Scientific Research Publishing

International Journal of Geosciences, 2026, 17(11), 976-984
https://www.scirp.org/journal/ijg
ISSN Online: 2156-8367
ISSN Print: 2156-8359

Methodology for Assessing the Joint Use of Groundwater in Transboundary Aquifers Using Mathematical Models

Irina Polshkova

Independent Researcher, Moscow, Russia
Email: z_irpol1@mail.ru

How to cite this paper: Polshkova, I. (2026) Methodology for Assessing the Joint Use of Groundwater in Transboundary Aquifers Using Mathematical Models. *International Journal of Geosciences*, **17**, 976-984.
https://doi.org/10.4236/ijg.2026.1711045

Received: August 27, 2026
Accepted: November 23, 2026
Published: November 26, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).
http://creativecommons.org/licenses/by/4.0/

Open Access

Abstract

This article presents the results of modeling transboundary groundwater flows in the border regions between Russia and Ukraine and between Russia and Kazakhstan. The models reproduce steady-state flow conditions and quantify changes in the hydrodynamic flow structure caused by anthropogenic stresses under disturbed conditions. Changes in transboundary horizontal groundwater flows induced by groundwater abstraction are calculated. Modeling is performed using the Aquasoft special-purpose software system.

Keywords

Transboundary Ground Water Flows, Mathematical Modelling

1. Introduction

In recent years, the use of transboundary waters has become particularly important for many countries, especially Russia, Kazakhstan and Ukraine.

A report by the United Nations Economic Commission for Europe notes that more than 150 major rivers and 50 lakes cross the borders of two or more countries. Western and Central Europe contain more than 100 transboundary groundwater basins. The term “transboundary aquifers” is therefore widely used to refer to aquifers that are currently exploited or considered prospective for development and are located in the border regions of two or more countries.

The use and protection of transboundary rivers are somewhat easier to manage than the use of transboundary aquifers. Most countries have well-developed monitoring networks for river discharge and water quality; in some countries, dedicated hydrometric stations at the border continuously measure river flow and

composition in automated mode. In such cases, the quantity and quality of water to be preserved for a neighboring state are regulated and monitored under existing international agreements.

The study and use of groundwater in transboundary aquifers are considerably more complex. Experience with regional assessments of the resources and quality of transboundary aquifers, particularly assessments of development prospects and permissible abstraction limits, remains extremely limited, even though such aquifers occur along nearly the entire state border of Russia. The first attempt to quantitatively assess the joint use of groundwater from a transboundary aquifer involved modeling the exploitation of the Lomonosov aquifer shared by Russia and Estonia [1].

Investigation and forecasting of hydrogeological processes in aquifers shared by neighboring states require accurate quantitative assessments, particularly where anthropogenic stresses on groundwater are increasing. Reliable forecasts are possible only when the groundwater flow system is considered as an integrated whole whose existence and properties are governed by recharge and discharge conditions; these areas may lie in two or more different countries. Modern hydrogeological practice enables the development of continuously operating groundwater-flow models that reproduce the hydrogeological conceptualization of the aquifer system in both plan and cross-section, as well as dynamically changing anthropogenic stresses. Mathematical modeling can forecast groundwater-surface water interactions and assess the risk of groundwater contamination caused by pollutants originating in a neighboring state.

Sustainable use of groundwater resources in transboundary aquifers is possible only through the development and joint operation of regional hydrogeological models. This approach requires the unrestricted exchange of information between neighboring states on hydrogeological and hydrological parameters, some of which may vary over time, as well as on the parameters and conditions of anthropogenic stresses.

Information on the locations of existing and planned potential sources of groundwater contamination is especially important.

Assessing whether a model adequately represents the process under study requires groundwater-monitoring data from all participating states for the entire period of aquifer exploitation. Consequently, one of the primary project objectives is to establish an interstate agreement governing the provision of the complete set of input and time-series data required for modeling.

