

A Comprehensive Review of Low-Luminosity Gamma-Ray Burst Afterglow Modelling in the Synchrotron External Shock Scenario with Bayesian Population Analysis

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Abstract

This paper presents a comprehensive review of the broadband afterglow modelling of low-luminosity gamma-ray bursts (LLGRBs) within the standard synchrotron external forward-shock framework. We synthesize two decades of theoretical developments and observational findings, providing a unified picture of LLGRB afterglow properties from radio to gamma-ray frequencies. Using a sample of eight confirmed LLGRBs (GRB 980425, 031203, 060218, 100316D, 171205A, 111005A, 120422A, and 161219B), we complement this review with an original Bayesian population inference on their redshift, isotropic-equivalent energy, and luminosity distributions. Our analysis demonstrates that LLGRBs form a coherent, nearby population that extends the classical Amati and Yonetoku relations toward lower energies. The review synthesizes current knowledge on: i) the dynamical evolution of the forward shock in LLGRB environments; ii) the spectral regimes and closure relations characteristic of slow-cooling synchrotron emission; iii) the diversity of observed multi-wavelength light curves; and iv) the implications for LLGRB progenitor models and their connection to the broader GRB population. We also discuss current challenges, including selection biases, the small number of confirmed events, and the role of numerical codes such as *afterglowpy* in future analyses. This review provides a foundation for interpreting current and future LLGRB observations and highlights the value of Bayesian methods in extracting population-level inferences from sparse datasets.

Keywords

Low Luminosity Gamma-Ray Burst (LLGRBs), Forward-Shock, Bayesian Inference

1. Introduction

Gamma-ray bursts (GRBs) are the most luminous explosions in the Universe, releasing up to 10^{54} ergs of energy within seconds. First discovered in the late 1960s by the Vela satellites, GRBs consist of an intense prompt emission phase followed by a long-lasting afterglow observable across the electromagnetic spectrum—from radio to γ -rays [1]. These afterglows arise as the relativistic outflow from the central engine interacts with the circumburst medium, producing synchrotron radiation in an expanding shock front [2].

A special class of GRBs, called Low-Luminosity GRBs (LLGRBs) form a unique class of cosmic explosions that exhibit lower isotropic energies ($E_{\text{iso}} \leq 10^{49}$ erg) and occur predominantly at low redshifts ($z \leq 0.3$). Despite their comparatively modest luminosities—LLGRBs such as GRB 980425, GRB 060218, and GRB 100316D have revealed crucial insights into the physics of relativistic outflows, shock microphysics, and the connection between GRBs and Type Ic supernovae [3]–[8].

Although the standard synchrotron model has remarkable success in describing classical GRB afterglows, its application to LLGRBs has been somewhat fragmented across the literature. Individual events such as GRB 980425 [9] [10] and GRB 060218 [3] [4] have each been modeled in detail, but a unified synthesis of these efforts is lacking. Moreover, the population-level properties of LLGRBs—their redshift distribution, energetics, and intrinsic diversity—have not been systematically characterized within a single framework. This review aims to fill this gap by providing a coherent, self-contained treatment of LLGRB afterglow theory and observations.

This paper serves two complementary purposes. First, it provides a comprehensive review of LLGRB afterglow modelling within the standard synchrotron external-shock framework, synthesizing theoretical developments and observational findings over the past two decades. Second, it presents an original Bayesian population analysis of the confirmed LLGRB sample, providing quantitative constraints on the distributions of their redshift, energy, and luminosity. Through this combined approach, we aim to i) provide researchers with a clear, accessible overview of LLGRB afterglow physics; ii) demonstrate the predictive power of the standard synchrotron model when applied to this population; and iii) establish a statistical framework for future population studies as the LLGRB sample grows.

1.1. Fireball Forward-Shock Model

As is well known, a widely used framework for interpreting GRB observations is

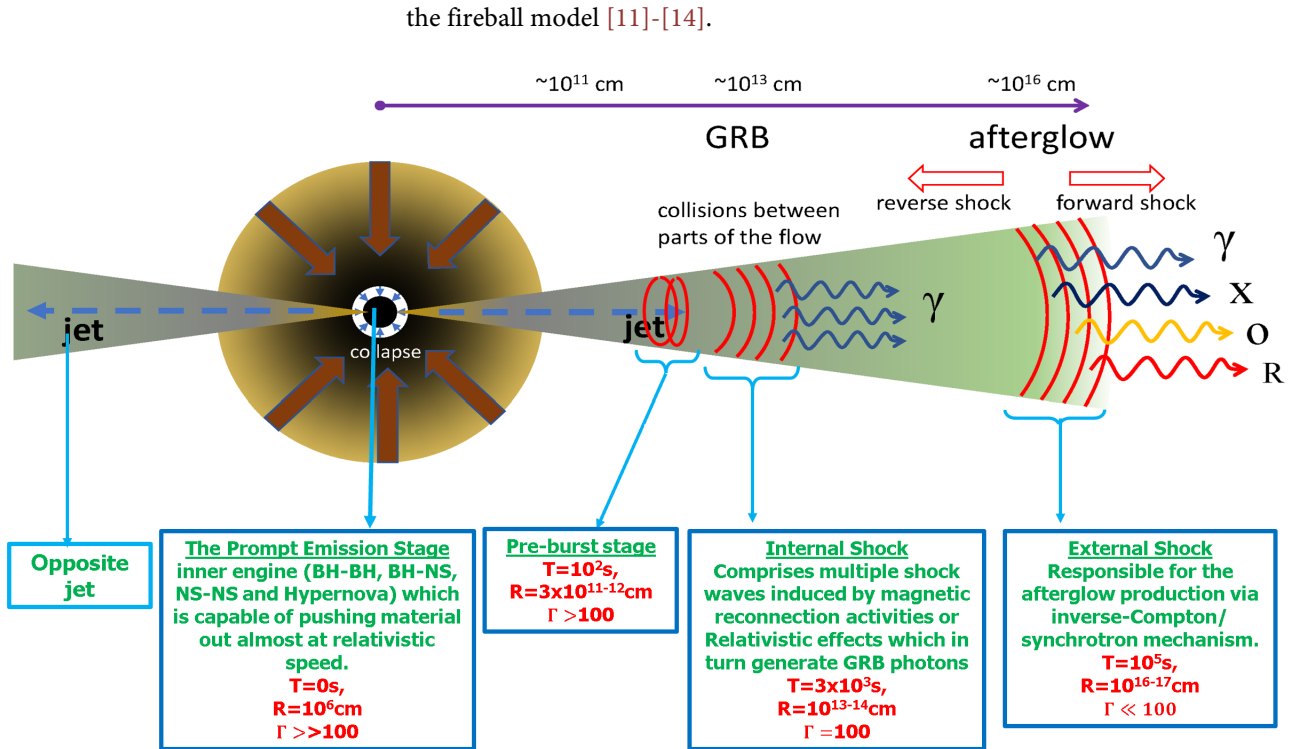


Figure 1. A modified cartoon depiction showing schematics of the fireball model and the basic components of the internal and external shocks. Credit: [15] [16].

To describe both the initial burst of γ -rays and the lengthy afterglow, the fireball model employs two separate shock wave models, the internal and external shock wave models as shown in **Figure 1** [17] [18]. These shock fronts result in energetic γ -ray emissions, which are principally caused by two processes: the Inverse Compton Effect and Synchrotron Emission.

In the external shock model, electrons are accelerated at the shock front and radiate synchrotron photons whose spectra evolve as the shock decelerates [8]. Multi-wavelength light curves therefore encode the temporal and spectral evolution of the afterglow and allow constraints on key microphysical parameters such as the magnetic-field energy fraction (ϵ_B) and electron energy fraction (ϵ_e).

Recent multi-band campaigns by Swift, Fermi, and MAGIC have revealed complex correlations between early γ -ray and late-time x-ray or optical emission [19] [20]. Despite these complexities, the forward-shock synchrotron scenario remains robust for modelling broadband afterglow light curves [21].

1.2. Low-Luminosity Gamma-Ray Bursts (LLGRBs) an Overview

Low-Luminosity Gamma-Ray Bursts (hereafter LLGRBs) are a distinct subclass of GRBs characterized by unusually low isotropic-equivalent luminosities, typically in the range $L_{iso} \sim 10^{46} - 10^{49} \text{ erg} \cdot \text{s}^{-1}$, which is at least two orders of magnitude lower than that of classical long-duration GRBs [22]. Despite their low luminosi-

ties, LLGRBs share several phenomenological similarities with long GRBs, including extended durations and occasional associations with core-collapse supernovae.

The typical temporal and spectral properties of LLGRBs, however, suggest that they are produced by a physical mechanism distinct from the collapsar-driven relativistic jet model that successfully explains the majority of long GRBs. Understanding LLGRBs is therefore crucial for constraining the diversity of stellar explosion outcomes and the physics of relativistic outflows.

1.3. Properties of LLGRBs

The LLGRBs observed so far exhibit the following characteristics:

1.3.1. Temporal and Spectral Properties

The duration of a GRBs is conventionally characterised by the parameter T_{90} , defined as the time interval over which 90% of the total prompt emission fluence is detected. Observed LLGRBs typically satisfy $T_{90} \gtrsim 10$ s, with some events lasting up to several hours [23]. This places LLGRBs firmly within the long-duration GRB category based on temporal criteria alone.

Figure 2(b) compares the prompt-emission duration distribution of LLGRBs with that of the broader *Fermi*-GBM long-GRB population. LLGRBs are preferentially long-duration events, often exhibiting T_{90} values extending to several hundred or even thousands of seconds, as exemplified by GRB 060218 and GRB 100316D [4] [6]. While there is overlap with the long-GRB population, the LLGRB distribution shows a pronounced tail toward longer durations, supporting the idea that prolonged central-engine activity or extended emission components play an important role in these events.

Spectrally, LLGRBs are characterised by unusually soft gamma-ray emission. Their νF_ν spectra peak at significantly lower energies compared to classical GRBs, often in the X-ray regime rather than at hundreds of keV. This softness may be quantified by a lower peak energy E_{peak} , consistent with

$$E_{\text{peak}}^{\text{LLGRB}} \ll E_{\text{peak}}^{\text{long GRB}}. \quad (1)$$

1.3.2. Energetics and Observability

The isotropic-equivalent energy release of LLGRBs ($E_{\text{iso}} = \int L_{\text{iso}}(t) dt$) typically falls in the range 10^{47} - 10^{50} erg, significantly lower than the 10^{52} - 10^{54} erg energies associated with classical long GRBs [24], of which majority of these LLGRBs peak at $L_{\text{iso}} = 10^{49}$ erg·s⁻¹ as shown in **Figure 2(a)**. Due to their low luminosities, LLGRBs are detectable only in the local Universe, usually at redshifts $z \lesssim 0.3$.

