether continuums can be composed of indivisible points continue to this day, features that were instrumental to the development of calculus. [21] Cavalieri's concept of indivisibles considered that a line can be made up of infinitely many points. However, Bell noted that by set-theoretical account, all mathematical entities are discrete. [21]

Within a relationalistic framework, a novel perspective enters the infinitesimal continuum debate. In a relationalistic framework, there are no infinitesimals. [7] For example, a temporal line has a measurable dimensional quantity of time, bounded by zero-dimensional (zero-D) points. [6] Since an Euclidean point has no parts, an adjacent array of indivisible zero-D points cannot form a continuum or line with a measurable dimensional quantity. It remains unexplored if a continuum can simply be a span of time bounded by two defined zero-time points, forming a given line, or duration, without internal points. A zero-time point within a relationalistic framework was described by Moore [6], but a formula has yet to define it.

Time in relativity is added to three-dimensional space, which accurately predicts both time and spatial coordinates for observed events in four-dimensional spacetime. [22] [23] Within this theoretical framework, time is mapped on a real number line as a continuous, unbounded, and infinitesimal quantity, where time can approach but not equal zero ($t \neq 0$), leading to singularities.

Time modeling in relationalism tracks time in the progressing present at a point. [6] [7] N. Maxwell's philosophical discussions on *probabilism* conflict with relativity, where he argues that "the universe is such that, at any instant, there is only one past but many possible futures". [24] Maxwell's work proposes the existence of a universal instant. Dieks, in his discussion of Maxwell's proposal, uses relativity-based arguments to disagree, concluding that "there seems to be no objection to the generalized flow of time doctrine within special relativity." [25] We suggest that the apparent contradiction in the literature may be resolved by interpreting Maxwell's approach in the context of relationalism. From this perspective, both Dieks and Maxwell may be seen as correct: Maxwell's philosophy instead aligns with relationalism, where the framework is causal (past and present inputs), and it does not utilize observers, inertial frames, and light clocks requiring space to measure time. [6] [7]

While recognizing and utilizing limits to matter in a real-world system as observed in quantum phenomena, the relationalistic approach generates finite, discrete-time measures and zero-time points [$t = 0$]. [6] [7] By separating time from space in causal modeling and expressing dynamic relationships at discrete zero-time points [7], this alternative approach avoids the mathematical implications of time dilation. It also avoids the mathematical outcome of the block universe model, where past, present, and future coexist in a four-dimensional spacetime, as well as the twin paradox.

This article uses celestial overtones to clarify key principles with set notation, exploring the unification of continuous time with discrete relational time for modeling purposes. We introduce a fundamental divisional structure of time and discuss its notation, drawing parallels with Mohist geometric translations. The connection between dimensional time and non-dimensional variables is explored, including non-bijective and bijective mappings and modular arithmetic-like features for cyclic time. Finally, we propose a link between Pythagorean triple triangles and dihedral symmetry, opening paths for further research.

2. Methods and Definitions

The rishta (rt) system utilizes object-oriented discrete corporeal geometry to model objects, object relations, relational time, and motion (cyclic orbital, cyclic axial rotational, and non-cyclic object-relative) from real-world systems. [6] [7]

Physical models are static state models, like an object-oriented discrete event simulation whereby one discrete state, or cell-like model on a Turing tape [26] [27], is expressed after another in sequence. Although the fixed conditions defining the model variables may remain consistent, the real-world measures—and consequently the output—transform each discrete state. Changes in datasets can only be observed at specific points anchored in time, with differences becoming noticeable only when one state is compared to the next.

For a real-world system with object-oriented geometric elements, the definition and application of numbers and symbols require consensus on notation, logic, and precision, as clear wording and symbolism are crucial for conveying concepts. Therefore, we will introduce options for consideration as we continue organizing the concept. Abstraction, or generalization that excludes specific objects or object conditions in explanations, is only used to simplify explanations and is also applicable for procedures associated with an object or method for an object being the same. ([28], p. 130)

2.1. Rishtar Elements

In this section, we present rishtar (rt) elements using a hierarchical structure. There are common features, especially an objectivation of object-oriented conditions. [7] The root node is a given real-world system separated into 1) temporal elements and 2) spatial elements. Let Rt_t be a non-empty set of *temporal* rishtar elements in a real-world system, and let Rt_s be a non-empty set of *spatial* rishtar elements.

Both sets Rt_s and Rt_t are initially empty sets and belong to a universal empty set prior to the Big Bang for a real-world system. With the first universe alpha signal, a particle of light, perhaps a temporal point, is defined as measurements are only taken at a temporal end-point; it is at the second signal that the first elements for each set are defined.

The rt temporal elements (**Figure 1**) are separated into two child nodes 1) a temporal line element with duration, or $t > 0$, and 2) a zero-temporal point element with no duration, or $t = 0$.

Internal temporal points can be used to identify adjacent *fte*(s) co-existing with a larger duration. For example, in a given single duration for a luniterranean year (1 temporal start- and end-point), this also co-exists with 13 lunations, giving 13-line elements with 1 temporal start-point, 12 internal points, and 1 temporal end-point.

2.2. Temporal Line Elements

A temporal line element (**Figure 1**) is an object-oriented finite geometrically represented duration, proportional to a dimensional quantity of time. The line element is continuous, with no points, and does not include the two flanking zero-D temporal point elements taken from geometric signatures of events anchored in linear time. The duration, or fundamental temporal element, is an open-like interval ($\varnothing_{t1}$, $\varnothing_{t2}$), with discrete and adjacent atomic temporal intervals (e.g., Plank time) mappable on a natural number line instead of a real-number line.

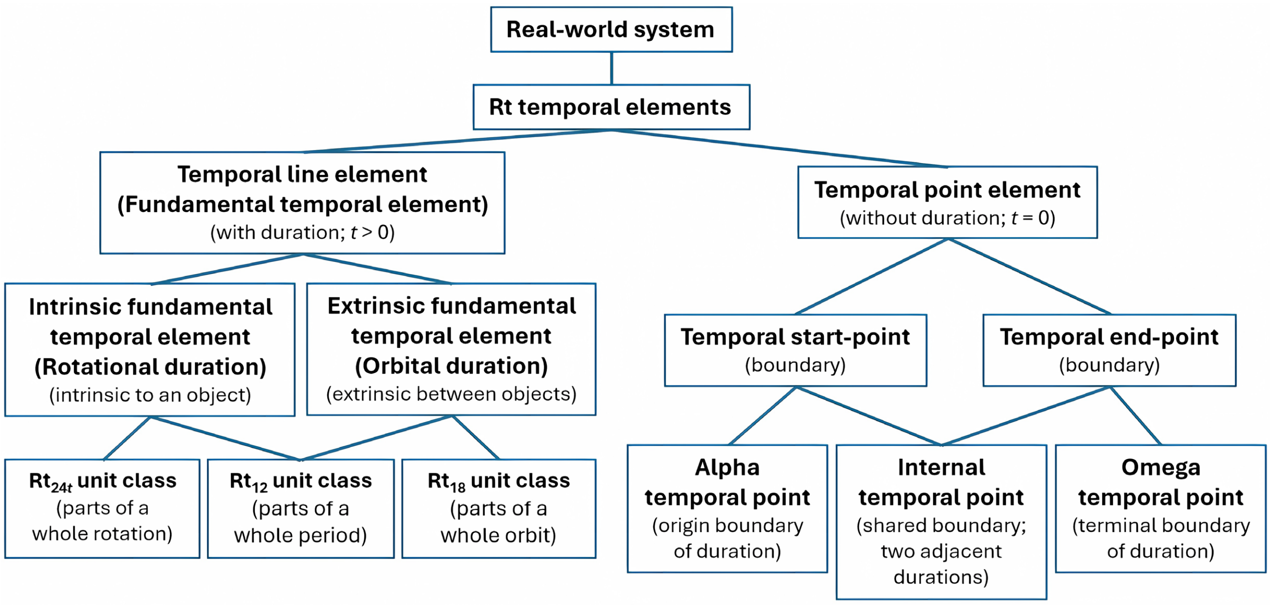

Figure 1. Hierarchical structure of temporal elements in a real-world system. Two sets of object-oriented elements are shown: line elements, representing a continuum with dimensional quantity, and point elements, with no duration. Line elements are divided into intrinsic variables and external variables, defined by signal markers. Line elements can be subdivided into dimensional quantities, while temporal point elements, such as start and end points, bind the line elements. t = time, Rt = rishtar. The period can be a sample or frame period.

