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A Derivation of the Empirical Anderson Formula for Earth Flyby Anomalies from the ASTG-Model

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Abstract

Following a detailed examination of the flyby anomaly phenomenon, we provide a clear derivation of Anderson *et al.*'s empirical formula based on the Azimuthally Symmetric Theory of Gravitation (ASTG-model). Our review concentrates on the foundational origins of Anderson *et al.*'s empirical formula. The ASTG-model, from which we will derive this formula, serves as a logical and natural extension of Sir Isaac Newton's central gravitational potential theory: $\Phi = \Phi(r)$. It proposes a gravitational potential that depends not only on the radial distance (r) but also on the azimuthal angle (θ), *i.e.*: $\Phi = \Phi(r, \theta)$. In the ASTG-model, it is suggested that the observed θ-dependence in gravitational potential arises from the spin angular momentum of the gravitating body in question. A significant and beneficial result of our derivation of Anderson *et al.*'s empirical formula is that—*to first order approximation*—it helps to partially address the '*nagging*' and '*bothersome*' free parameter problem that the ASTG-model encounters.

Keywords

ASTG-Model, Anderson Empirical Formula, Solar Gravitational Anomalies

1. Introduction

This work is the first instalment in a series subtitled '*Solar Gravitational Anomalies*', which aims to investigate the proposed Azimuthally Symmetric Theory of Gravitation (referred to as the ASTG-model [1]). As initially suggested in Ref. [1], the ASTG-model faces a significant challenge due to having an infinite number of free parameters, making it easily adjustable to fit any dataset. In this series, we will use data from Flyby Anomalies (FBAs) as well as from the unusual perihelion precession of Solar planetary orbits in an effort to address this issue, thereby positioning the ASTG-model as a theory subject to falsification. Our first objective will be to derive—*from the ASTG-model*—Anderson *et al.* [2]'s empirical formula for FBAs.

The first FBA occurred in 1990 when the United States of America's National Aeronautics and Space Administration (*NASA*) spacecraft, Galileo, conducted its initial gravitational sling-shot (gravity assist manoeuvre, or swing-by) past Earth in a hyperbolic orbit on December 8 at 20:34:34 UT. During this event, it not only experienced its first FBA [2]-[4] but also marked the first instance of such a puzzling phenomenon witnessed by humanity. This FBA was the beginning of a series of anomalies encountered by spacecraft performing Earth gravity assist manoeuvres. An Earth FBA involves not only an unexpected increase in the outgoing osculating hyperbolic excess speed but also an asymptotic speed increase at the perigee during these flybys. In general, a FBA refers to an unanticipated increase in the outgoing osculating hyperbolic excess speed and an asymptotic speed increase at the perigee during a spacecraft's flyby past a planet for the purpose of a gravity assist manoeuvre [2]-[4].

As already mentioned earlier, flyby anomalies have been observed in spacecraft such as *Galileo I*, *NEAR*, *Cassini*, *Rosetta I*, and *MESSENGER* that are sent to explore the mysteries of deep space. These spacecraft fly past Earth to utilize its gravitational field for manoeuvring and altering their trajectories. Besides redirecting their paths, this gravity assist manoeuvre is also a cost-saving strategy[1] for the spacecraft's payload. The FBA manifests as a change in both ranging and Doppler data. For these spacecraft, along their hyperbolic trajectory, they approach Earth with an incoming speed v_{∞}^{in} and exit with a speed of v_{∞}^{out}; according to spherically symmetric Newtonian and Einsteinian gravitation, it is expected that: $\Delta v_{\infty} = v_{\infty}^{\text{out}} - v_{\infty}^{\text{in}} \neq 0$.

However, reliable observations and analyses yield a completely different and surprising outcome that has puzzled scientists from the European Space Agency (*ESA*) and *NASA* for quite some time, as well as physicists in general. Contrary to all expectations, observations [see **Table 1**] show that: $\Delta v_{\infty} \neq 0$. This indicates that the incoming kinetic energy of the spacecraft: $K_{\infty}^{\text{in}} = \frac{1}{2} m_{\text{in}} \left(v_{\infty}^{\text{in}} \right)^2$, does not equal the outgoing kinetic energy: $K_{\infty}^{\text{out}} = \frac{1}{2} m_{\text{out}} \left(v_{\infty}^{\text{out}} \right)^2$, as would be anticipated under Newtonian and Einsteinian gravitational paradigms. There is a '*spooky and spurious*' energy transfer that occurs between the spacecraft and Earth's gravitational field, where m_{in} and m_{out} represent the spacecraft's mass at the incoming and outgoing asymptotes, respectively.