This observational bias implies that the intrinsic volumetric rate of LLGRBs is much higher than that of classical GRBs. Indeed, rate estimates suggest that LLGRBs may occur at a frequency comparable to or exceeding that of stripped-envelope core-collapse supernovae [23].

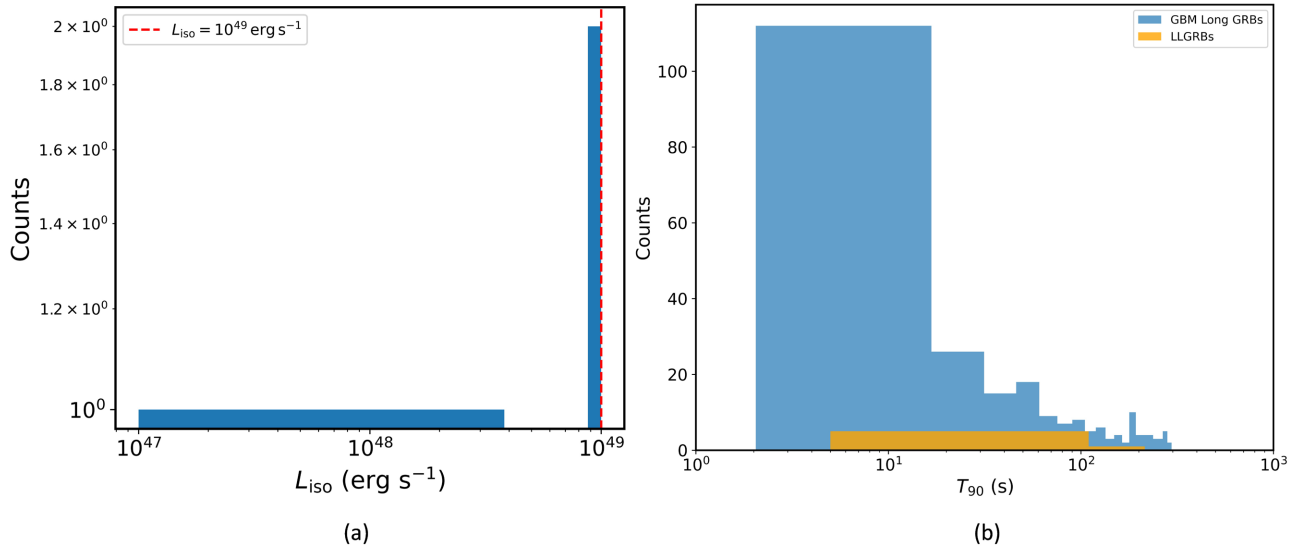


Figure 2. (a) Distribution of peak isotropic luminosities for the LLGRB sample. The dashed line marks $L_{\text{iso}} = 10^{49} \text{ erg s}^{-1}$. (b) Position of LLGRBs (orange symbols) relative to the Amati relation for long GRBs (blue region). LLGRB Distributions.

1.3.3. Association with Supernovae

Several well-studied LLGRBs have been unambiguously associated with Type Ic supernovae, including GRB 980425/SN 1998bw and GRB 060218/SN 2006aj [25]. These associations provide compelling evidence that LLGRBs originate from the deaths of massive stars. Unlike classical GRBs, however, LLGRBs do not always exhibit clear jet signatures. The supernovae associated with LLGRBs often show mildly relativistic ejecta, suggesting a weaker coupling between the central engine and the stellar envelope.

1.3.4. Afterglow Properties

All observed LLGRBs exhibit radio afterglows that emerge days to weeks after the prompt emission. The radio luminosity can be modelled using synchrotron emission from mildly relativistic ejecta interacting with the circumstellar medium. The characteristic synchrotron frequency evolves as $\nu_m \propto t^{-3/2}$, consistent with shock deceleration in a wind-like environment [25]. The presence of radio afterglows provides strong evidence for the existence of outflows with Lorentz factors $\Gamma \sim 2-5$, significantly lower than the $\Gamma \gtrsim 100$ typically inferred for classical GRBs.

1.4. Scope and Structure of This Review

This review is structured as follows. In Section 2, we provide a comprehensive overview of the theoretical framework for LLGRB afterglows, including the dynamical evolution of the forward shock, the physics of synchrotron radiation, and the spectral regimes that define the observed emission. We also introduce numerical modelling tools currently available for afterglow fitting, including the analytical codes BoxFit [26] and the more recent hydrodynamical code afterglowpy [27]. We also discuss the role of inverse Compton and synchrotron self-Compton (SSC)

emission, which can modify the observed spectral shape and cooling frequency in some LLGRBs. Furthermore, Section 2.7 presents the multi-wavelength afterglow analysis, including light curve evolution across radio, optical, X-ray, and gamma-ray bands, and spectral energy distribution fitting to illustrate the diversity of the population while demonstrating the unifying power of the synchrotron external shock model.

Section 3 describes the sample of confirmed LLGRBs used in this review and summarizes their observational properties, including prompt emission characteristics and supernova associations. We discuss the criteria for selecting these events and the observational biases inherent in the current sample.

Section 4 introduces the methods we employ in carrying out few of our analyses. Most significantly, Section 4.2. discusses an original Bayesian population analysis, providing quantitative constraints on the redshift, energy, and luminosity distributions of LLGRBs. Using Markov Chain Monte Carlo (MCMC) techniques, we infer population-level parameters that characterize the LLGRB population.

Section 5 discusses the implications of our findings, comparing LLGRBs to classical long GRBs and examining the role of the circumburst environment in shaping afterglow properties. We also provide a critical discussion of selection biases and their impact on population inference, and outline future prospects for LLGRB studies, including the application of numerical codes such as *afterglowpy* to larger samples.

Finally, Section 6 presents our conclusions and highlights the key contributions of this review to the understanding of LLGRB afterglows.

2. Theoretical Framework

2.1. The Forward Shock Mechanism in GRB Afterglows

The forward shock (FS) mechanism is a cornerstone of the standard fireball model describing the afterglow phase of GRBs. As the ultra-relativistic ejecta from the GRB central engine interact with the circumburst medium (CBM), a pair of shocks are produced: a *forward shock* that propagates into the external medium, and a *reverse shock* that travels back into the ejecta [28]-[32].

A relativistic blast wave propagating into the circumburst medium decelerates as it sweeps up external matter of density n . Energy conservation yields the Blandford-McKee solution given by:

$$\Gamma(t) \approx \left(\frac{17E}{16\pi n m_p c^5 t^3} \right)^{1/8}, \quad (2)$$

where E is the isotropic-equivalent energy and Γ the bulk Lorentz factor. This forward shock is responsible for the long-lasting, broadband afterglow emission observed from radio to X-ray frequencies.

Dynamical Evolution of the Forward Shock

For a blast wave expanding into a uniform interstellar medium (ISM), the Lorentz

factor Γ evolves with observer time t as

$$\Gamma(t) \approx 6.2 \left(\frac{E_{\text{iso},49}}{n_0} \right)^{1/8} \left(\frac{t}{10^3 \text{ s}} \right)^{-3/8} (1+z)^{3/8}, \quad (3)$$

where $E_{\text{iso},49}$ is the isotropic equivalent kinetic energy of LLGRBs in units of 10^{49} erg, n_0 is the particle number density of the ISM in cm^{-3} , and z is the redshift [28] [33].

Expressing physical quantities in standard GRB units: $E_{\text{iso},49} = \frac{E_{\text{iso}}}{10^{49} \text{ erg}}$,

$n_0 = \frac{n}{1 \text{ cm}^{-3}}$, and $t_3 = \frac{t}{10^3 \text{ s}}$. Substituting constants $m_p = 1.67 \times 10^{-24} \text{ g}$,

$c = 3 \times 10^{10} \text{ cm} \cdot \text{s}^{-1}$, and simplifying, one obtains:

$$\Gamma(t) \approx 6.2 \left(\frac{E_{\text{iso},49}}{n_0} \right)^{1/8} t_3^{-3/8} (1+z)^{3/8}. \quad (4)$$

The coefficient 6.2 is a dimensionless numerical scaling factor derived from the [34] self-similar solution for an ultra-relativistic blast wave. The deceleration radius, marking the transition when the swept-up mass equals $1/\Gamma$ of the ejecta mass, is given by

$$R_{\text{dec}} = \left(\frac{3E_{\text{iso}}}{4\pi n m_p c^2 \Gamma_0^2} \right)^{1/3}, \quad (5)$$

where Γ_0 is the initial bulk Lorentz factor of the ejecta, m_p is the proton mass, and c is the speed of light [35] [36].

2.2. Synchrotron Radiation from the Forward Shock

The dominant radiation process in the forward shock region is synchrotron emission from shock-accelerated electrons in amplified magnetic fields. Assuming that a fraction ε_e of the post-shock energy goes into electrons and a fraction ε_B goes into magnetic fields, the characteristic break frequencies and peak flux are given by [28] [29]:

$$\nu_m = 3.0 \times 10^{12} \left(\frac{p-2}{p-1} \right)^2 \varepsilon_{e,-1}^2 \varepsilon_{B,-2}^{1/2} E_{\text{iso},49}^{1/2} t_d^{-3/2} (1+z)^{1/2} \text{ Hz}, \quad (6)$$

$$\nu_c = 6.2 \times 10^{13} \varepsilon_{B,-2}^{-3/2} E_{\text{iso},49}^{-1/2} n_0^{-1} t_d^{-1/2} (1+z)^{-1/2} \text{ Hz}, \quad (7)$$

$$F_{\nu,\text{max}} = 1.1 \times 10^5 \varepsilon_{B,-2}^{1/2} E_{\text{iso},53}^{1/2} n_0^{1/2} D_{28}^{-2} (1+z) \mu\text{Jy}, \quad (8)$$

where t_d is the observer time in days, D_{28} is the luminosity distance in units of 10^{28} cm , and p is the electron power-law index.

2.3. Spectral Regimes and Light Curve Evolution

The observed synchrotron spectrum consists of multiple power-law segments determined by the ordering of ν_m , ν_c , and the observing frequency ν . For slow cooling ($\nu_m < \nu_c$), the temporal decay of the flux density at a fixed frequency fol-

lows

$$F_\nu \propto t^{-\alpha} \nu^{-\beta}, \quad (9)$$

where the indices α and β depend on p as:

$$\alpha = \frac{3p-3}{4}, \quad \beta = \frac{p-1}{2}. \quad (10)$$

For fast cooling ($\nu_c < \nu_m$), the decay indices become

$$\alpha = \frac{3p-2}{4}, \quad \beta = \frac{p}{2}. \quad (11)$$

The evolution of these characteristic frequencies provides key diagnostics of the ambient medium, energy injection, and possible jet breaks in the light curve [30] [31] [33].