Let *FTE* be a non-empty ordered set of fundamental temporal elements (*fte*) where $fte \in FTE$ and $FTE \in Rt_t$. Elements of set *FTE* are definable with a dimensional quantity of time [T].

Also known as a fundamental time element (*fte*), it is finite, discrete, and lacks breadth, with no gaps. A given object-oriented temporal line element is defined by a signal-based start and end point, representing a pairing between an object relation and its signals. Each *fte* has unique time quantities, anchored in linear time if the boundary signals are aperiodic. Temporal point elements have no time quantity, so adjacent *fte* have no gaps but do not overlap, with signal inputs from both past and present anchoring them in a causal system.

The temporal line element can be subclassified based on the type of motion used for signal [ρ] events. [7] Event signals are captured/received at a spatial point rather than having an observer calculate 4D spacetime coordinates. The two types of motion utilized 1) rotational motion: an intrinsic fundamental temporal element (object-specific), or 2) orbital motion: an extrinsic fundamental temporal element (object-relationship). Rotation is about an axis, and this alone does not provide a duration; only when paired with a signal does a cycle exist. Non-dimensional variables of rotation and orbit require association with a dimensional quantity of time for relevant measures.

A duration can be either a sample period (bound by two recurrent signals) or a frame period (a temporal start-point, one or more internal points, and a temporal end-point). [7] Asymmetrical motion with non-cyclic extrinsic properties, such as the motion of two stars relative to each other, do not model durations of time the same way, and further descriptions will be needed. [7]

The lowest hierarchical temporal line element is a unit created after the extension is known. A Rt_n unit is a ratio unit (ordered part) of a single and whole fundamental temporal element. [7] The magnitude of a noted Rt_n unit count expands from the temporal start-point and terminates at the temporal end-point from an existing temporal element in a causal system. For example, 2 Rt_n comes after 1 Rt_n and the magnitude of the quantity is the sum of each ratio united, or 2 of 86,400 seconds in a given Earth day comes after 1 of 86,400 seconds in a day if a given axial rotation period is composed of 84,000 unit parts of a whole.

2.3. Temporal Point Elements

A temporal point element is fixed in linear time, an anchored pillar in an otherwise continuous linear time. A temporal point is without duration, meaning there is no quantity of time measurable at this point. This is different than abstract infinitesimal time commonly used in mathematics. Instead, it is framed on the limits of the real-world system being modelled.

Let $P\varnothing_t$, be a non-empty set of zero-temporal points with no quantity of dimensional time as measured in a real-world system. Where were $P\varnothing_t \in Rt_t$.

At a designated object-relational discrete signal event [ρ], marking a temporal end-point, measurement(s) are taken, which anchor the zero-temporal point in linear dimensional time. The zero-time point is proposed as triggering a wave function collapse, framing a discrete zero-temporal point, [$\varnothing_t$].

$$|\psi\rangle \xrightarrow{\varnothing_t} |\phi\rangle \tag{1}$$

where $|\psi\rangle$ is before a collapse, $|\phi\rangle$ is after a collapse, and $\varnothing_t$ (or $t = 0$) is at the zero-temporal point.

2.3.1. Relational Timeline

A relational timeline is an object-oriented finite geometrical representation of an extended object-oriented duration proportional to a dimensional quantity of time. This timeline can also anchor both temporal line elements and temporal point elements. Each relational timeline is unique to a pairing between a given signal event and a spatial point element. For example, independent sidereal and solar timelines from the same point on Earth's surface co-exist with Earth and the rotational motion of the planet.

Let set TL be a non-empty ordered set composed of two proper subsets of a relational timeline, where $P\varnothing_{ty} \subset TL$, $FTE \subset TL$, and $TL \subset Rt_t$.

An alpha-omega timeline refers to the beginning and end of a relational timeline, which would only exist in a memory system stored in the present. If the omega signal has not occurred yet, the timeline is considered active or ongoing. We explore the Mohist related to "jìn 'to be exhaustive' means that nothing is not so" ([14], p. 89). "Mohist recognized the pertinence of jìn 'to be exhaustive' beyond spatio-temporal contexts" ([14], p. 89).

Let $P\varnothing_{ty}$ be a non-empty ordered set of zero-temporal point elements on an object-oriented relational timeline. Alpha temporal points [$\varnothing_{t\alpha}$] are temporal start point elements, internal temporal points [$\varnothing_{t(\alpha+y)}$], and omega terminal points [$\varnothing_{t\omega}$], are members of set $P\varnothing_{ty}$.

$$P\varnothing_{ty} = \left\langle \varnothing_{t(\alpha)}, \varnothing_{t(2)}, \varnothing_{t(3)}, \cdots, \varnothing_{t(\omega)} \right\rangle \tag{2}$$

where $\varnothing_{ty} \in P\varnothing_{ty}$ and y is an ordered mapping of a signal (or point) count on a relational timeline. $y \in \mathbb{N}$, not including zero. When y = alpha, this is the first signal, and when y = omega, this is the last signal of an alpha-omega relational timeline.

There are various model-specific applications that can use sample or frame periods that are dependent upon the designed model output. A suggestion for notation to consider as a guide is to show a particular sample period with relevant flanking points is shown. Vertical bars (see Equation (7)) represent three adjacent elements without overlap and without gaps, given the nature of the dimensional line and flanking non-dimensional points elements. Vertical bars represent the size of the inner line element.

$$\varnothing_{ty} \in P\varnothing_{t} \subset TL \tag{3}$$

$$P_{\varnothing t}, P_{\varnothing ty} \bigcap FTE = \varnothing \tag{4}$$

$$P_{\varnothing ty} = \left\langle \varnothing_{t1}, \varnothing_{t2}, \cdots, \varnothing_{ty} \right\rangle \tag{5}$$

$$FTE = \left\langle fte_1, fte_2, \cdots, fte_n \right\rangle \tag{6}$$

$$\varnothing_{t(y)} \left| fte_1 \right| \varnothing_{t(y+1)}, \quad fte_1 \in FTE, \ \varnothing_{t1}, \varnothing_{ty+1} \in P\varnothing_{t} \tag{7}$$

In relationalism modeling, excluding flanking points is crucial, similar to the Mohist translations of "having an interstice is (the sides) not reaching to the center." ([14], p. 113) In modeling, more than one zero-D object-oriented spatial point element (OCO) can overlap in a fixed state model output, which is only possible because points have no magnitude and no parts and do not interfere with line element quantities.

2.3.2. Spatial Line Elements

A spatial line element (**Figure 2**) is an object-oriented finite geometric represented extension, proportional to a quantifiable dimension of length. [7] The line element is continuous with no points, so it does not include the two flanking object-oriented spatial point elements that mark zero-D point locations; thus, it is an open-like interval ($\varnothing_{l1}$, $\varnothing_{l2}$) with discrete atomic intervals (e.g., Plank length) mappable on a natural number line instead of a real-number line. Flanking spatial point elements for *fte*(s) are fixed in relational space, anchored by a shared temporal point element.