As clearly illustrated in figure 2 in Ref. [2], this '*spooky and spurious*' energy transfer between the spacecraft and Earth's gravitational field first occurs at the

[1]A gravity assist manoeuvre can change the speed of a spacecraft without using its propellant; when applicable, especially combined with aerobraking, it can save significant amounts of fuel.

perigee—specifically, when the spacecraft reaches its closest approach to the planet. It has been observed that these spacecraft experience a previously unknown, mysterious, and unexplained asymptotic speed change. All of this information has been derived from the telemetry received from the spacecraft. When the shifts in the Doppler and ranging data are analysed, flyby anomalies manifest as a very small yet significant unaccounted speed increase of up to 13.46 mm·s^{-1} at perigee.

Table 1. Earth Flyby Parameters: Columns (1) and (2) gives the name of the spacecraft and the date it made its gravity assist manoeuvre. Columns (3), (4), (5), (6), (7) & (8) give the inclination, I_e, of the spacecraft's orbit relative to the Earth's equator; the incoming speed at infinity, v_∞^{in}; the radial distance at the perigee point, $\mathcal{R}_{\text{prg}}$; the osculating hyperbolic excess velocity at infinity, Δv_∞, and at the perigee, Δv_{prg}, and finally the velocity at the perigee point, v_{prg}. The values for eccentricity are adapted from [5].

Spacecraft	Date	I_e (1˚)	ϵ	v_∞^{in} (km·s^{-1})	$\mathcal{R}_{\text{prg}}$ (km)	Δv_∞ (km)	Δv_{prg} (mm·s^{-1})	v_{prg} (km·s^{-1})	δ_{in} (1˚)	δ_{out} (1˚)
[a]Galileo I	08/12/1990	142.90	2.4729	8.949	7356	+3.9200 ± 0.0800	+2.560 ± 0.050	13.738	−12.52	−34.15
[a]Galileo II	08/12/1990	138.90	2.3194	14.080	6703	−4.600 ± 1.0000	−9.200 ± 0.600	8.877	+34.26	−4.87
[a]NEAR	23/01/1998	108.80	1.8135	6.851	6939	+13.4600 ± 0.1300	+7.210 ± 0.070	12.739	−20.76	−71.96
[a]Cassini	18/08/1999	25.40	5.8525	1.601	7571	−2.0000 ± 1.0000	−1.700 ± 0.900	19.030	−12.92	−4.99
[b]Rosetta I	04/03/2005	144.90	1.3118	3.863	8354	+1.8000 ± 0.0500	+0.670 ± 0.020	10.517	−2.81	−34.29
[b]M'NGER	02/08/2005	133.10	1.3600	4.056	8736	+0.0200 ± 0.0100	+0.008 ± 0.004	10.389	+31.44	−31.92

Notes: [a]Operated by *NASA*. [b]Operated by *ESA*. [c]According to [9], the actual measured value of $\Delta v_\infty^{\text{in}}$ for Galileo-II is ~ +8 mm·s^{-1}—the value here appearing is reduced in magnitude after subtracting out an estimated atmospheric drag of ~+3.4 mm·s^{-1}.

As mentioned in the opening paragraph, the first FBA was detected during a meticulous examination of Doppler data shortly after the Earth flyby of the Galileo spacecraft on December 8, 1990. While the Doppler residuals (observed minus computed data) were expected to remain constant, the analysis uncovered an unexpected shift of 66 mHz, corresponding to a speed increase of 3.92 mm·s^{-1} at perigee. Investigations into this phenomenon at the Jet Propulsion Laboratory (JPL) [3], the European Space Operations Center (ESOC) in Darmstadt, Germany [6], the Goddard Space Flight Center (GSFC), and the University of Texas did not provide a satisfactory explanation for this occurrence, thus giving rise to the FBA problem—a problem whose resolution remains elusive.