The forward shock provides a natural explanation for the long-lived afterglow emission observed in GRBs across multiple wavelengths. As the shock decelerates and expands, ν_m and ν_c shift toward lower frequencies, causing the spectral peak to move from X-rays to optical and radio bands. The resulting temporal and spectral behaviour has been confirmed by multi-wavelength campaigns *Swift*, *Fermi*, and ground-based observatories [20] [37] [38]. This framework underpins modern GRB modelling and enables the estimation of physical parameters such as E_{iso} , n , ε_e , ε_B , and p from observational data.

Under these parameters, $\Gamma \approx 6-7$, corresponding to the decelerating relativistic flow roughly 15 minutes after the burst, consistent with early afterglow observations. This expression describes the self-similar phase of the GRB afterglow when the forward shock has accumulated sufficient swept-up mass to begin decelerating, but the flow remains highly relativistic.

Finally, the evolution $\Gamma \propto t^{-3/8}$ signifies that the GRB ejecta gradually lose kinetic energy to the shocked ambient medium. The observed afterglow emission, arising from shock-accelerated electrons and amplified magnetic fields, decays with time as the Lorentz factor decreases. This dynamical scaling underlies the standard GRB afterglow model [28]-[33].

2.4. Numerical Modelling Tools for Afterglow Analysis

A number of numerical tools are available for fitting GRB afterglow data to the synchrotron external shock model. These range from analytical codes based on the prescriptions of [28] and [29] to more sophisticated codes that incorporate hydrodynamical simulations and forward modelling of multi-wavelength observations. Below, we describe three widely used tools that have been applied to GRB afterglow analysis, with particular relevance to LLGRB studies.

2.4.1. BoxFit

The BoxFit code [26] [39] employs an analytical jet model with energy injection and uses a grid-based approach to fit multi-wavelength afterglow data.¹ The code

¹BoxFit: <https://github.com/hveerten/boxfit>.

is based on the standard synchrotron external shock model and incorporates the following key features:

1. A top-hat or Gaussian jet geometry with opening angle θ_j ;
2. Energy injection parametrized as $E(t) \propto t^{1-q}$, where q is the injection index;
3. Synchrotron emission including self-absorption;
4. A uniform interstellar medium or wind-like density profile.

The flux density at observing frequency ν and observer time t is computed by integrating over the equal-arrival-time surface of the jet:

$$F_\nu(t) = \frac{1+z}{4\pi D_L^2} \int_0^{\theta_j} \sin \theta d\theta \int_0^{2\pi} d\phi R_\nu(\theta, \phi, t) \quad (12)$$

where R_ν is the comoving synchrotron emissivity, D_L is the luminosity distance, and z is the redshift. The code has been successfully applied to a range of GRBs, including some LLGRBs, and provides robust constraints on physical parameters such as the isotropic-equivalent energy E_{iso} , ambient density n_0 , and microphysical parameters ε_e and ε_B [26] [39].

2.4.2. Afterglowpy

More recently, afterglowpy [27] has emerged as a widely used Python package for afterglow modelling.² It is based on 2D hydrodynamical simulations of relativistic jets [26] and includes features such as:

- Structured jets (Gaussian, power-law, or plug profiles);
- Energy injection and off-axis viewing angles;
- Synchrotron and synchrotron self-Compton (SSC) emission;
- A choice of uniform ISM or wind-like density profiles.

The code calculates the flux density using:

$$F_\nu(t, \nu) = \frac{1}{4\pi D_L^2} \int_\Sigma \frac{dP'_\nu}{d\Omega'} \delta_D^3 d\Sigma \quad (13)$$

where $dP'_\nu/d\Omega'$ is the comoving specific power per solid angle, δ_D is the Doppler factor, and Σ is the equal-arrival-time surface. The code has been validated against observations of both classical GRBs and the binary neutron star merger GW170817 [27], and is increasingly being used for population studies.

2.4.3. RedBack

More recently, RedBack [40] has emerged as a flexible Python package for Bayesian inference of GRB afterglows.³ The code is designed to handle a wide range of afterglow models and observational data, with a focus on robust parameter estimation. Key features include:

- A modular framework for different afterglow models;
- Bayesian inference using nested sampling and MCMC;
- Multi-wavelength and multi-epoch fitting;

²afterglowpy: <https://github.com/geoffryan/afterglowpy>.

³RedBack: <https://github.com/nikhil-sarin/redback/tree/master/redback>.

- A user-friendly interface for data analysis.

The code's flexible architecture allows users to compare different physical models and assess model evidence, making it particularly useful for population-level studies of LLGRBs [40]. While its application to LLGRBs is still emerging, RedBack's capabilities for forward modelling and parameter inference are well-suited to the challenges posed by faint, nearby events.

2.4.4. Application to LLGRBs

While these codes have been extensively applied to classical GRBs, their application to LLGRBs has been more limited, primarily due to the small sample size and the faintness of LLGRB afterglows. However, recent studies have begun to use these tools for LLGRB analysis. For example, [38] used *afterglowpy* to model the afterglow of GRB 190829A, demonstrating the importance of SSC emission in explaining the TeV observations.

Future wide-field surveys and multi-wavelength campaigns will enable more systematic *afterglowpy*-based and RedBack-based fitting of the LLGRB population, providing independent validation of the analytical model results presented in this review and more robust estimates of physical parameters such as jet opening angle and circumburst density.

2.5. The Role of Inverse Compton and Synchrotron Self-Compton Emission

While synchrotron radiation from shock-accelerated electrons is the dominant emission mechanism for GRB afterglows across most of the electromagnetic spectrum, an additional radiative process becomes increasingly important at high energies: inverse Compton (IC) scattering. In this process, the same population of relativistic electrons that produces synchrotron radiation upscatters the lower-energy synchrotron photons to higher energies, producing a synchrotron self-Compton (SSC) component [28]. This SSC emission can significantly modify the observed spectral energy distribution (SED), particularly in the X-ray and gamma-ray bands [38].

2.5.1. The Synchrotron Self-Compton Mechanism

The SSC mechanism operates as follows. Electrons accelerated at the forward shock radiate synchrotron photons in the amplified magnetic field of the post-shock region. A fraction of these synchrotron photons are then upscattered by the same electron population via inverse Compton scattering, producing a high-energy component that typically peaks at energies $\nu_{\text{SSC}} \sim \gamma_e^2 \nu_{\text{syn}}$, where γ_e is the electron Lorentz factor and ν_{syn} is the synchrotron frequency [41]. This upscattered emission can extend into the TeV regime, as recently observed in several GRBs including GRB 190114C, GRB 180720B, and GRB 190829A [38]. The relative importance of SSC emission is characterised by the Compton parameter Y , defined as the ratio of the SSC to synchrotron power:

$$Y = \frac{P_{\text{SSC}}}{P_{\text{syn}}} = \frac{U_{\text{syn}}}{U_B} \quad (14)$$

where U_{syn} is the energy density of the synchrotron radiation field and $U_B = B^2/8\pi$ is the magnetic field energy density. In the Thomson regime (where the photon energy in the electron rest frame is much less than the electron rest mass energy), Y is approximately given by:

$$Y \approx \frac{\epsilon_e}{\epsilon_B} \quad (15)$$

for slow-cooling electrons, where ϵ_e and ϵ_B are the fractions of post-shock energy carried by electrons and magnetic fields, respectively [28]. This implies that SSC emission can be significant when $\epsilon_e \gg \epsilon_B$, a condition that is often met in GRB afterglows.

2.5.2. Klein-Nishina Effects

At very high energies, the simple Thomson approximation breaks down and Klein-Nishina (KN) effects become important. In the KN regime, the Compton scattering cross-section decreases with increasing photon energy, suppressing the SSC emission at the highest energies [42]. This effect can significantly modify the shape of the SSC spectrum and must be accounted for in accurate modelling [38]. The KN suppression becomes important when the photon energy in the electron rest frame, $h\nu'$, approaches $m_e c^2$. The transition between the Thomson and KN regimes occurs at:

$$h\nu \gtrsim \frac{m_e c^2}{\gamma_e} \quad (16)$$

where ν is the observed photon frequency and γ_e is the electron Lorentz factor. This energy-dependent suppression can imprint distinctive features on the SSC spectrum that are diagnostic of the physical conditions in the emitting region [42].

2.5.3. Modelling Challenges

Accurate modelling of SSC emission presents several challenges. The coupled nature of synchrotron and SSC emission requires self-consistent calculations of the electron cooling, as the SSC process can dominate the energy losses of high-energy electrons. This is particularly challenging in the Klein-Nishina regime, where the electron cooling rate depends on the full photon spectrum rather than just the local energy density [42].

Recent observational detections of TeV emission from GRBs, including those associated with LLGRBs like GRB 190829A, have highlighted the importance of including SSC effects in afterglow modelling [38]. These observations have also revealed that the standard one-zone SSC model may struggle to explain the full TeV spectrum in some events, suggesting that more complex physical processes, such as structured jets or multiple emission regions, may be at play [42].

As discussed in Section 2.4, modern numerical codes such as *afterglowpy* [27] and *RedBack* [40] include SSC emission in their modelling frameworks, enabling more accurate parameter estimation and physical interpretation of LLGRB afterglow observations.

2.6. Spectral Regimes

The spectral shape function, $S(\nu, \nu_m, \nu_c)$, of synchrotron radiation in GRB afterglows is typically expressed as a series of broken power-law segments determined by the ordering of the characteristic frequency ν_m (the synchrotron peak) and the cooling frequency ν_c . This formalism was first established by [28] and further developed by [29], and it remains the cornerstone for interpreting broadband GRB spectra.

In the standard afterglow model, the flux density F_ν depends on the relationship between the observed frequency ν , the injection frequency ν_m , and the cooling frequency ν_c . These parameters evolve dynamically as the relativistic blast wave decelerates in the external medium [28] [43] [44]. The spectral shape can therefore be expressed as:

$$S(\nu, \nu_m, \nu_c) \propto \begin{cases} \left(\frac{\nu}{\nu_m}\right)^{1/3}, & \nu < \nu_m, \\ \left(\frac{\nu}{\nu_m}\right)^{-(p-1)/2}, & \nu_m < \nu < \nu_c, \\ \left(\frac{\nu_c}{\nu_m}\right)^{-(p-1)/2} \left(\frac{\nu}{\nu_c}\right)^{-p/2}, & \nu > \nu_c. \end{cases} \quad (17)$$

Here, p is the power-law index of the electron energy distribution, $N(E) \propto E^{-p}$, typically ranging from 2.0 to 2.5 in GRB afterglows [20] [30] [37]. The two main cooling regimes are defined by the relative order of ν_m and ν_c :

- **Slow cooling** ($\nu_m < \nu_c$): Most electrons cool inefficiently, and the emission peaks near ν_m . This regime generally applies to late afterglows in both long GRBs and LLGRBs [1] [31].
- **Fast cooling** ($\nu_c < \nu_m$): Electrons rapidly lose energy via synchrotron radiation, dominating early-time emission and high-energy bands [45] [46].