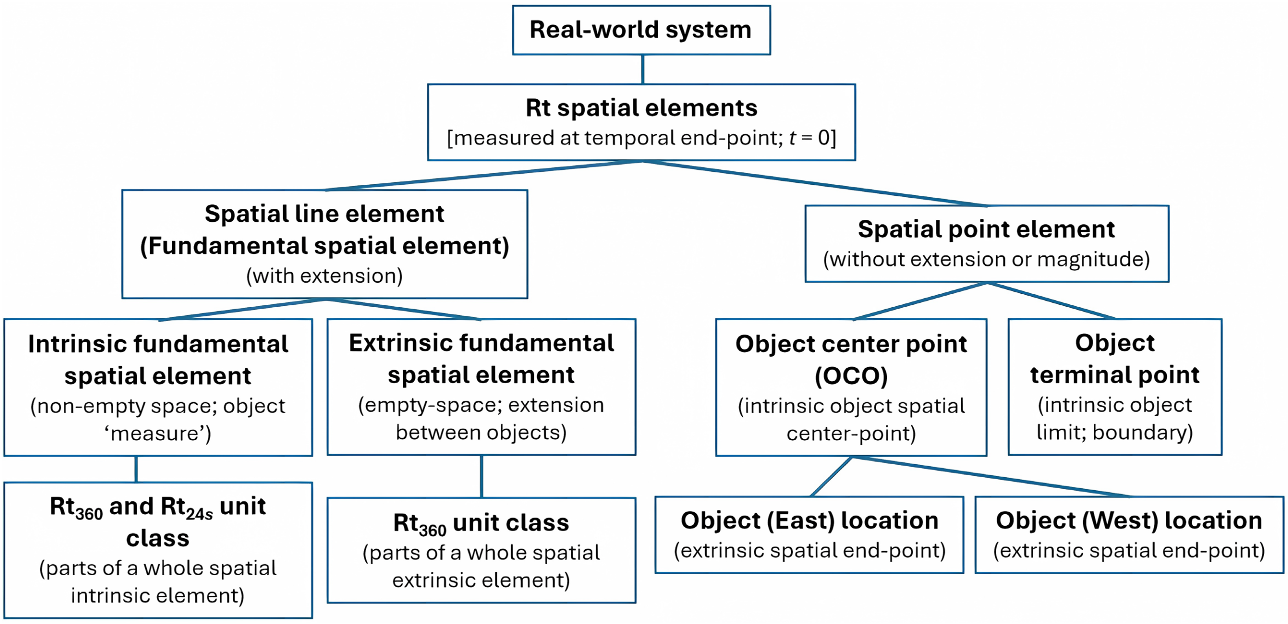

Figure 2. Hierarchical structure of rishtar spatial elements. In a real-world system, a set of object-oriented spatial elements includes both line elements and point elements. Line elements can be classified into intrinsic and extrinsic, both of which are dimensional quantities, divided and subdivided into unit multiples, or ordered parts of a whole for a given class of unit. Two independent objects are measured by a straight extrinsic fundamental element, whereby each object's OCO is the location point, binding the directional relational extension, or line element, between them. *t* = time, *s* = spatial, Rt = rishtar, OCO = Object centroidal origin [7].

Let *FSE* be a non-empty ordered set of fundamental spatial elements (*fse*) where $fse \in FSE$ and $FSE \in Rt_s$. Elements of set *FSE* are definable with a dimensional quantity of length [L] and can be scaled.

The spatial line element, or *fse*, can be subclassified based on the spatial point elements used to flank the extension. [7] The two types are 1) intrinsic, which is object-specific, for example a polar radius, and 2) extrinsic, which involves an object-relationship.

Thus far in modeling exercises, rotation can have both a spatial (Rt_{24s}, Figure 2) and temporal (Rt_{24t}, Figure 1) unit class classification; ongoing modeling tests will be needed to continue to support or dismiss this should a conflict arise. Non-dimensional variables of rotation and orbit require association with a dimensional quantity to be proportional to a 1D element.

An example of a model output is a geometric static physical model, or the real-world fixed temporal point, shown using a scaled *fse* (1:10^9) measurable by SI units (Figure 3). Positioning on a lattice aligned to North, a model architect can layer additional information that can be independent of the temporal point element, such as Earth's ecliptic plane angle of ~1.6˚.

A fundamental element shares consistency in many descriptions for jiān, or "interstices", discussed in Mohist canon works. [14] In the approach for relationalism,

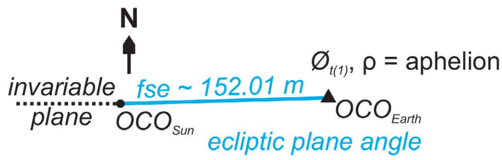

Figure 3. Event state model for Earth's aphelion. OCO = object-oriented origin, ρ = signal, t = time, m = meters, *fse* = fundamental spatial element in scale of 1:10^9, t_1 = temporal end-point (aphelion signal on July 5th, 2024 @ 05:06 UTC).

interstices are being initially explored as being consistent with an extrinsic fundamental spatial element. The interstice in this exploratory interpretation is consistent with that described as created by objects in relation ([14], p. 113) rather than being an actual physical entity itself. Intrinsic fundamental spatial elements are instead converted into Rt units that have magnitudes that fill object-specific space.

2.3.3. Spatial Point Elements

A spatial point is a geometric signature of an object-oriented location, without magnitude (e.g., point of no parts; zero-D). This means there is no measurable quantity at this point, including a zero-rest state mass. A center-point element is fixed at a defined object-oriented temporal end-point, an anchored pillar in an otherwise continuous state of motion for the object.

Let $S\varnothing_l$ be a non-empty set of zero-D spatial point elements with location only, where $\varnothing_l \in S\varnothing_l$.

Each object's fixed location is a spatial point element which can be 1) intrinsic, an object centroidal origin (OCO) [7], or centre point, and 2) extrinsic, meaning one OCO is in relation to another object's OCO (**Figure 3**). An intrinsic *fse* is bound by an OCO and an object-based terminal point of the same object, e.g., the Northern tip (polar radius on polar plane) or a point on the equator (equatorial radius on equatorial plane). [7] An extrinsic *fse* is flanked by the two OCOs, giving a directionality, and it is proposed as a notation for OCO East and OCO West. The extrinsic *fse* is empty of any intrinsic object information and vice versa.

In a system model, more than one point can overlap, a geometric superposition, possibly as locations for zero-D points with no parts. A single point can overlap (geometric superposition) more than one point is also speculatively similar to "yīng 'overlapping' means each is entailing the other." ([14], p. 119) These object-centroidal origins also share similarities with the Mohist word zhōng, or center point, as "extensions [units, or measuring rods] starting from this match one another" ([14], p. 104). We suggest this shares similarities worth noting for building a magnitude using Rt units in straight alignment, with equal lengths from this point, object-specific.

For a final comparison in this section, we consider Mohist translations for "duān, or 'end-point' is the element that, having no magnitude, comes foremost." ([14], p. 112) The translators go on to describe this element being both and end-point and a starting point. Discussed as "refer equally to the 'starting point' as well as the 'termination point' of a line or rod." ([14], p. 112) This duality is consistent with being an end-point in extrinsic *fte*(s), but also a starting point for intrinsic *fte*(s) used to build out from this center-point in a model. Additionally, in relationalism geometric modeling, this point is a fixed object location based on a cycle's temporal end-point, which also serves as the starting point for the next cycle, particularly when it's the primary cycle rather than a secondary one with a paired function remainder.

Dimensional quantities are measured from a geometric centroidal point at a temporal point signaled by a discrete event (non-continuous) taken from the boundaries of a real-world system. Like a quantum superposition, its location isn't fixed between measurements, so is that a temporal end-point is when spatial measurements exists and only when a specific signal event occurs.