Not all flyby encounters have resulted in an anomalous effect. For instance, no anomaly was detected during the second Earth flyby of the Galileo spacecraft in December 1992, marking the first null result [7]. It is generally believed (assumed) that this null result is due to any potential velocity increase being masked by uncertainties related to atmospheric drag, given the spacecraft's lower altitude of 303 km [8] [9]. The theory of flyby anomalies (FBAs) that we will present in this work predicts larger asymptotic perigee speed changes for orbits with smaller perigee distances. Thus, the assumption of atmospheric drag may indeed explain why the expected asymptotic perigee speed change was not observed.

In a more recent case of no flyby anomalies detected, the third and final Rosetta

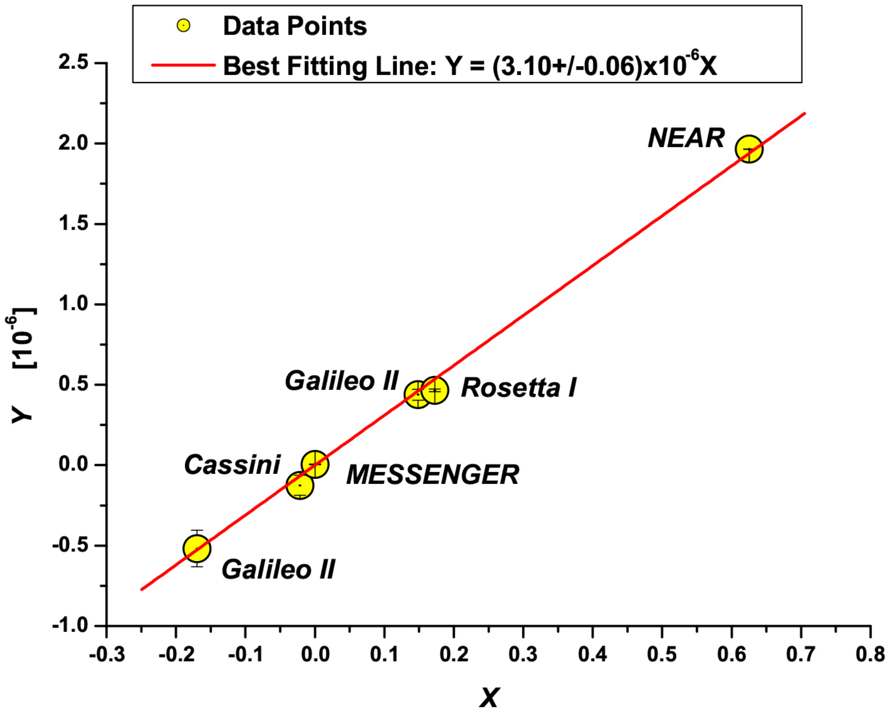

Figure 1. **Anderson (2008)'s Graph:** of $Y = \left(\Delta v_{\infty} / v_{\infty}^{\text{in}}\right)$ *vs* $X = \left(\cos\delta_{\text{in}} - \cos\delta_{\text{out}}\right)$ showing a very good fit to the data for the Flyby Anomalies of *Galileo I, II*; *NEAR*, *Cassini*, *Rosetta I* and *MESSENGER*.

encounter on November 11, 2009, yielded no anomaly. Such observations only deepen the puzzle—the mystery persists. What causes these mysterious occurrences? On a more positive note, on January 23, 1998, the Near Earth Asteroid Rendezvous (*NEAR*) spacecraft experienced an anomalous speed increase of 13.46 mm·s^{-1} following its Earth encounter. Similarly, *Cassini-Huygens* gained about 0.11 mm·s^{-1} in August 1999, and Rosetta experienced an increase of 1.82 mm·s^{-1} after its Earth flyby in March 2005. However, an analysis of the *MESSENGER* spacecraft, which was studying Mercury, did not reveal any significant unexpected velocity changes.