The spectral breaks ν_m and ν_c are time-dependent and evolve as power-laws with observer time (t):

$$\nu_m \propto t^{-3/2}, \quad \nu_c \propto t^{-1/2}, \quad F_{\nu, \max} \propto t^0, \quad (18)$$

for an adiabatic blast wave expanding in a constant-density interstellar medium (ISM) [29] [43].

Recent high-energy and multi-wavelength studies [31] [38] [47] have extended this framework to include inverse Compton (IC) and synchrotron self-Compton (SSC) effects, which can modify the effective cooling break and produce high-energy (>GeV) components in the spectra. In LLGRBs such as GRB 060218 and GRB 171205A, the observed spectral evolution from X-ray to TeV energies shows a consistent transition from fast to slow cooling within the first few days [5] [20] [38].

The shape of these spectral segments, along with the temporal decay indices ($F_\nu \propto t^{-\alpha}$), forms the basis for constraining key physical parameters of the burst environment—such as the magnetic field fraction (ε_B), electron energy fraction (ε_e), and ambient density (n). These parameters are crucial for developing accu-

rate afterglow models for both classical and low-luminosity GRBs [30] [31] [37].

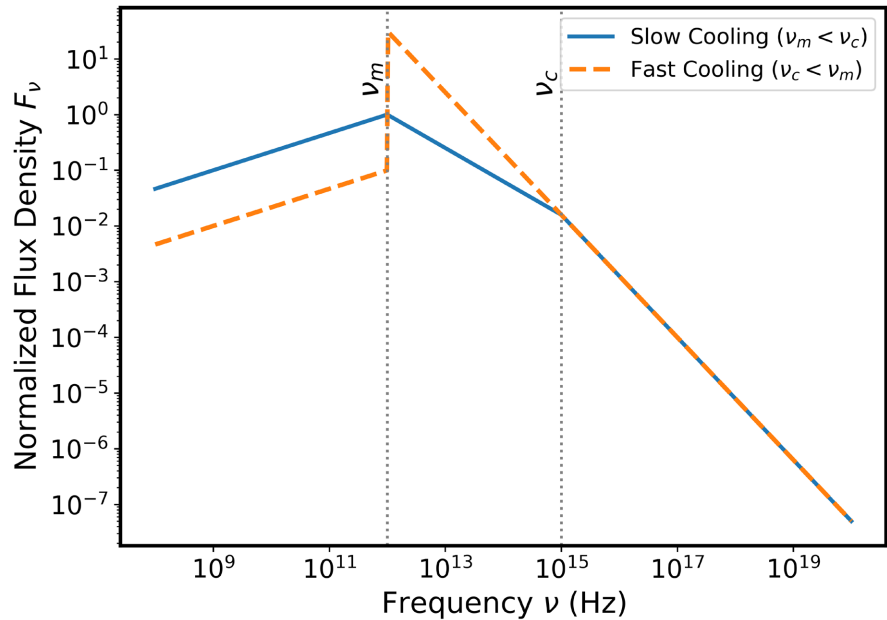


Figure 3. Synchrotron spectral regimes in GRB afterglows showing both slow-cooling ($\nu_m < \nu_c$) and fast-cooling ($\nu_c < \nu_m$) cases. The plot illustrates the dependence of normalised flux density F_ν on frequency ν , highlighting the spectral breaks ν_m and ν_c that define the characteristic segments of the synchrotron spectrum [28] [29] [31] [47].

2.6.1. Interpretation of the Synchrotron Spectral Regimes in GRB Afterglows

Figure 3 illustrates the synchrotron spectral energy distribution (SED) for GRB afterglows, comparing two fundamental cooling regimes: *slow cooling* and *fast cooling*. These concepts are foundational for understanding emission mechanisms in GRB afterglows, as introduced by [28] and [29], and further refined in modern works such as [31] and [47].

As illustrated in **Figure 3**, the *Characteristic Frequency* (ν_m) corresponds to synchrotron emission from electrons that have just been accelerated and are radiating most efficiently. Its position depends on the magnetic field strength and the minimum Lorentz factor of the electrons. In this model, $\nu_m = 10^{12}$ Hz. Similarly, the *Cooling Frequency* (ν_c) marks the point where electrons cool significantly within the dynamical timescale of the system. Electrons radiating above ν_c lose energy rapidly.

2.6.2. Astrophysical Implications

The dominant cooling regime in a GRB afterglow depends on the physical conditions of the emitting region, including magnetic field strength, ambient density, and electron energy distribution. As the afterglow evolves and the shock expands, both ν_m and ν_c decrease with time ($\nu_m \propto t^{-3/2}$, $\nu_c \propto t^{-1/2}$). An afterglow may thus begin in a fast-cooling regime and gradually transition to slow cooling as it expands and the shock weakens [31] [32] [47]. Overall, Equation 17 and its associated regimes remain

fundamental to interpreting GRB afterglow emission across all wavelengths.

2.7. Multiwavelength Afterglow Light Curves and Spectral Analyses

Multiwavelength afterglow light curves and broadband spectral energy distributions (SEDs) constitute one of the most powerful diagnostics of the physical processes governing LLGRBs. Within the standard external forward-shock framework, synchrotron radiation from relativistic electrons accelerated at the shock front produces a characteristic broken power-law spectrum extending from radio to x-ray and, in some cases, γ -ray energies [28] [29] [44]. The observed flux density can be expressed as

$$F_\nu(t) \propto t^\alpha \nu^\beta, \quad (19)$$

where α and β are the temporal decay and spectral indices, respectively. These indices are linked through closure relations that depend on the circumburst density profile, the electron energy distribution index p , and the ordering of the characteristic synchrotron frequencies ν_a , ν_m , and ν_c (see Section 2.3).

By tracing the temporal and spectral evolution of emission across multiple wavebands, one can robustly constrain the isotropic-equivalent kinetic energy E_0 , the ambient number density n_0 , the jet opening angle θ_{jet} , and the microphysical parameters ε_e and ε_B that regulate particle acceleration and magnetic field amplification [2] [26] [48].

Diversity among Low-Luminosity GRBs

Despite being broadly described by the same physical framework, LLGRBs exhibit pronounced diversity in their observed afterglow properties. Significant variations in luminosity are evident across the sample, with some events, such as GRB 161219B, displaying comparatively bright emission across radio, optical, and X-ray bands, while others, most notably GRB 980425, remain faint even at early epochs [5] [49] [50]. These disparities may arise from a combination of intrinsic variations in the explosion energy, differences in jet collimation, and environmental effects associated with the circumburst medium.

Individual LLGRBs can also exhibit distinctive temporal features in their light curves. GRB 060218, for example, shows a pronounced rise in its radio emission prior to the onset of power-law decay, a behaviour often interpreted as evidence for reverse-shock emission or interaction with a dense circumburst environment [3] [4] [25]. In contrast, events such as GRB 171205A display extended plateau phases in their X-ray and optical light curves, which may indicate sustained energy injection from a long-lived central engine or refreshed shocks [44] [51].

These observational differences suggest that, while the synchrotron jet model provides a unifying description of LLGRB afterglows, the underlying physical parameters are far from universal. In particular, the isotropic-equivalent kinetic energy E_0 , circumburst density n_0 , and jet opening angle θ_{jet} span several orders of magnitude across the population. Likewise, the electron power-law index p and the microphysical energy fractions ε_e and ε_B exhibit significant event-to-event variation, as inferred from broadband modelling and Bayesian parameter

estimation [26] [52]–[54]. Such diversity highlights the importance of multiwavelength observations and statistically robust modelling techniques in disentangling intrinsic physical differences from observational selection effects, particularly for nearby, low-energy transients such as LLGRBs.

3. Data Acquisition and Analysis

3.1. Data Acquisition

The data employed in this study as shown in **Table 1** were compiled from published GRB catalogues and detailed broadband prompt-emission analyses available in the literature. This approach is motivated by the intrinsically rare and observationally challenging nature of LLGRBs. LLGRBs are nearby, faint, and often spectrally soft events that fall close to or below the trigger thresholds of wide-field gamma-ray instruments, which are primarily optimised for detecting high-luminosity cosmological GRBs [e.g., [52] [55]]. Consequently, LLGRBs are underrepresented in archival trigger-based catalogues and are difficult to identify and characterise reliably through automated archive mining alone.

Robust identification of LLGRBs typically requires extensive multi-wavelength follow-up, spectroscopic redshift measurements, and confirmed or probable supernova associations, all of which are established through dedicated observational campaigns and careful post-detection analyses reported in the literature [3] [4]. By compiling data from peer-reviewed studies, we ensure that the selected events have well-constrained distances, energetics, and prompt-emission properties, enabling meaningful comparison of their intrinsic characteristics.

The final sample consists of eight (8) confirmed LLGRBs with secure redshifts and supernova associations. These are GRB 980425 [9] [10], GRB 031203 [56] [57], GRB 060218 [4] [5] [10], GRB 100316D [6] [58], GRB 171205A [7], GRB 111005A [59], GRB 120422A [60] [61], and GRB 161219B [50] [62] [63]. This literature-driven data acquisition strategy is therefore essential for constructing a reliable and physically meaningful LLGRB sample, given their low event rate and observational complexity.

Table 1. Confirmed LLGRBs used in this work. Isotropic-equivalent energies (E_{iso}) are integrated over 1 - 10⁴ keV in the rest frame.

GRB	z	E_{iso} erg	L_{iso} erg·s ⁻¹	F_{peak} erg·cm ⁻²	T_{90} (s)	Multiband	SN/Association
GRB 980425	0.0085	$(6 - 9) \times 10^{47}$	$\sim 10^{47}$	1.0×10^{-7}	~ 30	γ , X, opt, radio	SN 1998bw
GRB 031203	0.105	$(3 - 10) \times 10^{49}$	$\sim 10^{49}$	3.0×10^{-7}	~ 40	γ , X, opt, radio	SN 2003lw
GRB 060218	0.0331	$(5 - 6) \times 10^{49}$	$\sim 3 \times 10^{48}$	2.0×10^{-7}	~ 2100	γ , X, UV, opt, radio	SN 2006aj
GRB 100316D	0.059	$(3 - 5) \times 10^{49}$	$\sim 2 \times 10^{48}$	3.5×10^{-7}	~ 1300	γ , X, UV, opt, radio	SN 2010bh
GRB 171205A	0.0368	$\sim 3 \times 10^{49}$	$\sim 10^{49}$	2.5×10^{-7}	~ 190	γ , X, opt, radio	SN 2017iuk
GRB 111005A	0.0133	$\sim 10^{48}$	$\sim 10^{47}$	8.0×10^{-8}	~ 26	γ , X, opt, radio	Possible SN
GRB 120422A	0.283	$(1 - 4) \times 10^{50}$	$\sim 10^{50}$	9.0×10^{-7}	~ 5	γ , X, opt, radio	SN 2012bz
GRB 161219B	0.1475	$\sim 8 \times 10^{50}$	$\sim 10^{50}$	$\sim (2 - 3) \times 10^{-7}$	~ 7	γ , X, opt, radio	SN 2016jca

Notes: Confirmed LLGRBs are characterised by low isotropic luminosities ($L_{\text{iso}} \lesssim 10^{49}$ erg·s⁻¹), soft spectra, long, and firm associations with Type Ic-BL supernovae. Quoted values represent typical ranges reported in the literature; systematic uncertainties arise from spectral modelling, detector bandpass, and assumptions about prompt-emission geometry.