2.4. Temporal Rt_n Units

A Rt_n unit is a ratio unit (ordered part) of a single and whole fundamental temporal element, similar to Egyptian unit fractions. [6] We note this as being similar to Mohist translations for sǔn, to lessening, or "an element of a composite whole." ([14], p. 92) Divisional structure of Rt units is aligned with ancient divisional structures that include creation unit classes for rotation (D_{24}), orbit (D_{18}), time (D_{12}), length (D_{360}), and scale (D_{10}). [7]

The accuracy of a fundamental element's duration is limited by technology, not its definition. Thus, partitioning a larger element into smaller parts can closely match standard measurements (e.g., SI units), with precision affected by modern rounding rules and unit definitions.

2.4.1. Division of Time

The divisional system for a fundamental temporal element, a continuous duration with no points, begins by organizing first-order partitions into a dihedral symmetry class—a group of symmetries for a regular polygon, including rotation and reflection. [7] This first division defines an object-oriented class for representative and comparative geometric forms from the same class. Subsequent subdivisions are then made using ordered radix groups (**Table 1**). In the context of a single radix group, a unit is ordered according to its position in the numeral system, counting as a summation of a magnitude, not a stand-alone unit independent of another unit.

For temporal modeling, we guide the divisional system based on ancient mathematical approaches. [6] For example, Babylonians divided an Earth's orbit into 360 degrees or units, but they did not divide the Earth orbital period using the sexagesimal (base-60) system. Consistent with dimensional analytics, spatial units can not be equated to temporal units. Therefore 360 is applied as a division for spatial elements and 12, 18, are applied as a division of temporal elements with a subdivision that can equal 360 parts of a whole. For example, $N_t = 12(30) = 360$ and $N_t = 18(20) = 360$. [6] [7] (**Figure 1** and **Table 1**).

Table 1. Temporal divisions/subdivisions for ratio units and n-polygon angle.

Unit class	n-Polygon	Interior angle	Divisional structure for unit class
Rt_{12}	Dihedral-12	30˚	$12(30^x)$
Rt_{18}	Dihedral-18	20˚	$18(20^x)$

By expanding the radix subdivisions, e.g., $12(30^x)$ where x is a natural number, the temporal interval can be shortened to either near, or at the temporal atomic interval limits of the real-world system (e.g., Plank time). The approach eliminates rounding errors from decimal notation. An atomic interval is the smallest indivisible duration between two discrete signals based on real-world system constants.

2.4.2. Term Statements, Modula, and Geometry

As previously discussed, [6] a geometric term statement for the magnitude of the dimensional quantity of a Euclidean temporal element includes the necessary conditions to describe the element. The counting system does not follow the fundamental laws of arithmetic because each number is not abstract.

Numbers are a part of a symbolic representation of a dimensional geometric element with real-world quantities and properties. Fundamentally, there is only geometry. Thus, numbers and symbols are simply codified expressions to make real-system relationships, scales, and quantities comprehensible for model construction. Symbols and measurement technology change over time, but the identity in the identical geometry remains.

Temporal term statement being

$$W\frac{k_t}{N_t}\text{Object}_{\text{condition}}\text{Rt}_n\left[\rho\right] \tag{8}$$

where W is a count of the number of whole periods, k_t is the temporal Rt unit count part of a whole period, N_t is the number of temporal elements in the set of units or multiplicity of the set. Note that when N is used, it is a general notation that is applicable to either N_t or N_s. Rt refers to Rt unit, n identifies the symmetry of the unit (also the first-order division of the whole), and $[\rho]$ is the assigned discrete signal for cycle counts. [6] [7]

Here, we consider parallels to modular arithmetic with variables from the Rt term notation; we present below with object-oriented data and dimensional quantities omitted.

$$\text{Improper Fraction}\left(\text{mod}\,N_t\right)=\left(\frac{W\times N_t+k_t}{N_t}\right)\text{mod}\,N_t \tag{9}$$

Example using $N_t = 12$, $W = 3$, and $k_t = 6$, we apply the modulo operation and express this as 42 mod 12 = 6, with a final representation being:

$$\left(\frac{3\times 12+6}{12}\right)\text{mod}12=\frac{42}{12}\text{mod}12=6 \tag{10}$$

In this example, $N_t = n = 12$ in a term statement.

$$3\frac{6}{12}\mathrm{Earth}_{\mathrm{orbit}}\mathrm{Rt}_{12}[\rho] \tag{11}$$

The same information is expressible using static state Euclidean geometry. The temporal line represents a unique cyclic duration (**Figure 2**), a temporal element. The temporal duration (an element from the domain) is mapped to a geometric line as a codomain, and in this example, arbitrary length represents a whole duration. Note that a Cartesian coordinate system is not needed. The same count of parallel lines can be represented by whole W and parts of a whole, k_b, in term statements (**Figure 4**). [6] [7] An inherent feature of the geometry is that for an actual concrete record (**Figure 4**), the three whole lines would not be equal in length as each Earth orbit has a unique duration. In a physical model application, the OCO for Earth can be used as an object center-point to fill in using a straight alignment of intrinsic spatial Rt units for a magnitude.

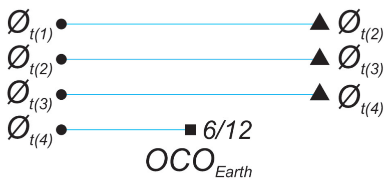

Figure 4. Geometric temporal element memory. Mapping three recurrent *fte* temporal elements and half of a fourth marked by the placement of $\mathrm{OCO}_{\mathrm{Earth}}$, time as the domain, and proportional length as a codomain.

We note that missing from the term is the temporal anchored position in linear time from the real-world system, which today can be in reference to the Gregorian calendar, or other global calendar systems, and SI units for time.

2.5. Long Count Notation

Because the variable W in the terms statement is limited for applications requiring a longer count, such as a calendar system, a novel long count structure is introduced.

$$\langle bx'_z\rangle\langle bx'_2\rangle\langle bx'_1\rangle \| D_n \mid \langle bx_1\rangle\langle bx_2\rangle\langle bx_3\rangle\langle bx_z\rangle\cdot\langle 10_1\rangle\langle 10_2\rangle\langle 10_z\rangle \tag{12}$$

where z is a group order counting up using natural numbers, and x represents a base-x group. The prime symbol to the left of the double vertical lines signifies a whole count group for the recurrent *fte* elements.

To the right of the double vertical line (see Equation (12)), we represent the divisional structure of the fundamental element. It starts with the dihedral symmetry class for the Rt_n unit, followed by potential subdivisions using ordered radix groups (numerical order) marked by the radix point. Decimal positions—tenths, hundredths, thousandths, etc.—are grouped by base-10 on the right side of the radix point. Progressing from right to left beyond the double vertical bars, we see the order of groups and counts past recurring fundamental temporal elements (or cycles) which need defining.

Important to consider is the radix application. To demonstrate, we take a fundamental spatial element's divisional structure. Here, ||360|⟨60⟩⟨60⟩, whereby N_s = 1,296,000. However, in comparison to conventional applications, the radix point increases the multiplicity of the set, whereby ||360|⟨60⟩⟨60⟩·⟨10⟩, is N_s = 12,960,000.

For example, we present the primary lunation cyclic count of a luniterranean calendar's [6] short and long count system in geometric form (**Figure 5**), inspired by Mesoamerican timekeeping divisional structures [29] and the Aztec calendar stone physical structure. Shown is a full moon after 58 lunation cycles (**Figure 5**), which is the equivalent to 4 luniterranean years, the 6th moonth of 13 moonths, and the 9th of 18 divisions of a moonth. Note we only present "moonth" to avoid confusion about the 12 months term from the modern Eurocentric Gregorian Earth orbit calendar.