2. Rationale for the Modeling Methodology

The governing equation describing groundwater flow in the Aquasoft special-purpose software system [2] can be written in a form with a straightforward physical interpretation: under natural conditions, the sum of flow rates at each point in space is zero; under disturbed conditions, it equals the change in storage:

$$\sum_i Q_{xi}^n + \sum_i Q_{yi}^n + \sum_i Q_{zi}^{n-1} + \sum_i Q_{zi}^{n+1} + \sum_i Q_{IIi}^n + \sum_i Q_{IIIi}^n + \sum_i Q_{wi}^n = \sum_i Q_{ci}^n ,$$

where

- $Q_{xi}^n = \frac{\partial}{\partial x}\left(T_{xi}^n \frac{\partial H_i^n}{\partial x}\right)$: horizontal flow along the X axis (m/day);
- $\sum Q_{yi}^n$: horizontal flow along the Y axis;
- $\sum Q_{zi}^{n-1}$, $\sum Q_{zi}^{n+1}$: vertical flows from adjacent aquifers;
- $\sum Q_{wi}^n$: infiltration;
- $\sum Q_{IIi}^n$: groundwater abstraction rate specified by second-kind boundary conditions;
- $\sum Q_{IIIi}^n = \left(H_{si}^n - H_i^n\right)G_i^n$: groundwater-surface water exchange, H_{si}^n: surface-water level elevation, G_i^n: streambed conductance specified by third-kind boundary conditions;
- $\sum Q_{ci}^n = S_i^n \frac{\partial H_i^n}{\partial t}$: change in storage under transient flow conditions;
- S: aquifer storage coefficient;
- H_i^n: unknown hydraulic head at point i; n is the aquifer number.

The first four terms of this equation describe the structure and hydrodynamic parameters of the water-bearing sequence; the remaining terms describe the parameters and response of external stress sources.

A geological-environment monitoring system can serve as the information base for a mathematical model if automated data exchange is available between the monitoring system and the model grid database. In this ideal case, continuously updated monitoring information is also available to the mathematical model database, providing both control of the geological environment and continuous updating of forecasts in response to additional anthropogenic impacts.

A cartographic database is essential for the development and operation of mathematical models. Accordingly, one function of the modeling software must be to present input and output information in a form compatible with standard GIS package formats.

Observed data on external stress sources serve two purposes:

1) to reproduce model interaction with the external environment in real time;

2) to provide observations, such as discharge at hydrometric cross-sections or spring flow rates, for validating the model against the hydrogeological process under study.

It should be noted that the Aquasoft special-purpose software system does not use first-kind boundary conditions, because such conditions simulate infinite flow rates from recharge or discharge sources near a specified point. The equation is solved by the finite-difference method using nonrectangular quadrilateral blocks. All parameters are assigned at grid nodes with exact geographic coordinates.

This approach demonstrates an important capability of mathematical modeling: deriving new information on groundwater conditions. For example, individual components of the overall water balance can be calculated and used to compile corresponding tables of aquifer groundwater flow at each computational time step. In addition, calculating individual components of the hydrodynamic-flow

balance for any set of model nodes makes it possible to investigate and forecast the effects of anthropogenic stresses on the subsurface hydrosphere.

The following methodology is proposed for assessing permissible joint groundwater use by comparing mathematical-modeling results with undisturbed flow conditions.

The proposed criteria are:

- the degree of groundwater depletion;
- the magnitude of depletion of the groundwater component of river flow;
- a forecast of the extent of groundwater-contamination zones, including compliance with sanitary protection-zone dimensions during operation of well fields located in neighboring states.

Modeling-based optimization of joint groundwater-use schemes according to the proposed criteria requires the following outputs:

➢ ***groundwater-depletion assessment***:

1) aquifer-specific potentiometric-surface maps at selected time steps, showing the direction, rate, and travel time of cone-of-depression propagation relative to administrative boundaries;

2) groundwater-level drawdown curves at internal points of the hydrodynamic flow system located along administrative boundaries;

3) curves showing changes in horizontal groundwater flow as a cone of depression propagates into a neighboring state along the administrative boundary.

➢ ***surface-water depletion assessment***:

1) curves showing changes in groundwater-surface water exchange at the same selected time steps relative to undisturbed flow conditions;

2) the magnitude of depletion of the groundwater component of river flow for any surface-water sources specified in the model.

The total change in water exchange, which determines the depletion of the groundwater component of river flow, can be evaluated by comparing disturbed and natural groundwater-flow conditions using the following relationship. The percentage increase in recharge from surface water and the percentage decrease in groundwater discharge to surface water are calculated relative to natural conditions.

Summation is performed either over all model nodes at which surface-water sources are specified or over the nodes representing an individual watercourse or a selected reach.

➢ ***groundwater-contamination assessment***:

1) maps showing the travel times and extent of groundwater-contaminant fronts from surface sources relative to the sanitary protection zones of operating or planned well fields;

2) identification of the probable locations of contamination sources when pollutants are detected in groundwater samples.