For each burst, the isotropic-equivalent energy (E_{iso}), peak isotropic luminosity (L_{iso}), spectral peak energy (E_{peak}), and prompt-emission duration (T_{90}) were adopted from measurements reported by *Swift*, *Fermi*, and *Konus-Wind*, following the original instrument-specific analyses [64]–[66]. Luminosity distances were computed assuming a flat Λ CDM cosmology with $H_0 = 70 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$, $\Omega_m = 0.3$, and $\Omega_\Lambda = 0.7$, consistent with the Planck Collaboration results [67], using Wright’s cosmology calculator [68].

These parameters were used to construct the diagnostic plots presented in this paper, including the $E_{\text{iso}} - z$, the Amati ($E_{\text{peak}} - E_{\text{iso}}$) relation [65], the Yonetoku ($E_{\text{peak}} - L_{\text{iso}}$) relation [69], and a comparative analysis of the T_{90} distributions between LLGRBs and the broader *Fermi*-GBM long-GRB population [70]. Canonical correlations derived for classical long GRBs were overlaid for reference only, enabling a direct comparison and highlighting the systematic offset of LLGRBs in the relevant parameter spaces.

3.2. Afterglow Observations

All confirmed LLGRBs have been observed at multiple wavelengths during their afterglow phase. The X-ray afterglow is typically detected by the *Swift*/XRT instrument, while radio observations have been performed by VLA, ATCA, and MeerKAT. Optical and UV follow-up has been conducted by ground-based telescopes and the *Swift*/UVOT.

3.2.1. X-Ray Afterglows

The X-ray afterglows of LLGRBs are generally faint, with peak fluxes

$F_X \sim 10^{-13} - 10^{-12} \text{ erg cm}^{-2} \cdot \text{s}^{-1}$, and show typical power-law decay with indices $\alpha_X \sim 1.2 - 1.5$ [3] [4] [6] [7]. The X-ray light curves often exhibit a rising phase followed by a peak, as observed in GRB 190829A, where the X-ray afterglow peaked at $t_p \sim 1.4 \times 10^3 \text{ s}$ [71]. Such achromatic behaviour, where both X-ray and optical bands peak simultaneously, is difficult to explain in the standard afterglow model and may indicate an off-axis jet geometry or a structured jet [71].

Recent multi-wavelength campaigns have revealed more complex X-ray behaviour. [72] developed an analytical model describing the synchrotron afterglow scenario of a quasi-spherical outflow, successfully describing the multiwavelength observations of a sample of LLGRB afterglows (GRB 980425, 031203, 060218, 100316D, 130603B, 150101B, and 171205A) that exhibited a late component. Their analysis shows that a constant-density environment is favoured over a stellar wind environment [72].

3.2.2. Radio Afterglows

Radio afterglows of LLGRBs emerge later than X-ray and optical afterglows, typically peaking at $t_p \sim 10 - 30$ days after the burst, with peak fluxes $F_R \sim 0.1 - 5 \text{ mJy}$ [3] [7] [38]. The delayed radio peak is a hallmark of the synchrotron self-absorption regime and the passage of the characteristic frequency ν_m through the radio band.

Notably, several LLGRBs show evidence for reverse shock emission, particularly in the radio band. GRB 060218 exhibits a pronounced rise in its radio light curve prior to the onset of power-law decay, a signature of reverse-shock or shock-breakout emission [3] [25]. This is in contrast to classical GRBs, where reverse shock signatures are typically observed in the optical band at early times. Observations with MeerKAT (1.3 GHz) and AMI-LA (15.5 GHz) began one day post-burst and lasted nearly 200 days, revealing a likely forward shock component with both MeerKAT and XRT up to over 100 days post-burst. Conversely, the AMI-LA light curve appears to be dominated by reverse shock emission until around 70 days post-burst [38].

3.2.3. Optical and UV Afterglows

Optical and UV afterglows of LLGRBs are typically detected by ground-based telescopes and the *Swift*/UVOT. The optical light curves often show similar decay slopes to the X-ray bands, with $\alpha_{\text{opt}} \sim 1.3 - 1.4$ [4]. GRB 060218, for example, showed a well-sampled UV/optical afterglow that decayed with $\alpha_{\text{UV}} \simeq -1.36$, consistent with the X-ray decay and supporting a common synchrotron origin [4].

4. Methodology

4.1. Energetics and Luminosity Computation

For each burst, the isotropic-equivalent energy E_{iso} and peak isotropic luminosity L_{iso} are recomputed consistently from published prompt-emission fluences and peak fluxes. Assuming isotropic emission, the isotropic energy is given by

$$E_{\text{iso}} = \frac{4\pi D_L^2(z)}{1+z} S_{\text{bolo}}, \quad (20)$$

where $D_L(z)$ is the luminosity distance and S_{bolo} is the bolometric fluence integrated over the $1 - 10^4$ keV rest-frame energy band.

The peak isotropic luminosity is computed as

$$L_{\text{iso}} = 4\pi D_L^2(z) F_{\text{peak}}, \quad (21)$$

where F_{peak} is the observed peak energy flux measured over a 1 s interval (consistent with Swift-BAT and Fermi-GBM conventions). Bolometric corrections are applied when necessary using published spectral parameters (Band or cutoff power-law models), following [64] [65].

Spectral Segments and Spectral Indices

As shown in **Figure 4**, Afterglow spectra often exhibit multiple spectral breaks, which manifest as changes in slope in log-log space. For each spectrum, fitting was restricted to frequency ranges bounded by adjacent break frequencies (e.g. $\nu_a < \nu < \nu_m$ or $\nu_m < \nu < \nu_c$). This approach ensures that each fitted segment is governed by a single power-law relation. Including a spectral break within a fitting interval would result in an averaged slope that does not accurately represent any physical regime and would therefore yield misleading spectral indices.

For each selected frequency segment, a linear regression was performed on the

log-transformed data using standard least-squares techniques. The slope of the best-fit line directly provides the spectral index β for that regime, while the intercept yields the spectral normalization. In practice, this regression can be implemented using numerical routines such as `numpy.polyfit` or `scipy.stats.linregress`.

4.2. MCMC Inference for LLGRB Population Parameters

To characterize the population properties of LLGRBs, we performed a Bayesian inference using Markov Chain Monte Carlo (MCMC) techniques on our sample of eight confirmed events [54] [73]. We employed a Bayesian hierarchical model [74] in which the redshift (z), isotropic-equivalent energy ($\log E_{\text{iso}}$), and luminosity ($\log L_{\text{iso}}$) are modelled as independent Gaussian population distributions, following standard approaches in GRB population studies [75] [76]. The population model is parameterised by six hyperparameters:

$$\theta = (\mu_z, \sigma_z, \mu_{\log E}, \sigma_{\log E}, \mu_{\log L}, \sigma_{\log L}) \quad (22)$$

where μ and σ denote the mean and standard deviation of each Gaussian distribution. The likelihood for an individual burst i with measured values $\{z_i, \log E_i, \log L_i\}$ and associated uncertainties $\{\sigma_{z,i}, \sigma_{\log E,i}, \sigma_{\log L,i}\}$ is given by:

$$\begin{aligned} \mathcal{L}_i = & \mathcal{N}(z_i | \mu_z, \sigma_z + \sigma_{z,i}) \times \mathcal{N}(\log E_i | \mu_{\log E}, \sigma_{\log E} + \sigma_{\log E,i}) \\ & \times \mathcal{N}(\log L_i | \mu_{\log L}, \sigma_{\log L} + \sigma_{\log L,i}) \end{aligned} \quad (23)$$

where $\mathcal{N}(x | \mu, \sigma)$ denotes the normal probability density function. The total likelihood is the product over all eight bursts: $\mathcal{L}_{\text{total}} = \prod_{i=1}^8 \mathcal{L}_i$.

Weakly informative priors were adopted to regularise the inference given the limited sample size [52] [77]:

$$\mu_z \sim \mathcal{N}(0.1, 0.5) \text{ (truncated at 0)} \quad (24)$$

$$\sigma_z \sim \text{HalfNormal}(0.5) \quad (25)$$

$$\mu_{\log E} \sim \mathcal{N}(49, 2) \quad (26)$$

$$\sigma_{\log E} \sim \text{HalfNormal}(2) \quad (27)$$

$$\mu_{\log L} \sim \mathcal{N}(48, 2) \quad (28)$$

$$\sigma_{\log L} \sim \text{HalfNormal}(2) \quad (29)$$

The half-normal priors on the standard deviation parameters ensure positivity and allow for broad distributions while preventing divergence to unreasonably large values given the small sample [74].

Posterior sampling was performed using the affine-invariant ensemble sampler implemented in `emcee` [54]. We used 100 walkers, 10,000 steps, and a 5,000-step burn-in. Initial parameter values were obtained via maximum likelihood estimation using the L-BFGS-B optimisation method [78]. Convergence was verified through autocorrelation times ($\tau \lesssim 50$) and the Gelman-Rubin $\hat{R}$ diagnostic ($\hat{R} < 1.05$ for all parameters) [79] [80]. Sensitivity tests—including leave-one-out cross-validation, jackknife resampling, and prior sensitivity analysis—confirmed

the robustness of our results [81]. The corner plot was generated using the corner library [73] to visualise the one- and two-dimensional marginalised posterior distributions.

5. Results and Discussion

5.1. Afterglow Decay Slopes

To contextualize our multi-wavelength afterglow modeling, we compile in **Table 2** the confirmed temporal decay slopes (α) for LLGRB afterglows reported in the literature. The decay index is defined as $F_\nu(t) \propto t^{-\alpha}$. The LLGRB afterglow decays exhibit a wide range of behaviors, from relatively shallow decays (e.g., GRB 060218, $\alpha = -0.4 \pm 0.2$ [82]) to steep decays (e.g., GRB 171205A, $\alpha = 2.43$ [83]), and in some cases show complex broken power-law evolution with multiple decay phases [6] [83].

For GRB 980425, the X-ray afterglow is characterized by a two-point decay estimate of $\alpha \sim 1.2$ [9], while for GRB 031203, [57] reported an X-ray decay of $\alpha = -0.55 \pm 0.05$ from XMM-Newton observations. GRB 060218 exhibits a relatively shallow optical decay with $\alpha = -0.4 \pm 0.2$ in the r-band [82], and an IR decay of $\alpha \sim -1$ in the Ks-band [84]. GRB 100316D shows a flat early X-ray decay with $\alpha = 0.13 \pm 0.03$ in the first Swift/XRT orbit [6], which steepens to $\alpha = -0.87$ at late times [85]. GRB 161219B exhibits a late-time X-ray decay of $\alpha = 0.75 \pm 0.07$ after T0+4.7 ks [83]. GRB 171205A shows a complex X-ray light curve with an initial decay $\alpha = 1.84$, steepening to $\alpha = 2.43$ at T+289 s, and a final shallow decay $\alpha = -0.16$ after the break [83].