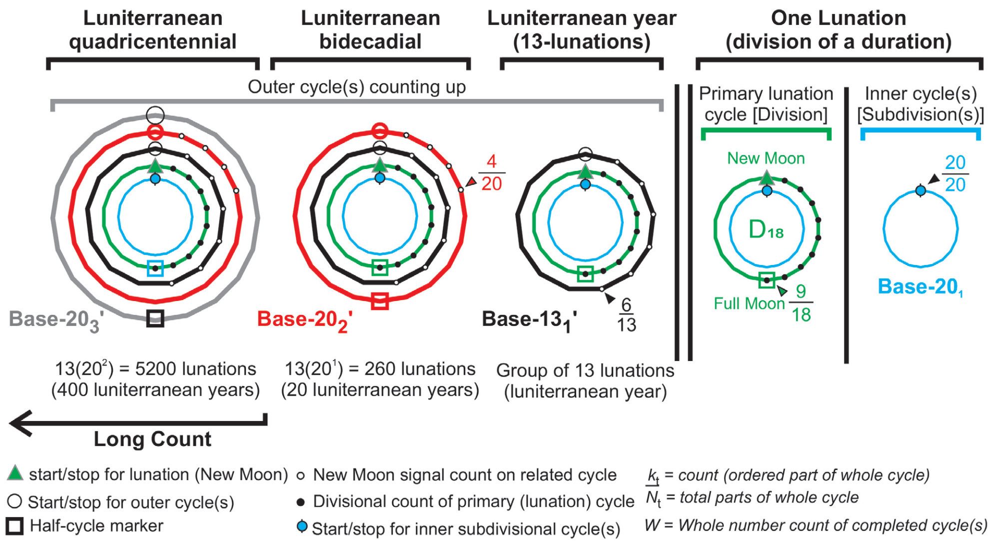

Figure 5. Close form geometric expression of ordered lunations from a luniterranean calendar system. Geometric representation of ⟨20⟩⟨20⟩⟨13⟩||D18|⟨20⟩.

2.6. Dimensional Quantities Associated with Non-Dimensional Variables

A temporal Rt unit is a geometric element, but it can also be associated with a quantity of dimensional time as measured by a standard unit for time. For a single *fte*, the divisional structure of time adds symmetry, group order, and numerical order consistent for each *fte*. However, when signals are aperiodic, the dimensional quantity of time each cyclic duration would be different and represented by the slight change in geometric proportions.

Associating a non-dimensional variable, such as a rotation, with a dimension of time involves mapping the proportional geometric structure of a duration taken from a defined object-relative condition. This process effectively imbues the geometric proportion with temporal properties, allowing it to be both compared to other geometric proportions and measurement of dimensional quantities that correspond to the proportions. How time is mapped will determine how the geometric proportions are represented in the geometric output.

Using term statement notation, we show an example of the duration of Earth's orbit using a perihelion signal; each orbit can be measured by a dimensional quantity of time to be about 365 days, 6 hours, 13 minutes, 53 seconds (31,558,433 seconds) and mapped to a rishtar fundamental temporal element.

$$1\frac{0}{360}\text{Earth}_{\text{orbit}}\text{Rt}_{12}\left[\text{perihelion}\right] \approx 365\text{ days}, 6\text{ hours}, 13\text{ minutes}, 53\text{ second} \quad (13)$$

The same approach is applicable to a frame period and calendar systems. For example, a luniterranean calendar system's primary frame period is 13 lunations with a secondary cycle of Earth's rotation (384-day regular year), creating one unique luniterranean year. [6] We used Rt_{18} units for a sample period (lunation/orbit). In this example, $N = 18(20) = 360$ and the aperiodic signal is a New Moon conjunction event.

$$13\frac{0}{18(20)}\text{Moon}_{\text{lunation}}\text{Rt}_{18}\left[\text{New Moon}\right] \approx 383.900\text{ days} \quad (14)$$

The same frame period can be treated as a single fundamental temporal element and expressed using the Rt_{12} unit class whereby one luniterranean year that uses the New Moon signal for the temporal start point and end-(stop) point can also be divided into 12 equivalent units of time (**Figure 1**). To reduce the interval duration, increasing precision), the multiplicity of the set can increase, $N = 12(30^x)$.

$$1\frac{0}{12}\text{luniterrean}_{\text{year}}\text{Rt}_{12}\left[\text{New Moon}\right] \approx 383.900\text{ days} \quad (15)$$

The object-oriented method mapping is universal for Rt_n unit class designations. This can include each unique Earth sidereal rotational duration, with divisions of $N = 24(60^2) = 86{,}400$.

$$\begin{aligned} &1\frac{0}{86400}\text{Earth}_{\text{rotation}}\text{Rt}_{24}\left[\text{sidereal}\right] \\ &\approx 86164.095\text{ seconds} \approx \frac{12}{12}\text{Earth}_{\text{rotation}}\text{Rt}_{12}\left[\text{sideral}\right] \end{aligned} \quad (16)$$

Existing examples from Mars24 time are already used. [30]

$$1\frac{0}{86400}\text{Mars}_{\text{rotation}}\text{Rt}_{24}\left[\text{solar}\right] \approx 88775.244\text{ seconds each sol} \quad (17)$$

3. Modeling

In relationalism modeling, geometric outputs can be studied in a way consistent with object-oriented discrete event simulations yet include geometric dimensional analytics. Relational elements from a temporal end-point are identities that can be selectively layered, scaled, and mapped where past and present relational inputs are expressed in geometric outputs.

3.1. Arc Modeling Temporal Elements

An arc length configuration is a setup where the distance along a curve (like a rope of fixed length) is held constant, maintaining the curve's shape and total length. For modelling fundamental temporal elements, we apply an arc-like principle where the temporal element is the domain mapped to a spatial element; it creates both open (Figure 6(A)) and closed geometric forms (Figure 6(B)). The example in the figure uses temporal elements as the domain.