On a promising and optimistic note, Anderson *et al.* [2] had previously deduced an empirical relationship predicting a FBA of up to about 1 mm·s^{-1} for the November 13, 2009, Rosetta Earth encounter. However, this prediction did not materialize. Instead, what was measured was approximately 0.004 ± 0.044 mm·s^{-1}, which, for all practical purposes, constitutes a null result. This measurement carries an error margin of 11000%. The empirical relationship that Anderson *et al.* [2] identified is:

$$\frac{\Delta v_{\infty}}{v_{\infty}^{\text{in}}} = \kappa_{\text{A}}^{\oplus}\left(\cos\delta_{\text{in}} - \cos\delta_{\text{out}}\right) = -\kappa_{\text{A}}^{\oplus}\Delta\left(\cos\delta\right), \tag{1}$$

where: $\kappa_{\text{A}}^{\oplus} = \left(3.10 \pm 0.06\right) \times 10^{-6}$, δ_{in} and δ_{out} are the dimensionless Anderson parameter (constant), the incoming and outgoing osculating asymptotic velocity vectors respectively, and: $\Delta\left(\cos\delta\right) = \cos\delta_{\text{out}} - \cos\delta_{\text{in}}$. Anderson *et al.* [2] have proposed a possible relation of $\Delta v_\infty / v_\infty^{\text{in}}$ to the Earth's spin—they (Anderson *et al.* [2]) noticed that:

$$\kappa_{\text{A}}^{\oplus} = \frac{2\mathcal{R}_{\oplus}\omega_{\oplus}}{c_0} = (3.10 \pm 0.06) \times 10^{-6}, \tag{2}$$

where: $\omega_{\oplus} = 7.29 \times 10^{-5}$ rad/s and $\mathcal{R}_{\oplus} = 6.40 \times 10^{6}$ m, are the angular frequency of the Earth and the radius of the Earth, respectively (see *e.g.* Stacey 1992, in Ref. [2]).

The graph of $Y = (\Delta v_\infty / v_\infty^{\text{in}})$ *versus* $X = (\cos\delta_{\text{in}} - \cos\delta_{\text{out}})$ is shown in **Figure 1**. This graph clearly demonstrates an impressively good fit to the data for the flyby anomalies of *Galileo I, II*; *NEAR*, *Cassini*, Rosetta-I, and *MESSENGER*. The R^2-value, or Coefficient of Determination (COD), for the best fitting line: $Y = (3.10 \pm 0.06) X$, is 99.815%. This is not merely a good fit but an outstanding one. Despite efforts (by researchers such as Ashby [10], Mbelek [11], and Carlos [12]) to derive the formula of [2] as presented in Equation (1), there has yet to be a substantial physical foundation, as an accepted physical theory has not provided the necessary groundwork. Within the framework of the ASTG-model [1], we will attempt to derive the Anderson formula [Equation (1)] in Section § (5), which is the primary focus of this work. Our modest hope is that, if accepted, this effort will lend support to the ASTG-model.

In closing this introductory section, we must quickly acknowledge that numerous studies have been conducted all in an effort to understand the flyby anomalies (see *e.g.*, Refs. [5] [11]-[34]). Despite these efforts, a clear understanding of the cause of the FBA remains elusive. In the next section, we will briefly explore some of the potential causes of FBAs.

2. Possible Causes

The FBA problem is included in the *Wikipedia List of Unsolved Problems in Physics*[2] under the Astronomy and Astrophysics section. This highlights its significance not only for space navigation but also for fundamental physics, as resolving this issue could greatly enhance our understanding of both our terrestrial environment and the underlying gravitational phenomena. Due to its importance, a variety of solutions have been proposed from different areas of theoretical physics. We will briefly discuss some of the frequently mentioned solutions in the literature. However, none of these studies has yet provided a universally accepted explanation for the true cause of the FBAs. It is quite possible that within this diverse range of ideas, some suggested effects may indeed contribute to the origins of these anomalies.