The travel time of the contaminant front is calculated using a piston-flow approximation in a mass-transport model, which additionally requires effective-po-

rosity data. At each point in the modeled area, the contaminant-velocity components V_X, V_Y, *and* V_Z are calculated from the flow gradient in the corresponding directions obtained by solving the groundwater-flow problem. Travel-time isochrones define the configuration of the contaminant front.

Calculated travel times in days are stored in the model database and displayed as an isochron map that, depending on the selected mode, may represent either contaminant-front propagation or well-field sanitary protection zones.

This approximation is sufficient for a conservative worst-case assessment of the rate of groundwater-quality deterioration.

3. Results and Discussion

The methodology described above was used to assess the joint exploitation of aquifers in Belgorod Region, Russia, and Kharkiv Region, Ukraine, as well as aquifers shared by Russia and Kazakhstan.

➢ MODEL 1

The Russia-Ukraine hydrodynamic model covers an area of 248 × 276 km with a grid spacing of 1 km. The model nodes are georeferenced, allowing all observed data from the three original models to be integrated into a single regional model.

The principal exploited aquifers represented in the model are the Quaternary-Pliocene, Maastrichtian-Turonian, Albian-Cenomanian, and Bathonian-Callovian aquifers (terminology according to [3]). Undisturbed flow conditions were reproduced for 1969.

Selected modeling results obtained using the proposed methodology are presented for the Albian-Cenomanian aquifer, which is subject to the greatest stress.

Transient groundwater flow was simulated from 1970 through 2009 using a time step of 365 days. Groundwater abstraction was specified in stages for 1970, 1980, and 1990 in Russia and for every odd-numbered year from 1975 through 2003 in Ukraine.

Table 1 presents data for the largest well fields during the periods of maximum groundwater abstraction.

Table 1. Groundwater abstraction at major well fields specified in the model (m^3/day).

year	Recharge from rivers	Discharge to rivers	Abstraction Aquifer 1	Abstraction Aquifer 2	Abstraction Aquifer 3	Abstraction Aquifer 4	Total abstraction
1970	297100.0	−2895643.0	0.0	0.0	0.0	0.0	0.0
1980	380198.0	−3092887.0	−5600.0	−142735.0	−337633.0	−2000.0	−487968.0
1991	416498.0	−2821991.0	−12640.0	−140649.0	−411201.0	−17898.0	−582388.0
1996	399894.0	−2756362.0	−12640.0	−131186.0	−384720.0	−17898.0	−546444.0
2009	383275.0	−2634687.0	−19660.0	−85030.0	−422116.0	−20622.0	−547428.0

Figure 1 presents comparative maps of groundwater levels under natural and disturbed conditions at the time of maximum groundwater abstraction.

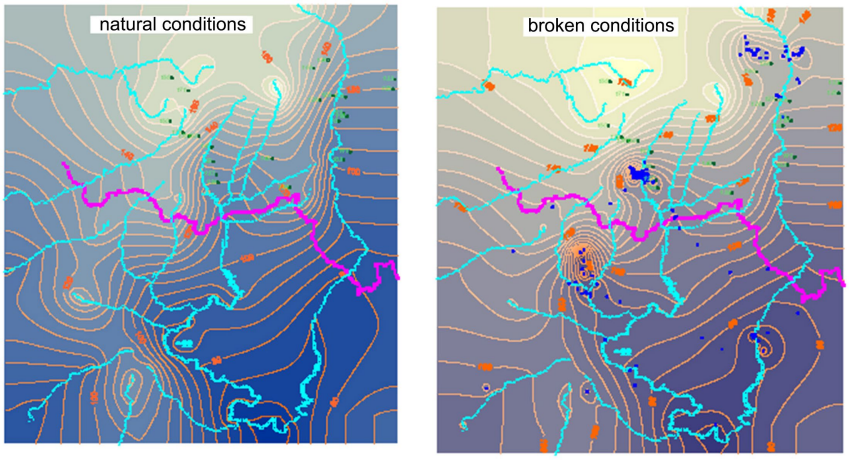

Figure 1. Groundwater-level maps for the Albian-Cenomanian aquifer under steady-state flow conditions and as of 1981.

Maximum groundwater-level drawdown within the cones of depression occurred in 1980-1981.