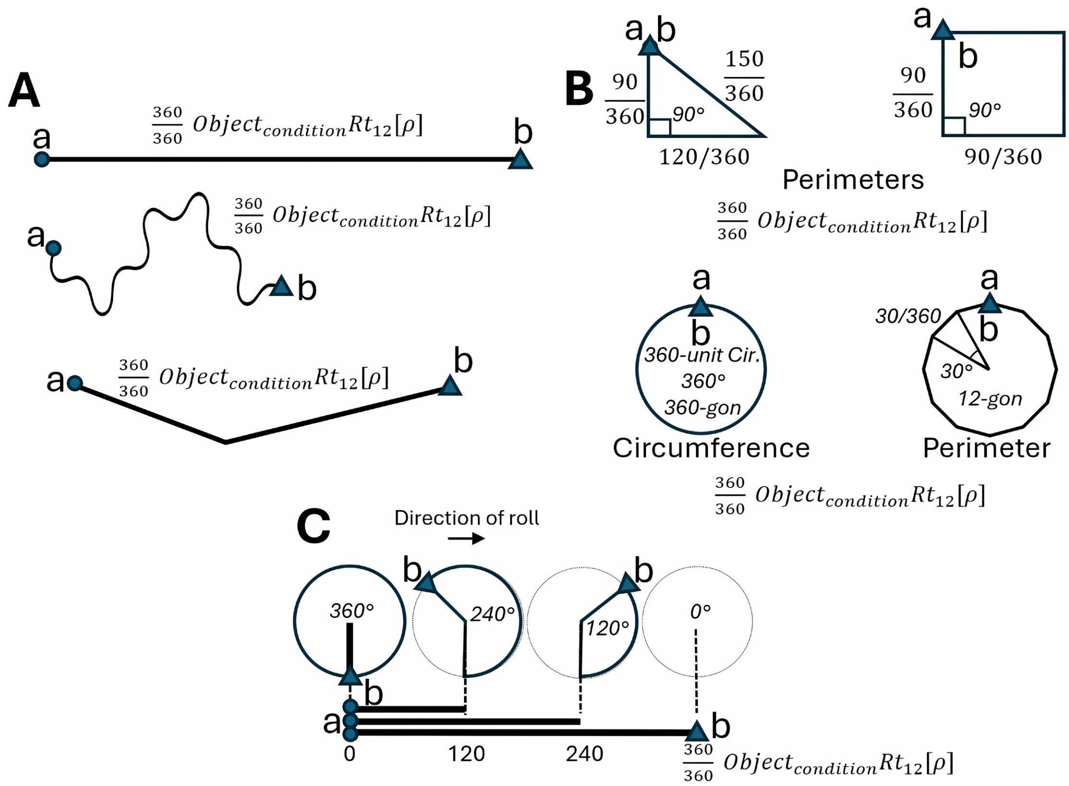

Figure 6. (A) open and (B) closed geometric forms; Pythagorean triple [3:4:5], square, circle, and 12-gon; (C) circumference measure from an unwrapped circumference, shown partitioned into $N_t = 360$ Rt_t units.

For closed-group geometric structures, the approach allows for a transfer of symmetries from Rt_n units to a geometric shape, including a type of modular arithmetic for temporal cycles. We introduce two types of scenarios for 2D (or 3D) forms 1) symmetric vertex alignment and 2) asymmetric vertex alignment.

Symmetric vertex alignment is where the number of vertices (V) in the geometric structure equals the order of the symmetry group (n) in the Rt_n unit ($V = n$). For Rt_{12} examples include a dodecagon ($V = 12$, sides = 12) or icosahedron ($V = 12$, edges = 30, and faces = 20). Asymmetric vertex alignment is where the number of vertices in the geometric structure does not equal the order of the symmetry group of the Rt_n unit ($V \neq n$), as seen in a triangle with three vertices.

For closed forms, consistent with an Euler circuit using vertexes and edges, a temporal cyclic path, or the complete *fte*, is from the temporal start point to temporal end-points in time, $\varnothing_t$. Whereby the geometric superposition of two zero-temporal points, $\varnothing_{t(y)}$ and $\varnothing_{t(y+x)}$ close the geometric cyclic form of time. Application of a 1D path ensures both order and consistency to geometric modeling cyclic intervals of time where a signal both ends one path so it can be modelled to begin the next module, or object-oriented cycle of time.

A circle is generally defined as having no vertices. However, in this context, a vertex is created by the geometric superposition for two defined zero-temporal points that bind the *fte*. A temporal circle model can correspond to a *fte* partitioned into $N = 12(30) = 360$ Rt_{12} units (**Figure 1**), meaning the circumference has 360 temporal line element units. In context describing a circle rather than a polygon with equal sides, the length of an arc corresponds to part of the whole circumference.

$$s = k_t \tag{18}$$

where k_t = the partial count (magnitude), or Rt_t unit count, and s = arc temporal quantity, which both correspond to a part of a whole duration (*fte*). In contrast, a spatial line element uses a length dimension, not a temporal dimension, where Rt_{360} unit can be divided into $N_s = 360$, $N_s = 360(60) = 21{,}600$, or $N_s = 360(60)(60) = 1{,}296{,}000$.

A circle in this scenario is not necessarily on a 2D plane, it can be twisted into a helical form, for example. In helix geometry (or regular polygons using vertices instead of twists), a unit length of $N_t = 360$ Rt_t units for a complete cycle can use a basic formula to define the first sub-divisional structure of temporal metrics.

$$bx_1 = \frac{N_t}{n} \tag{19}$$

where $N_t = 360$, n = equals dihedral symmetry group (D_n) = number of twists (or vertices), and bx_1 = first subdivisional radix group = equals the k_t count (magnitude) between twists (or edges). For example, when $n = 12$, bx_1 equals 30 and when $n = 18$, bx_1 equals 20. Even though a temporal element may be modelled in a higher-dimensional geometric space, it is still considered 1D because its points are parameterized by a single variable.

3.2. Mapping Space and Time

The rishta system uses independent sets for spatial and temporal elements. This approach allows for bijective and non-bijective mapping of elements. We discuss non-bijective (spatiotemporal, temporospatial) and bijective mapping of spatial-temporal elements from constants. Various propositions from the Mohist canon align with spatial and temporal contingency ([14], pp. 85-99, 125-129). However, an in-depth comparison is beyond the scope of this article.

3.2.1. Length Element Mapped to Time Element (Spatiotemporal)

We define a function that maps an element from a set of spatial elements (domain) to a corresponding element in a set of temporal elements (codomain).

$$f : FSE \to FTE \tag{20}$$

In an example that combines mapping with term statements (without scaling), we present the following where the mapping uses $k_t = N_t$:

$$fse \to fte : N_s \text{ Object}_{\text{Condition}} \text{Rt}_{360}^{N_s} [\gamma \text{ or } \rho] \to \frac{k_t}{N_t} \text{Object}_{\text{Condition}} \text{Rt}_{12} [\rho] \quad (21)$$

where subscript s refers to spatial properties and subscript t refers to temporal properties. The 1D spatiotemporal mapped line can be represented by a k_t count where N_t can be subdivided for greater precision in time, e.g., $N_t = 12(30^6) = 8{,}748{,}000{,}000$.

In a real-world example, we introduce a many-to-one mapping, where an average or an astronomical unit (AU = 149,597,870,700 m) is mapped to the Earth's sidereal rotational period (~86,164.0905 seconds).

$$149{,}597{,}870{,}700 \text{ meters} \to 86{,}164.0905 \text{ seconds} \quad (22)$$

Expanding upon this, we introduce the concept of temporal scaling a spatial element, where we *divide* the spatial element by the temporal element to create a type of ratio, whereby a spatial quantity is associated with a temporal quantity, measurable by SI units expressed as an absement, L·T.

$$\text{Temporal scaling factor}\left[\frac{fse}{fte}\right] = \frac{N_s \text{ Object}_{\text{Condition}} \text{Rt}_{360}^{N_s} [\gamma \text{ or } \rho]}{\frac{k_t}{N_t} \text{Object}_{\text{Condition}} \text{Rt}_{12} [\rho]} \to [\text{L}] \cdot [\text{T}] \quad (23)$$

For example,

$$\frac{1296000 \text{ Earth}_{\text{orbit}} \text{Rt}_{360}^{1296000} [\text{AU}]}{1\frac{0}{86400} \text{Earth}_{\text{rotation}} \text{Rt}_{24} [\text{sideral}]} \to 1{,}736{,}197.409 \text{ m} \cdot \text{s} \quad (24)$$

The equation also allows us to use Rt units, or a part of a whole extension, to scale even further, where $N = 360(602) = 1{,}296{,}000$. Adjusting k_s count equal to one, the output changes accordingly.

$$\frac{1 \text{ Earth}_{\text{orbit}} \text{Rt}_{360}^{1296000} [\text{AU}]}{1\frac{0}{86400} \text{Earth}_{\text{rotation}} \text{Rt}_{24} [\text{sideral}]} \to 1.33966 \text{ m} \cdot \text{s} \quad (25)$$

In contrast to Maxwell's approach to addressing problems of ratios of units, whereby a ratio of length over time has equality to velocity (Ch 2) [31], each element remains independent. The ratio is simply a mapped relationship between elements from two independent sets, more consistent with being a scaling factor for geometric elements. We contend that the conventional definition of velocity as length divided by time has imposed a perceptual constraint, leading to a more limited approach in mathematical modeling within dimensional analysis.