2.1. Thermal Recoil Effects

Thermal recoil forces have been a significant factor in the analyses of spacecraft anomalies related to their telemetry. Following the publications by Turyshev *et al.*

[2]https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_physics

[35] [36], and Rievers [37], there is a strong belief in the mainstream narrative that thermal recoil forces are responsible for the Pioneer Anomaly (PA) [38] [39]. However, not everyone is convinced; for example, in Ref. [40] we have argued that the PA may not solely be due to thermal recoil forces and that the findings of Turyshev *et al.* [35] [36], and Rievers [37] do not conclusively resolve the issue. Indeed, the search for alternative solutions to this problem is ongoing. We believe that a dedicated mission aimed at thoroughly investigating the PA is the most logical and credible approach to definitively settle this matter. Such a mission would provide an opportunity to rigorously test most, if not all, of the proposed ideas.

In the context of FBAs, thermal recoil effects were thoroughly examined by Rievers [37], who analysed the thermal radiation pressure on a model of the Rosetta spacecraft. They found that this pressure was too small to account for the anomalies. Furthermore, Rievers [37] produced results that were incorrect in sign (as noted by Acedo [5]), leading to the dismissal of thermal effects as a potential cause of the FBA problem. Importantly, Rievers [37] did not attempt to derive Anderson *et al.* [2]'s empirical formula; their focus was solely on how thermal recoil forces might contribute to the observed anomalies.

2.2. Terrestrial Effects

Terrestrial (or conventional) effects encompass a range of factors, including but not limited to ocean and solid Earth tides, atmospheric drag, the electric charge and magnetic moment of the spacecraft, Earth albedo, solar wind, and spin-rotation coupling. Lämmerzah & Dittus [18] have examined all of these effects. The Earth's gravitational field is affected by its irregular shape and varying density due to geological features, which can cause unexpected changes in a spacecraft's trajectory and velocity during a flyby. Additionally, the gravitational influences of the Moon and the Sun can create tidal effects that may alter the spacecraft's path, especially during close approaches to Earth. Beyond gravitational effects, fluctuations in the Earth's magnetic field can impact charged particles and electronic systems on spacecraft, potentially affecting their trajectories and contributing to observed anomalies.

Although spacecraft generally operate at high altitudes where atmospheric drag is minimal, any residual atmospheric effects can still impact their velocity, especially during low-altitude flybys. Variations in atmospheric density caused by weather patterns or solar activity can alter the spacecraft's trajectory, potentially contributing to observed anomalies. While terrestrial effects are unlikely to fully explain these anomalies [26], they underscore the complexity of gravitational and magnetic interactions between the spacecraft and the Earth-Moon system. This complexity highlights the necessity for precise measurements in spacecraft navigation to mitigate these influences entirely.

2.3. Electrostatic Effects

Atchison & Peck [26] have examined potential spacecraft electrostatic charging effects through the lens of the Lorentz force and rules them out as a possible cause of anomalies. Similarly, Lämmerzah & Dittus [18] and Lämmerzah [19] briefly consider the influence of the Earth's magnetic field on a charged spacecraft, concluding that electrical charge on the spacecraft is unlikely to be the cause of the observed effects. In line with the considerations of Lämmerzah & Dittus [18], Lämmerzah [19] and Atchison & Peck [26], the current model explores the possibility of solar wind-induced electrostatic charge accumulation on the spacecraft, which is then subjected to a Lorentz force due to the Earth's magnetic field.

2.4. Extensions of the General Theory Relativity

Extensions of Einstein's General Theory of Relativity (GTR) have been explored by various researchers (see *e.g.*, Refs. [5] [28] [30] [41]-[43]). Acedo [30] [44] has examined the FBA phenomenon from diverse theoretical perspectives, advocating for the investigation of alternative gravitational models to gain a better understanding of this enigmatic issue. Meanwhile, Pinheiro [28] [43] propose that topological torsion currents may significantly contribute to our understanding of the FBA. Additionally, Hafele [41] [42] suggests a revised interpretation of gravitational effects during spacecraft flybys, arguing that incorporating time-retardation offers a more accurate explanation for the anomalies observed in spacecraft trajectories.

These extensions of General Theory of Relativity (GTR) are closely linked to Gravitomagnetism (GM). Specifically, GM is a theoretical framework that expands on the principles of GTR to incorporate effects analogous to electromagnetism, but involving gravitational fields. It explains how moving masses can generate gravitational fields in a manner similar to how moving electric charges create magnetic fields. This theory (see *e.g.*, Refs. [5] [23]) has been proposed as a potential explanation for various phenomena (see *e.g.*, Ref. [23]), including the flyby anomalies observed in spacecraft missions.