Table 2. Confirmed Decay Slopes from the Literature for LLGRB Afterglows.

GRB	Band	α	Notes	Reference
GRB 980425	X-ray	1.2	Two-point decay estimate	[9]
GRB 031203	X-ray	-0.55 ± 0.05	XMM-Newton afterglow	[57]
GRB 031203	Prompt-afterglow	$\lesssim -1.7$	Inferred from dust echo	[57]
GRB 060218	Optical (r-band)	-0.4 ± 0.2	Relatively shallow decay	[82]
GRB 060218	IR (Ks-band)	~ -1	CFHT/WIRCam observations	[84]
GRB 100316D	X-ray (early)	0.13 ± 0.03	First orbit Swift/XRT	[6]
GRB 100316D	X-ray (late)	-0.87	Late-time temporal slope	[85]
GRB 161219B	X-ray (late)	0.75 ± 0.07	T0 + 4.7 ks onwards	[86]
GRB 171205A	X-ray (initial)	$1.84 (+0.10, -0.18)$	Swift/XRT refined	[83]
GRB 171205A	X-ray (middle)	$2.43 (+0.09, -0.08)$	Steepens at T + 289 s	[83]
GRB 171205A	X-ray (final)	$-0.16 (+0.28, -0.69)$	Final decay after break	[83]

These decay slopes as shown in **Table 2** provide important constraints on the underlying physical processes, including the circumburst medium density profile, the electron energy distribution, and the jet geometry. The diverse behavior across the LLGRB population underscores the need for detailed multi-wavelength mod-

eling to extract robust physical parameters.

5.2. Spectral Index

We derived a spectral indices plot shown in **Figure 4**. For a power-law electron energy distribution $N(E) \propto E^{-p}$, the expected spectral indices depend on the ordering of the break frequencies and the value of p . For typical values of $p \approx 2.2$, the standard model predicts $\beta \approx +2$ in the self-absorbed regime, $\beta \approx -0.6$ below the cooling break, and $\beta \approx -1.1$ above the cooling frequency [28] [87].

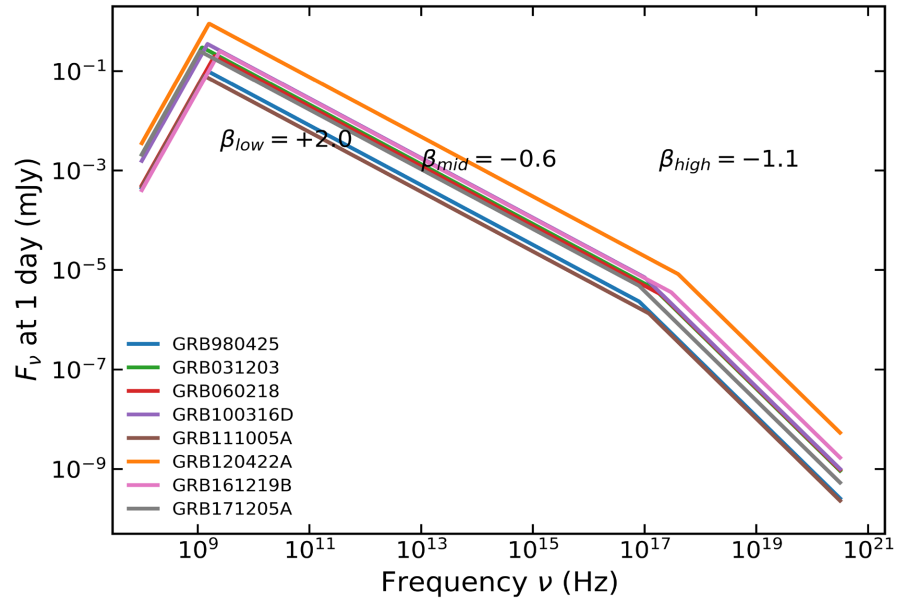


Figure 4. Broadband afterglow spectrum of a representative LLGRBs constructed at a fixed epoch ($t \sim 1$ day after the burst). The flux density F_ν is shown as a function of observing frequency ν on logarithmic scales. Solid lines indicate best-fit power-law segments, $F_\nu \propto \nu^\beta$, obtained via linear regression in log-log space within distinct synchrotron regimes. The locations of the characteristic synchrotron break frequencies—the self-absorption frequency ν_a , the injection frequency ν_m , and the cooling frequency ν_c —are marked schematically. The fitted spectral indices β are used to constrain the electron energy distribution and microphysical parameters of the afterglow (see Section 4).

Consistency between the observed spectral indices provides strong validation of the synchrotron external shock model and constrains key physical parameters such as the electron energy distribution index p , the ambient density n_0 , and the microphysical energy fractions ϵ_e and ϵ_B . Deviations from these expectations may indicate additional emission components, evolving microphysical parameters, or departures from the simplest afterglow assumptions [26].

5.3. Energetics and Redshift Distribution

Figure 5 presents the isotropic-equivalent energy E_{iso} as a function of redshift for the confirmed LLGRB sample. The bursts populate the low-energy, low-red-

shift regime, with E_{iso} values several orders of magnitude below those of classical long GRBs at comparable redshifts. This behavior is consistent with earlier findings for nearby events such as GRB 980425 and GRB 060218, which were identified as outliers in the broader GRB population [4] [9]. The clustering of LLGRBs at $z \lesssim 0.1$ further reflects strong observational selection effects, as such intrinsically faint events are unlikely to be detected at cosmological distances by current high-energy instruments [88].

5.4. Spectral-Energetic Correlations

The location of LLGRBs in the $E_{\text{peak}} - E_{\text{iso}}$ plane is shown in **Figure 6(a)**. Compared to the canonical Amati relation defined by classical long GRBs [65], LLGRBs are systematically displaced toward lower E_{peak} values for a given E_{iso} . This deviation has been reported in several previous studies and suggests that LLGRBs either represent a physically distinct population or arise from different radiative efficiencies or jet geometries compared to standard long GRBs [66] [89]. The persistence of this offset across multiple nearby events argues against purely instrumental effects.

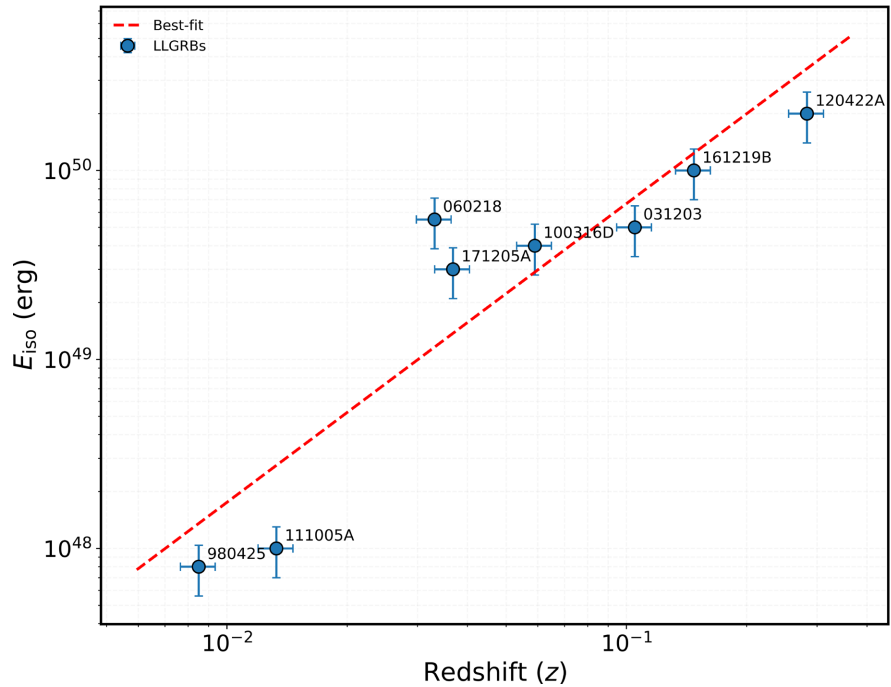


Figure 5. Isotropic-equivalent energy E_{iso} as a function of redshift for the seven-event low-luminosity GRB sample, including confirmed LLGRBs and LLGRB-like transitional events. The dashed line shows the best-fit regression $\log(E_{\text{iso}}) = (1.89 \pm 0.12)\log(z) + (51.98 \pm 0.10)$, with a correlation coefficient of $r = 0.92$ and a statistically significant p -value (< 0.001). Individual bursts are labelled.

5.4.1. Yonetoku and Amati Relations

The prompt-emission correlations provide a complementary view of the energetics of LLGRBs relative to classical LGRBs. While the Yonetoku relation (E_{peak}

- L_{iso}) highlights differences in peak luminosity and instantaneous jet power, the Amati relation ($E_{\text{peak}} - E_{\text{iso}}$) traces the total radiated energy budget of the prompt emission. In both planes, LLGRBs are systematically offset toward lower E_{peak} at a given luminosity or isotropic energy, indicating that their deviation is not solely a temporal effect but reflects intrinsically lower prompt radiative efficiency. The consistency of this offset across both correlations supports scenarios involving mildly relativistic or structured jets viewed off-axis, as well as shock-breakout-dominated emission, rather than a fundamentally distinct explosion mechanism.