3.2.2. Time element mapped to length element (temporospatial)

In a corresponding mapping, a duration is mapped to a proportional length element, resulting in a length that corresponds to a time period applicable for comparative modelling. This relationship is represented by the function:

$$f : FTE \to FSE \quad (26)$$

3.2.3. Bijective Mapping a Spatial-Temporal Constant

The speed of light in a vacuum, c (defined as 299,792,458 m/s), is properly described as being one of seven defining constants. ([32], p. 128) Discrete displacement of light particles can also be represented using SI units and absement, or 299,792,458 m·s.

A bijective mapping using a constant, such as the speed of light in this example, results in simultaneous spatial extension and temporal duration relationship. This ensures that the mapping is both injective and surjective, thus establishing a bijection. Consequently, a perfect one-to-one correspondence between a length and a time quantity when defined from an origin to a terminal point. This relationship can be expressed as a function, where either time or space is used as a measure with the constant.

For light, let L_c be a non-empty set of undefined 1D extensions of space, where element $l_c \in L_c$. Let T_c be a set of undefined 1D durations of time in a real-world system, where element $t_c \in T_c$. In this function, discrete displacement can be a measure of time or of length.

$$f:[L_c \leftrightarrow T_c] \tag{27}$$

Therefore, for light,

$$l_c \to f[l_c] = t_c \text{ and } t_c \to f^1[t_c] = l_c \quad (\text{here } f^1 \text{ is the reciprocal function}) \tag{28}$$

where c is a relational quantity element for light in a vacuum. The approach can adjust for differences in the speed of light through a given medium, such as water.

3.3. Measure of Time

We introduce absolute point-time as being a uniform flow of time measurable from any independent privileged zero-D point in the universe relative to itself. This concept differs from both the Newtonian Absolute Time model [33], which models time uniformly across 3D space, and Einstein's Theory of Relativity [22] [23], where time is measured through space and influenced by the observer's velocity and gravitational fields useful for different modeling purposes.

A so-called "ideal" time is measured without accounting for relativistic effects, as it operates outside a relativistic framework at a point relative only to itself. A duration of relational time is measured by ideal time and becomes objectified in a relationalistic framework.

Abstraction of time in a mathematical system with infinite and infinitesimal is proposed to flow through a privileged point. By definition, relational-time does not exist without objectivation in a relationalistic framework. It is a hypothesis that abstract and continuous time exists outside of this framework. Philosophically, this raises the question of whether time can exist in a progressing present if there is no memory of the past [change] or anticipation of the future [change]. However, for a real-world system, relationalism uses this hypothesized uniform flow and binds it to the limits and scale of matter within a real-world system itself (e.g., wave function collapse representing a temporal point element, t = 0 quantifiable time).

The flow of time can be measured from a privileged point limited; however, precision and accuracy are limited to instrumentation technology, and definitions of a standardized unit system. It is also limited to a methodology. The flow of time is modelled as being uniform at each point relative to itself, durational measures from multiple privilege points can serve as both proportional geometric measures.

We introduce co-existing infinite and infinitesimal continuous time along with discrete relational time with an invariant and constant scale from a real-world system. This opens a theoretical possibility of overlapping and co-existing relational timelines from two distinct real-world systems. Consider real-world system A with a constant with bijective mapping that has an atomic temporal line element that is proportionally greater than the entire alpha-omega relational timeline of a real-world system B. In a discrete event model of system A, at a temporal endpoint defined by one atomic temporal line element to another, system B would have started, progressed, and ended. This suggests the theoretical possibility of an invariant scale for real-world systems within an infinite range of real-world possibilities.

The evolution of time in a system model follows the Principle of Indiscernibles, where each object-oriented relationship has a unique temporal point and corresponding spatiotemporal coordinates. Understanding the zero-D geometric signature coordinates from one event to another can lead to a repository of data applicable for non-causal operations for building predictions that can be tested to the limits of technology.

4. Findings

Pythagorean Triple Triangles and a Relationship with Dihedral Symmetry

In study of closed geometric structures (**Figure 2**) using ancient divisional structures (e.g., $12[30^x]$, $24[60^x]$, $18[20^x]$, $360[60^2]$), we discovered an interesting relationship.

We introduce an exploratory formula with variables being e (scaling factor), perimeter (P), p (side), q (side), and r (hypotenuse) are each natural number measures where the perimeter is equal to N (multiplicity of elements representing the whole fundamental element). The problem is Diophantine in nature, but it adds an additional criterion related to the divisional structure of units. N can be either the division unit ($N = n$) of the Rt_n unit, or a subdivided unit inclusive of radix group(s) where $N > n$, regardless, $N = k = P$. As this was found in both N_t and N_s, N is used in generality.

The scaling factor e for Pythagorean triples maintains a consistent geometric relationship between the triangle's perimeter and dihedral rotational symmetry. We introduce e as a function of the parts (k) for the whole fundamental element and the perimeter of the triangle.

Pythagorean Triple Condition:

$$p^2 + q^2 = r^2 \tag{29}$$

Perimeter P of the triangle:

$$P = p + q + r \tag{30}$$

Determining e:

Value for e is the ratio of the total number of parts N of a fundamental element to the perimeter P of the triangle, scaled by a factor:

$$e = \frac{N}{P} \tag{31}$$

Applying to an example where, $N = 360$ for application in a 3-4-5 triangle (**Figure 1**):

$$P = 3 + 4 + 5 = 12 \tag{32}$$

$$e = \frac{360}{12} = 30 \tag{33}$$

Additional examples with $N = 360$ include other Pythagorean triples; for instance, $P = 30$ for the 5-12-13 triangle where $e = 12$. Also, $P = 40$ (for 8-15-17) where $e = 9$. Lastly, (3) $P = 90$ (for 9-40-41) where $e = 4$.

$$P = e(p) + e(q) + e(r), \text{ where } P, e, p, q, r \in \mathbb{N} \text{ and } P = N = k \tag{34}$$

when $N = n$, the interior angle θ of a n-polygon can be seen as a division of the total angular symmetry, directly linking P, e, and the dihedral group's n-polygon structure. The introduced scaling factor for a subset of Pythagorean triples resembles a form of modular arithmetic pending formal description.

5. Discussion

In this article, we advanced the physical theory of relationalism to support constructing static geometric models of real-world systems suitable for dimensional analytics and discrete event simulations. Our focus was on the independent dimension of time for a real-world system at a scale relevant to that system. We clarified definitions, introduced set theory notation, described the divisional structure of time using ancient timekeeping systems, and discussed unit classes associated with differentiated types of motion. Additionally, we introduced a novel long-count notation relevant to calendar systems.

For modeling, we applied arc length properties for a whole duration to construct both open and closed geometric forms, mapping 1D time elements to a length codomain. This approach also explored mapping temporal and spatial elements as either non-bijective or bijective, demonstrating how the absement of light can be represented as a bijective geometric relationship between elements from spatial and temporal sets.

The mathematical model of time-integrated the flow of infinitesimal and infinite time with discrete time in a real-world system. We did this by introducing an absolute point-time, where time flows uniformly at a point relative to itself anywhere in the universe. Discrete intervals of relational time are bound to the fundamental constants of the real-world system with indivisible atomic temporal line elements (e.g., Plank time) and temporal point elements (e.g., modelled after wave function collapses in quantum physics). Relational time becomes a tracible memory in Euclidean temporal space [6] of a defined order of relationally defined durations, flanked between object-oriented signals and bound to the limits of the real-world system. Time moves through space through privileged points. Those points can change discrete relational spatial coordinates over discrete durations of relational time.