In summary, it is important to note that while no single extension of General Theory of Relativity (GTR) has universally resolved the FBA problem, the exploration of various theoretical frameworks—ranging from gravitomagnetism to topological effects—can provide valuable insights. Each approach contributes to a deeper understanding of gravitational interactions, and these efforts may ultimately lead to a comprehensive explanation of the FBA. Therefore, ongoing research in this direction is essential for refining these theories and potentially reconciling them with empirical evidence. Notably, these attempts do not typically aim to derive [2]'s empirical formula but rather focus on modifications to the Newtonian potential that can reproduce the observed anomalies.

2.5. Inertia Effects

McCulloch [20] [21] [45] proposes a mechanism based on the hypothesis that the inertia of material bodies arises from a quantum mechanical effect related to a form of radiation known as Unruh radiation. This Unruh radiation varies with the acceleration of the material body due to a *Hubble-scale Casimir effect*. The author argues that this proposed quantum effect can explain the Pioneer Anomaly (PA) [38] [39], which is now considered to be a resolved issue (see *e.g.*, [35] [36] [46]).

Apart from addressing the PA, McCulloch [20] [21] [45]'s *Quantized Inertia Hypothesis* qualitatively reproduces the latitude dependence of the FBAs and shows good quantitative agreement with three of the six measured FBAs. Specifically, McCulloch [20] derives a formula that exhibits a cosine dependence similar to that of Anderson *at al.* [2], and his formula reproduces the observed anomalies within acceptable margins of error.

In a manner akin to Milgrom [47]-[50]'s Modified Newtonian Dynamics (MoND), McCulloch [51]-[53] further demonstrates that his theory can explain the Anomalous Flat Rotation Curve Problem [54]-[59] without invoking the existence of darkmatter. While McCulloch [20] [21] [45]'s ideas are intriguing, not all researchers are convinced that this theory can adequately account for the FBAs.

2.6. Radar and Doppler Effects

Mbelek [11] and Petry [60] attempt to explain the FBA using the transversal Doppler effect, with Mbelek [11] employing Einstein [61]'s Special Theory of Relativity and Petry [60] utilizing Einstein [62] [63]'s General Theory of Relativity. However, according to Acedo [5], these efforts are dismissed as potential causes because the spacecraft movements are not consistently transversal to the Earth's surface. Additionally, the anomaly is observed in both ranging data and Doppler data, further complicating the applicability of the transversal Doppler effect as an explanation.

According to Guruprasad [33], discrepancies in range measurements between the Space Surveillance Network radars and the Deep Space Network during the 1998 Earth flyby of NEAR, as well as inconsistencies in ESA's Doppler and range data during Rosetta's 2009 flyby, reveal a consistent excess delay or lag equal to the instantaneous one-way travel time of the telemetry signals. These lags effectively account for all aspects of the FBA and are identified as symptoms of chirp d'Alembertian travelling wave solutions. This phenomenon is related to traditional sinusoidal waves through a rotation of the spectral decomposition, attributed to clock acceleration caused by Doppler rates during the flybys. Consequently, the lags connect to special relativity while also yielding distance-proportional shifts similar to those observed in cosmology, but at short ranges.

2.7. Phenomenological Approach

Phenomenological approaches (*e.g.*, Refs. [64]-[67]) have been developed as alternatives to [2]'s empirical formula (Equation (1)). In their work, [64] examines the effects of atmospheric drag and solar radiation pressure on the spacecraft. They rule out atmospheric drag as a significant factor due to its insufficient impact, identifying solar radiation pressure as a viable candidate for contributing to the FBA. However,

[64] also emphasizes the lack of comprehensive tracking data and attitude information, which limits their ability to draw definitive conclusions about the causes of the anomaly. Notably, they do not attempt to derive Anderson *et al.* [2]'s empirical formula but instead focus on the values obtained from observations.