Depletion of the groundwater component of river flow was assessed for 2009 relative to 1979. As shown in Table 1, water exchange between groundwater and surface water is correlated with changes in total water use: an increase in water use leads to greater recharge of ground at the expense of surface water and, consequently, the outflow into rivers decreases. Then the amount of water use decreases the reverse process occurs.

Groundwater abstraction exceeding 1980s levels may produce a deeper cone of depression and cause substantial depletion of the Albian-Cenomanian aquifer. Figure 2 shows the cones of depression; their outermost contour represents 1 m of drawdown.

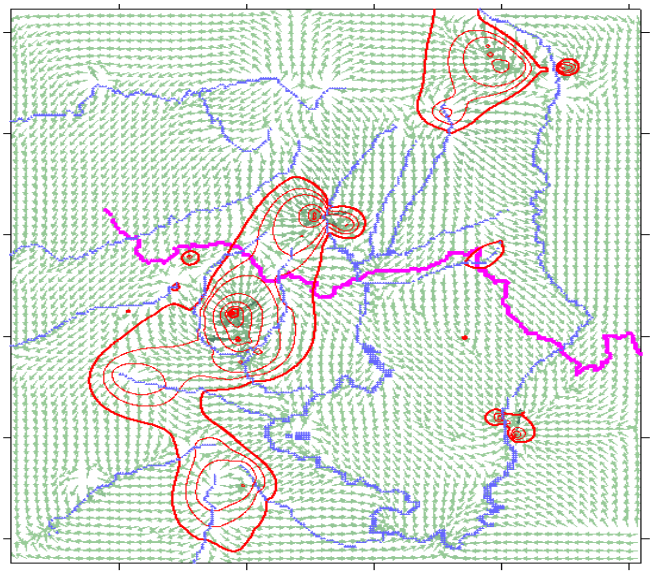

Figure 2. Cones of depression in the Albian-Cenomanian aquifer as of 1981. The minimum drawdown is 1 m.

➢ MODEL 2

A model of the southeastern part of the West Siberian Artesian Basin provides a second example of applying the proposed methodology. Within the modeled area, which measures 800 × 550 km, a single hydrodynamic flow system is divided by the administrative border between Russia and Kazakhstan. The northern part of the model includes Novosibirsk and Omsk Regions and Altai Territory; the southern part covers northern Kazakhstan.

The model grid consists of 170 × 140 nodes and represents three aquifers vertically, including the upper Atlym aquifer and two underlying aquifers in Cretaceous deposits. The model boundaries coincide with natural hydrodynamic-flow boundaries where the aquifers pinch out in the recharge area. Calculations are performed using nonrectangular blocks. Natural conditions are reproduced for the early 1950s, while disturbed conditions represent a 50-year period of Cretaceous-aquifer exploitation. The predictive scenario shown as an example assumes total groundwater abstraction of approximately 530000 m^3/day from the two Cretaceous aquifers. This operating regime produces a cone of depression that extends across the administrative border into Russia (Figure 3).

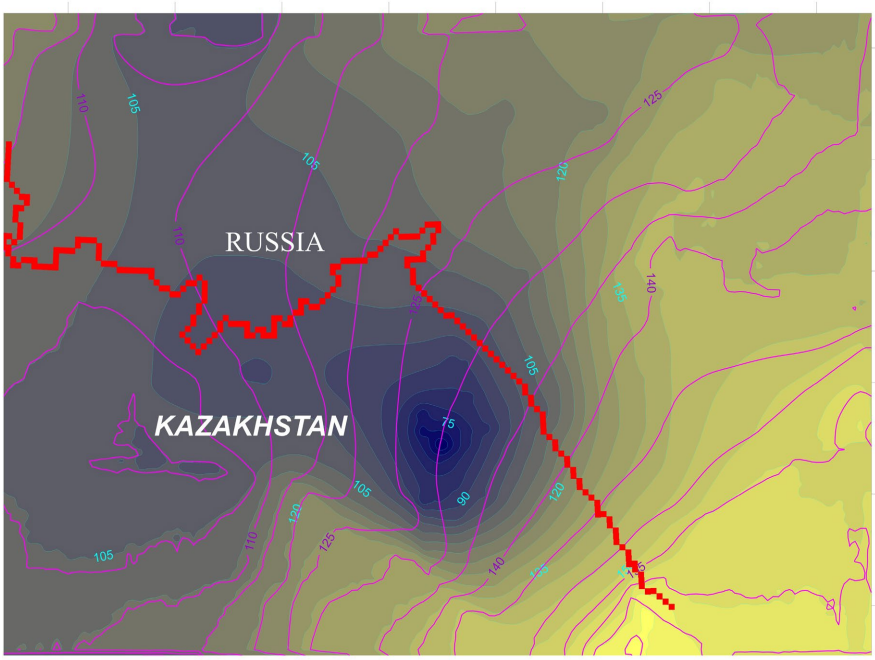

Figure 3. Groundwater-level map for the Lower Cretaceous aquifer (predictive scenario).