Building on these concepts, the article presented several options for expressing time, in both term statements as well as open and closed geometric forms. By decoupling time from space, the approach allows for the precise mapping of temporal elements without relying on continuous space-time assumptions and offers new ways to map independent elements of space and time. Additionally, the article explored methods to associate dimensional quantities of time with non-dimensional variables, creating a framework where time can be quantified and analyzed independently from spatial dimensions. These innovations provide a more versatile means of modeling time.

Looking forward, a relational approach to modeling time is proposed to reduce the computational burden for modeling matter, space, time, and motion. For example, in the application of object-oriented discrete event simulation (OODES) modeling, changes in a real-world system are modelled at distinct points in time, the signal events marking zero-temporal points in linear time. In OODES, the system is represented as a collection of objects, each with its own attributes and behaviors, well suited for relational modeling. In summary, the advantages of relationalism lie in its ability to simplify time measurement and emphasize discrete and finite structures.

Beyond applications in physics, we propose applications for the field of historical study of time in metrology. Consistencies are seen through the study of ancient civilization's timekeeping systems. For example, ancient China used 12 Shichen units in 1 day, as did the Etruscans and early Romans. In the approach to model time with Rt_{12} units, any sample or frame period of a duration can be divided into $N_t = 12$ parts of a whole duration, be it one Earth solar day or one Earth orbit. This structure is reminiscent of the invented Julian calendar, which was introduced in 45 BCE, with 12 months and 28 - 31 days each month. The divisional structure of 12(30) is also in alignment with the historic tracking of Earth's axial precession, which corresponds with the origins of ancient constellations divided into 12 segments, each representing a zodiac sign with 30 degrees. [34]

Julian-style calendars are distinctive in two key ways: first, they use a secondary cycle (Earth's axial rotation, with a 365-day regular year and a 366-day leap year) to measure a primary duration (one Earth orbital period), which results in a misalignment between spatial and temporal coordinates for each orbit. Second, the system lacks a defined start/stop signal, instead assigning the arbitrary date of January 1st, which is asynchronous with the natural cycle.

At the time of this article, we maintain that a subdivisional structure that uses a sexagesimal (base-60) system is related to spatial line elements rather than a temporal line element. For example, Rt_{360} spatial units divided and subdivided as $N = 360(60^2) = 1,296,000$, and Rt_{24s} units as $N = 24(60)(60) = 86,400$. Modeling experiments (data not shown) shows consistency with using either spatial Rt_{360} and/or Rt_{24s} unit class for build-out from an object center-point. Clarification of dimensional quantities in units is critical for geometric dimensional analytics.

Historically, the Babylonians utilize a 360-degree system with sexagesimal subdivisions defining 60 arcminutes each degree and 60 arcseconds each arcminute for astronomical purposes. This article proposes a consistent approach to the application of an arc-length model. We do acknowledge Babylonians divided Earth's orbit into $N = 360$ "degrees", but they did not use add on the sexagesimal subdivisional system for temporal applications, only spatial applications, as far as we can find in the literature.

In the context of a Eurocentric Julian-style calendar, assumptions are documented they rounded days for simplicity, and had "inaccurate approximations" ([35], p. 81), considering a lack of harmony with the Eurocentric Catholic introduced 365-day Gregorian calendar. We propose that current interpretations in the literature are inconsistent with the precision of timekeeping and mathematics used in ancient Egyptian and Babylonian civilizations and suggestive of confirmational biases rather than giving consideration that alternative methods may have been used. This can be better appreciated by considering the alternative model for time presented in this article, which finds only the Julian calendars as the outliers to ancient pre-Greek calendars thus studied. [6]

The models for a time introduced here are hypothesis-generating for ancient timekeeping systems. They provide an alternative model which can include the study of the Babylonian 360-"degree" units in an Earth orbit, rather than a vague 360 days in a year. Consequently, we thus far continue to support a temporal framework that aligns with the 2D geometric form of a dihedral n-polygon for dividing and subdividing a duration into 360 units: $N_t = 12(30)$ and $N_t = 18(20)$, as discussed in the article.

We also acknowledge the familiar and modern use of $N = 24(60)(60) = 86,400$ for seconds as a temporal division/subdivisional structure to map an Earth solar day and Mars sol. However, we consider a scenario where this application was a more recently evolved approach from Babylonia and Egypt. Thus, until more modeling tests are conducted, we will consider Rt_{24t} (t = temporal) (**Figure 1**) a designation by convention that will require further discussion. Therefore, we propose $N = 24(60)(60)$ should be applied as a spatial subdivisional metric (N_s) to prevent potential confusion for geometric modeling and corresponding dimensional analytics.

We suggest more studies into a more sophisticated approach to using the Rt_{24t} unit class for rotation and temporal mapping, which may be needed in geometric models, including the application of discrete Archimedean spirals.

Three key findings emerged in this article, raising questions for future consideration. The first involves the temporal scaling factor using one astronomical unit (AU) scaled by one Earth sidereal day duration (see Equations (23), (24)). Unexpectantly, the resulting scaled spatial element (1736.2 km) closely aligns with the Moon's semi-minor axis. The result has a 0.011% difference (~200 meters) from published NASA datasets, 1736.0 km [36]. Consistent with this metric, previous work demonstrated how semi-minor axes are used as *fse* for building out object representations on plane-specific models. [7]

The second finding revealed that certain closed-form Pythagorean triple triangles show a consistent scaled relationship between their perimeters and the rotational symmetry of dihedral n-polygons. Both time and length divisional structure show consistent findings. Further study is needed on incircles, inradii, and areas to catalog shared characteristics.

Finally, within the context of motion as framed in this article, we explored Mohist cannon A51, or "bì, 'being inevitable' means being unable to get rid of." [14], p. 99) The canon has been translated "as something 'not stoppable', *i.e.*, that will come about absent any external intervention that might interrupt it." ([14], p. 99) Considering the translation and context, we suggest this finding is a hypothesis generating around comparisons to Newton's first law of motion. [33]

6. Conclusions

In conclusion, this paper advances relational-time modeling continuous and abstract time bound by zero-D temporal points that were defined using quantum phenomena. The new tools for a relationalistic framework continue to advance for unexplored applications in physics.

We introduced what is termed *absolute point-time,* which flows continuously and uniformly through a zero-D privileged point relative only to itself. Slightly different than Newton's absolute time that spans across abstract 3D space. The concept harmoniously co-exists with the well-studied relativistic framework for spacetime. Future work will be required to explore applications for modeling discrete quantum state phenomena.

We discussed a new method for dimensional analysis whereby spatial extensions and temporal durations are separated into independent sets. The approach offered various ways to map elements with different dimensional quantities, including the bijective relationship for the speed, or absement, of light.

By integrating set theory and geometry, we developed a hierarchy of elements and clarified the difference between unit ratios and more commonly known standard units, offering innovative ways to model space and time. Unexpected findings revealed that ancient divisional structures created a scaled relationship among specific Pythagorean triples that preserved the rotational symmetry of dihedral n-polygons, requiring further investigation.

Our approach suggests consistency with Aristotle's view that a continuum cannot be made up of indivisibles. We defined a continuum, or whole duration or extension, as being a line element with a measurable quantity bound between two zero-D point elements. In this framework, zero-D points in time can be sequentially anchored onto a 1D temporal line, separating two adjacent continuous durations of relational time.

This article also paves the way for new approaches to studying and reinterpreting ancient metrology, including the divisional structure of ancient timekeeping and calendars. The approach challenged Eurocentric Julian-style harmonistic interpretations of ancient calendars. Additionally, it introduces a previously unreported connection between relationalism and the Chinese Mohist canon, which also offers novel considerations for future research.

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AI Disclaimer

This paper has parts that have been reviewed and edited with the assistance of ChatGPT, an AI language model developed by OpenAI, for grammar and wording improvements. The content, arguments, and conclusions are the original work of the author(s), and any errors or omissions remain the responsibility of the author(s).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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