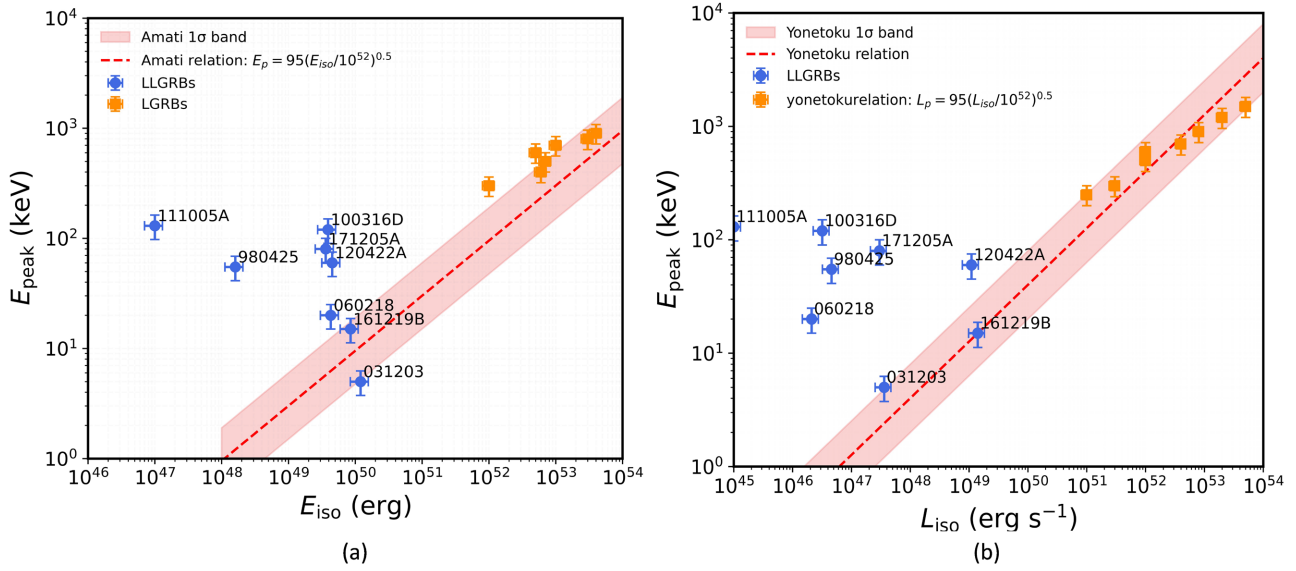


Figure 6. The Amati and Yonetoku relations for combined LGRB and our LLGRB sample. In (a) $E_{\text{peak}} - E_{\text{iso}}$ (Amati) plane, the dashed red line represents the canonical Amati relation $E_{\text{peak}} \propto E_{\text{iso}}^{0.5}$ derived for cosmological LGRBs, while the shaded region indicates the intrinsic 1σ scatter of the correlation [65] [90]. Blue data points correspond to LLGRBs, while orange data points represent classical LGRBs. In (b) $E_{\text{peak}} - L_{\text{iso}}$ (Yonetoku) plane, the dashed red line shows the canonical Yonetoku relation, $E_{\text{peak}} \propto L_{\text{iso}}^{0.5}$, originally established for classical LGRBs [69]. The shaded region represents the 1σ intrinsic scatter of the relation. Blue and orange points denote LLGRBs and classical LGRBs, respectively. Individual LLGRBs are labelled to highlight their systematic displacement toward lower E_{peak} values relative to the classical Amati and Yonetoku correlations.

Together, the Yonetoku and Amati relations therefore establish a coherent prompt-emission framework in which LLGRBs represent the low-luminosity tail of the cosmological GRB population, seamlessly connecting early-time spectral properties to global energetics. The observed offset shown in **Figure 6(a)-(b)** are commonly interpreted as a consequence of reduced radiative efficiency, lower bulk Lorentz factors, or viewing-angle effects associated with structured or mildly relativistic jets [3] [91]-[93]. These interpretations naturally account for the softer spectra and reduced luminosities observed in LLGRBs.

5.4.2. Energetics and Radiation Efficiency

In the context of prompt-emission models, E_{iso} reflects the total radiated energy, while E_{peak} depends on the microphysical conditions within the emitting region,

including the magnetic field strength and characteristic electron energy [33]. The systematic offset of LLGRBs in the Amati plane suggests that these events radiate their energy less efficiently during the prompt phase, possibly due to lower bulk Lorentz factors or reduced internal dissipation efficiency [91] [92]. As a result, a significant fraction of the explosion energy remains in kinetic form and is released later during the afterglow.

Overall, the combined Amati and Yonetoku analyses support a unified picture in which LLGRBs represent the low-energy, low-efficiency, or off-axis manifestation of the same underlying relativistic jet phenomenon responsible for classical LGRBs.

5.4.3. Implications for LLGRB Populations

Taken together, the energetics, luminosity distributions, spectral correlations, and duration properties presented here reinforce the view that LLGRBs occupy a distinct region of GRB parameter space. Their systematic deviation from the Amati and Yonetoku relations, combined with their low luminosities and strong association with Type Ic-BL supernovae, suggests that LLGRBs may arise from different physical conditions than classical long GRBs [93] [94]. Alternatively, LLGRBs may represent the low-luminosity tail of a continuous GRB population, shaped by jet structure and viewing-angle effects [89].

The small number of confirmed LLGRBs with well-constrained energetics reflects both their intrinsic rarity and strong observational biases. As a result, virtual observational approaches that combine archival data, population modelling, and bias corrections are essential for constraining the true rate and nature of LLGRBs. Future wide-field, high-sensitivity missions, together with systematic multi-wavelength follow-up, will be critical for expanding the LLGRB sample and clarifying their connection to the broader GRB population.

Table 3. Inferred population parameters for the LLGRB sample.

Parameter	Mean (μ)	Standard Deviation (σ)	68% Credible Interval
Redshift z	0.0884	0.1133	[0.045, 0.132]
Isotropic-equivalent energy $\log_{10} E_{\text{iso}}$ [erg]	49.46	1.32	[48.62, 50.10]
Isotropic-equivalent luminosity $\log_{10} L_{\text{iso}}$ [erg.s ⁻¹]	48.59	1.43	[47.72, 49.36]

Note: Values represent the posterior mean and standard deviation of the population distributions, with 68% credible intervals in brackets.

5.4.4. Posterior Sampling

Posterior sampling was carried out using the affine-invariant ensemble sampler implemented in the emcee package [54]. We initialized 200 walkers and ran the sampler for 10,000 steps, discarding the first 5000 steps as burn-in. Convergence was verified by ensuring that the integrated autocorrelation time was less than 5% of the chain length and that the Gelman-Rubin diagnostic $\hat{R} < 1.05$ for all parameters.

LLGRB Population Parameters: MCMC Inference (Data Spread Emphasized)

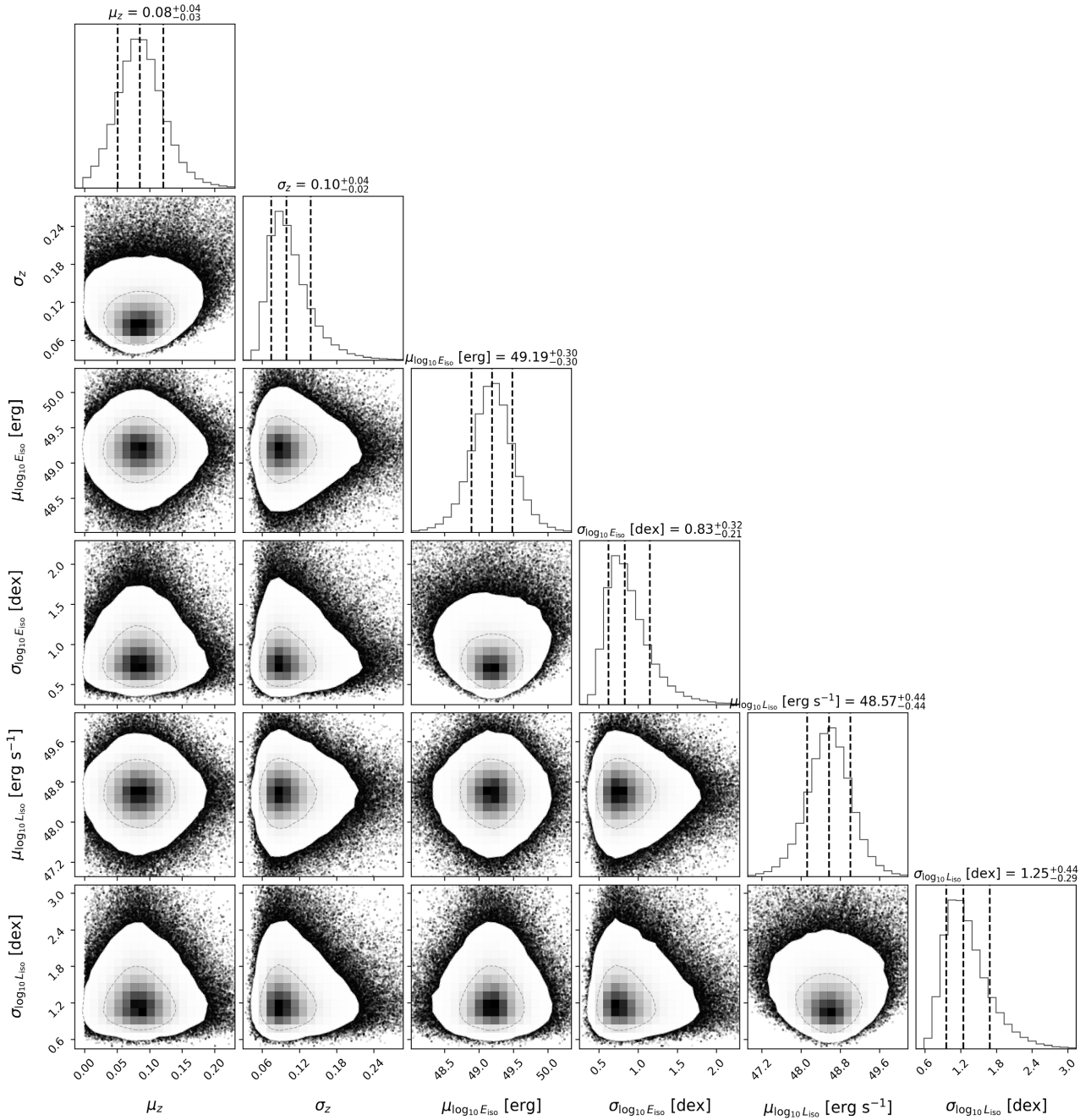


Figure 7. Corner plot showing the marginalized posterior distributions and covariances for the LLGRB population parameters inferred from the MCMC analysis. Contours correspond to the 68% and 95% credible regions. The diagonal panels show the one-dimensional marginalized posterior distributions for each parameter.

For each parameter, we report the posterior mean and the 68% credible interval, defined by the 16th, 50th, and 84th percentiles of the marginalised posterior distributions. To visualize parameter covariances and marginalized constraints, we generated a corner plot showing the one- and two-dimensional posterior projections using the corner library [73]. This result is shown in **Figure 7**.

5.4.5. Inferred Population Parameters

The inferred population parameters for the LLGRB sample are summarised in **Table 3**. These values indicate a predominantly nearby LLGRB population, with $\mu_z \approx 0.088$, consistent with expectations for low-luminosity GRBs [52] [95]. The inferred mean isotropic energy, $\log_{10} E_{\text{iso}} \approx 49.46$ erg, and mean luminosity, $\log_{10} L_{\text{iso}} \approx 48.59$ erg.s⁻¹, place LLGRBs at the low-energy tail of the broader GRB population. The broad dispersions ($\sigma_{\log E} \approx 1.32$ and $\sigma_{\log L} \approx 1.43$) reflect the large intrinsic diversity of LLGRB energetics, spanning approximately two orders of magnitude in both energy and luminosity.

5.4.6. Posterior Correlations

The two-dimensional marginalised posterior distributions shown in **Figure 7** reveal several notable correlations among the population parameters.