Groundwater-level drawdown gradually extends for up to 300 km along the international border and reaches 40 m after 40 years of well-field operation (Figure 4). The horizontal groundwater flow captured by the cone of depression at nodes where drawdown is nonzero increases from 60,000 to 135,000 m^3/day.

The proposed methodology for assessing the joint use of groundwater in transboundary aquifers through mathematical modeling was developed using the Aquasoft special-purpose software system [2].

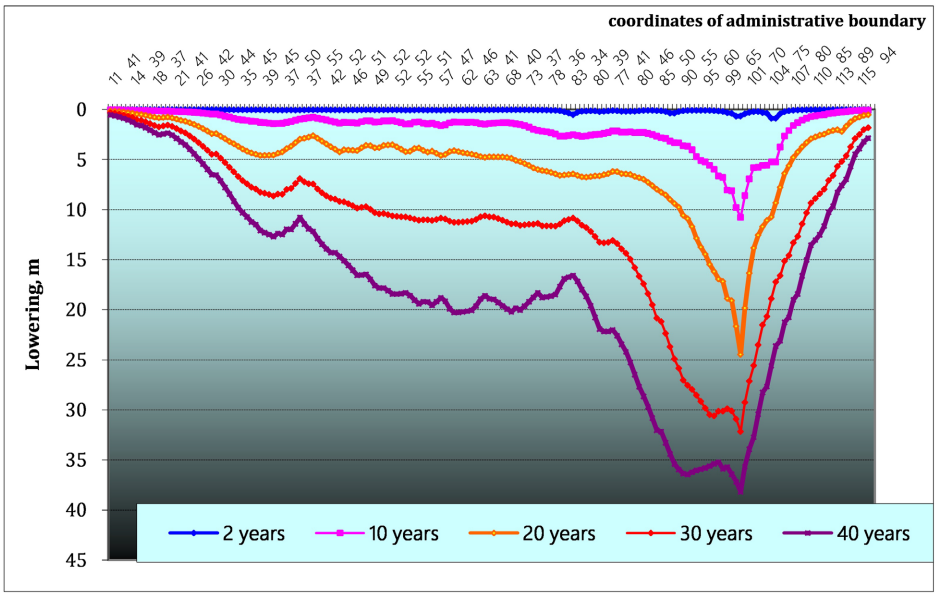

Figure 4. Groundwater-level drawdown in the Lower Cretaceous aquifer at points along the administrative boundary.

4. Conclusions

Mathematical hydrodynamic models used to forecast groundwater conditions should form a subsystem of a comprehensive subsurface-monitoring system.

Under this approach, geological surveys and government authorities obtain a practical tool for assessing current and forecast conditions in the subsurface hydrosphere and for promptly regulating anthropogenic stresses.

Model development should be entrusted to professional hydrogeological organizations with access to all required information, including archival data. Such organizations should include a unit responsible for verifying that the model adequately represents the hydrogeological processes occurring in the area.

Models may be operated by multiple users or clients according to their objectives, for example by well-field operating services or organizations responsible for forecasting environmental changes in the hydrosphere, such as waterlogging or deterioration of groundwater quality.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Mironova, A.V., Mol'skii, E.V. and Rumynin, V.G. (2006) Transboundary Problems Associated with Groundwater Exploitation near the Russian-Estonian State Boundary: Case Study of the Lomonosovskii-Voronkovskii Aquifer. *Water Resources*, **33**, 386-395. https://doi.org/10.1134/s009780780604004x

[2] Polshkova, I.N. (2023) Study of Groundwater Using Hydrodynamic Models. In: *Human-Assisted Intelligent Computing*. https://doi.org/10.1088/978-0-7503-4801-0ch25

[3] Frenkel, D.A. (1999) Development of a Hydrogeological Information and Computer System for Subsurface Resource Management in the Kursk Magnetic Anomaly, Belgorod Region. Geolink Joint Venture CJSC.