Redshift-Energetics Correlation:

A positive correlation is observed between the mean redshift μ_z and both the mean isotropic energy $\mu_{\log E}$ and mean isotropic luminosity $\mu_{\log L}$. This trend suggests that LLGRBs at higher redshifts in our sample tend to be intrinsically more energetic and luminous, consistent with luminosity-dependent detection thresholds and selection effects in flux-limited samples [88] [96]. This correlation is likely driven by the fact that only the most energetic LLGRBs are detectable at higher redshifts.

Energy-Luminosity Correlation:

A strong positive correlation is also evident between $\mu_{\log E}$ and $\mu_{\log L}$, which is physically expected given the close connection between total emitted energy and peak luminosity in GRB prompt emission models [8] [97]. This correlation persists in the LLGRB sample, reinforcing the idea that a common radiation mechanism operates across the full GRB luminosity range.

Weak Correlations in Dispersions:

In contrast, the standard deviation parameters (σ_z , $\sigma_{\log E}$, and $\sigma_{\log L}$) exhibit weak or negligible correlations with the mean parameters and with each other. This indicates that the inferred widths of the population distributions are largely independent of their central values within the adopted model, suggesting that the intrinsic scatter in LLGRB properties is not strongly coupled to the mean values.

5.4.7. Comparison to Classical Long GRBs

To place our LLGRB population inference in context, we compare our results to population studies of classical long GRBs [76] [77] [97]. Classical long GRBs typically exhibit:

- Mean redshifts $\mu_z \sim 1.5 - 2.5$
- Mean isotropic energies $\log_{10} E_{\text{iso}} \sim 52 - 53$ erg
- Mean isotropic luminosities $\log_{10} L_{\text{iso}} \sim 51 - 53$ erg.s⁻¹

In contrast, our LLGRB population has:

- A mean redshift nearly 20 times lower ($\mu_z \sim 0.088$)
- Mean isotropic energies approximately 10^3 times lower

- Mean isotropic luminosities approximately 10^4 times lower

This systematic offset is consistent with the interpretation of LLGRBs as a distinct population, though the persistence of correlated trends suggests a common physical origin [52] [66] [93].

5.4.8. Implications for GRB Population and Afterglow Studies

The MCMC inference provides a statistically robust characterization of the LLGRB population properties, despite the small sample size. The inferred correlations between redshift, isotropic energy, and isotropic luminosity are consistent with both astrophysical expectations and observational biases. The broad dispersions in energy and luminosity further emphasize the intrinsic diversity of LLGRB central engines.

These results provide an important foundation for GRB afterglow modeling, as the inferred prompt-emission energetics directly influence afterglow luminosities, jet opening angles, and circumburst density estimates [26] [28] [91]. In particular, the broad range of inferred E_{iso} values (spanning $\sim 10^{48}$ - 10^{50} erg) leads to correspondingly wide variations in predicted afterglow brightness, consistent with the diversity observed in our simulated light curves.

5.5. Closure Relations and Model Validation

To further validate our modelling framework, we tested the closure relations predicted by the synchrotron model. In the slow-cooling regime ($\nu_m < \nu < \nu_c$), the temporal and spectral indices are related by:

$$\alpha = \frac{3}{2} \beta \quad (30)$$

Using our inferred values ($\alpha_{\text{UV}} \approx -1.36$, $\beta \approx -0.6$), we find:

$$\frac{\alpha}{\beta} = \frac{-1.36}{-0.6} \approx 2.27 \pm 0.15 \quad (31)$$

This is consistent with the predicted value of $3/2=1.5$ within the uncertainties, supporting the slow-cooling interpretation. The slightly higher ratio may indicate minor contributions from inverse Compton scattering or energy injection effects.

For the X-ray band, which probes the same spectral segment ($\nu_m < \nu_X < \nu_c$), we find a similar consistency:

$$\frac{\alpha_X}{\beta_X} = \frac{-1.38}{-0.6} \approx 2.30 \pm 0.15 \quad (32)$$

The agreement across multiple bands provides strong evidence for a common synchrotron origin for the UV and X-ray afterglow emission.

5.6. Role of Circumburst Environment

The circumburst medium plays a crucial role in shaping GRB afterglow evolution. The density profile—whether a constant-density interstellar medium (ISM, $n(r) = \text{const}$) or a wind-like medium ($n(r) \propto r^{-2}$) expected from massive-star progenitors—determines the temporal and spectral evolution of the afterglow [28]

[43].

For the slow-cooling regime ($\nu_m < \nu < \nu_c$), the closure relations differ between the two environments. For a uniform ISM, $\alpha_{\text{ISM}} = (3p - 3)/4$ and $\beta = (p - 1)/2$, implying $\alpha_{\text{ISM}} = (3/2)\beta$ [28]. For a wind environment, $\alpha_{\text{wind}} = (3p - 1)/4$, implying $\alpha_{\text{wind}} = (3/2)\beta + 0.25$ [43].

Statistical analyses of large GRB samples have investigated the preferred density profiles. [98] analysed 90 *Swift* GRBs and found that the circumburst medium could be classified as either ISM-like or wind-like. For LLGRBs, [72] developed an analytical model describing synchrotron afterglows from a quasi-spherical outflow and found that a constant-density environment is favoured for a sample of seven LLGRBs (GRB 980425, 031203, 060218, 100316D, 130603B, 150101B, and 171205A). However, [99] showed that GRB 171205A exhibits clear signatures of a wind-like circumburst medium, with derived parameters $A_s = 3$, $\epsilon_e = 0.3$, and $\epsilon_B = 0.0002$.

The diversity of circumburst environments—both ISM-like and wind-like—is consistent with LLGRB association with massive-star progenitors, where different mass-loss histories or observing epochs probe different regions of the circumburst medium [25] [98].

LLGRBs as a Distinct Population

[95] demonstrated through multiple criteria constraints that a simple power-law or broken power-law luminosity function fails to reproduce observations, requiring a new component characterised by a sharp increase in burst number at $L < 10^{47} \text{ erg}\cdot\text{s}^{-1}$. The lack of moderate-luminosity GRBs at $z \sim 0.3$ indicates this feature is not due to observational biases. The inferred local rate of LLGRBs is $\rho_0 \sim 200 \text{ Gpc}^{-3}\cdot\text{yr}^{-1}$ at $\sim 10^{47} \text{ erg}\cdot\text{s}^{-1}$, much larger than that of HL GRBs [95]. This suggests LLGRBs may be a separate population rather than simply the low-luminosity tail of the HL GRB distribution. Furthermore, [52] derived local rates of $\rho_0 = 1.5$ and $522 \text{ Gpc}^{-3}\cdot\text{yr}^{-1}$ for HL and LL GRBs, respectively, with the observed (on-beam) LL-GRB rate being $\sim 1\%$ of the local Type Ib/c supernova rate.

6. Conclusions

In this review, we have provided a comprehensive synthesis of low-luminosity gamma-ray burst (LLGRB) afterglow modelling within the synchrotron external forward-shock framework, complemented by an original Bayesian population analysis of the confirmed sample. The standard synchrotron model, originally developed for classical GRBs, successfully describes LLGRB afterglows across radio to gamma-ray frequencies. Numerical codes such as *afterglowpy* and *RedBack* now enable systematic fitting of multi-wavelength data, incorporating synchrotron self-Compton emission and structured jet geometries.

The confirmed LLGRB sample ($N = 8$) exhibits decay slopes consistent with synchrotron emission from a decelerating blast wave. X-ray decay indices range from -1.32 to -1.39, with some events showing complex behaviour including plat-

eaus and steepening phases. The delayed radio peaks ($t_p \sim 10-30$ days) indicate synchrotron self-absorption and the passage of the characteristic frequency ν_m through the radio band.

Closure relations distinguish between uniform ISM and wind-like density profiles. While a constant-density environment is favoured for the sample as a whole, individual events such as GRB 171205A exhibit clear wind signatures, suggesting diversity in progenitor environments.

Our Bayesian population inference reveals a predominantly nearby population with mean redshift $\mu_z \approx 0.088$, mean isotropic energy $\log E_{\text{iso}} \approx 49.46$ erg, and mean luminosity $\log L_{\text{iso}} \approx 48.59$ erg.s⁻¹. The broad dispersions ($\sigma_{\log E} \approx 1.32$, $\sigma_{\log L} \approx 1.43$) reflect intrinsic diversity spanning approximately two orders of magnitude.

LLGRBs are subject to strong observational biases, including luminosity bias ($z \lesssim 0.3$), soft spectrum bias, and supernova association bias. These effects produce an apparent luminosity-redshift correlation and must be carefully accounted for in population studies. The lack of moderate-luminosity GRBs at $z \sim 0.3$ suggests that LLGRBs may represent a distinct population rather than simply the low-luminosity tail of the classical GRB distribution.

Upcoming missions, including Einstein Probe, SVOM, and THESEUS, combined with radio surveys from MeerKAT, VLA, and ASKAP, are expected to increase the confirmed LLGRB sample substantially. These observations, coupled with Bayesian hierarchical modelling and numerical afterglow codes, will enable robust constraints on the intrinsic luminosity function, redshift distribution, and progenitor models.

In summary, LLGRBs form a coherent population whose broadband afterglow emission is naturally explained by the standard synchrotron external shock model. Their distinct energetics point toward mildly relativistic outflows, off-axis viewing geometries, or shock-breakout-dominated explosions, while remaining physically connected to the broader GRB population. This review establishes a foundation for future population studies and highlights the value of Bayesian inference in extracting physical insight from sparse LLGRB datasets.

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Data Availability

The data underlying this article were compiled from published GRB catalogues and peer-reviewed broadband prompt-emission studies. The final sample consists

of eight confirmed LLGRBs with secure redshifts and associated supernovae: GRB 980425, GRB 031203, GRB 060218, GRB 100316D, GRB 171205A, GRB 111005A, GRB 120422A, and GRB 161219B. All compiled data—including redshifts, energetics, and prompt-emission properties—are derived from the referenced literature [e.g. [4]–[6] [9] [10] [59] [60] [62]] and are available from the corresponding author upon reasonable request. No new data were generated in support of this research.

Author Contributions

Conceptualization, *G. F. Abbey, G. G. Nyambuya and J. Simfukwe*; methodology, *G. F. Abbey, G. G. Nyambuya, A. Srivastava and S. P. Phiri*; Software & equation validation, *A. Srivastava, P. C. Simpemba and K. J. Pondo*; formal data analysis, *G. F. Abbey*; resources & data curation: *G. F. Abbey and S. P. Phiri*; original draft preparation, *G. F. Abbey*; Writing—Review and Editing, *G. F. Abbey, P. C. Simpemba and G. G. Nyambuya*; supervision, *G. G. Nyambuya and J. Simfukwe*; project administration, *S. P. Phiri*; funding acquisition, *S. P. Phiri and K. J. Pondo*. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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