On the other hand, Busack [65]-[67] explores the FBA problem through the lens of an empirical asymmetric gravitational field, aiming to simulate and predict anomalies in spacecraft velocity during flybys. He highlights the potential of this approach to enhance understanding of gravitational dynamics in space missions. However, Busack [67] introduces a somewhat phenomenological term with a subtle—*albeit, tailor-made*—kink in its structure, seemingly designed to fit the anomalies occurring at the perigee and asymptotes.

In addition to this limitation, the theory has three degrees of freedom (*i.e.*, constants: A, B, and C), providing ample flexibility to fit observations. Notably, Busack [67]'s model does not reproduce [2]'s empirical formula but does align with the values of some observations [*e.g.*, Galileo (I), NEAR, and Rosetta (I)], while failing to account for others [*e.g.*, Cassini, Galileo (II), and *MESSENGER*].

2.8. Darkmatter

In an effort to find a solution, Adler [13] [22] [25] [68] examines the controversial concept of darkmatter [54]-[59] as a potential explanation for the FBAs. In his theory, Adler [13] [22] [25] [68] posits that FBAs may result from the scattering of spacecraft nucleons by darkmatter particles orbiting the Earth. He suggests that the observed speed decreases could arise from elastic scattering, while the observed speed increases could result from exothermic inelastic scattering, which imparts an energy impulse to a spacecraft nucleon.

One significant setback of Adler [13] [22] [25] [68]'s intriguing model is that it has eight free parameters. In applying this theory to real data, Adler [22] used these eight parameters to fit only six data points. As he himself acknowledges, this presents a serious statistical weakness, as the theory requires a statistically significant number of data points for its results to be deemed credible and meaningful. To address this limitation, Adler [13] obtained five additional flyby data points from further Earth Gravity Assist (EGA) manoeuvres of Rosetta II *&* III and EPOXI[3] I, II & III.

Regarding the potential role of darkmatter in explaining the FBAs, Adler [13] concludes that this model remains a possible explanation but requires further evaluation with new data if it becomes available in the future. Furthermore, it is noteworthy that Adler [13]'s model predicted an anomaly of 11.30 mm·s^{-1} for Juno's EGA on October 9, 2013, at the asymptotes. This anomaly was not observed, suggesting a strong possibility that the model may not align with the physical realities of our present world.

[3]EPOXI is a compilation of NASA Discovery Program missions utilizing the already '*in flight*' Deep Impact Extended Investigation (DIXI) spacecraft and the Extrasolar Planet Observation and Characterization (EPOCh). The name EPOXI is derived from combining the pre and suffixes of the acronyms EPOCh and DIXI.

2.9. Exotic Causes

Far from the realms of conventional physics, some researchers have explored exotic concepts such as extra dimensions (*e.g.*, Refs. [13] [14] [22] [25]). Gerrard & Sumner [14] specifically examines the implications of extra dimensions, while Adler [13] [22] [25] focus on darkmatter as a potential explanation for the FBAs. Despite these various approaches, none of the studies mentioned have produced a universally accepted explanation for the underlying causes of FBAs. It remains very much possible that within this complex mix of ideas, the suggested effects may collectively contribute to our understanding of the origins of the anomalies.

3. ASTG-Model

The Azimuthally Symmetric Theory of Gravitation (hereafter ASTG-model) was first presented in Ref. [1] and its novel idea is the in-cooperation of spin of the massive gravitating body in question into the structure of the gravitational field produced by this body. From a banal mathematical standpoint, the ASTG-model is no more than the azimuthal solutions of the Poisson-Laplace equation applied to the scenario of Newtonian gravitation. The Newtonian solutions are naturally extended from the one dimensional (r) central field potential: $\Phi = \Phi(r)$, to two dimensions (r,θ): $\Phi = \Phi(r,\theta)$. At first glance, this theory appears as nothing more than the mundane azimuthally symmetric solutions of the well known Poisson-Laplace equation, namely:

$$\nabla^2\Phi = 4\pi G\varrho, \tag{3}$$

where: $G = 6.67430(15)\times 10^{-11}\ \mathrm{kg}^{-1}\cdot\mathrm{m}^3\cdot\mathrm{s}^{-2}$ (CODATA-Value, 2022) is Newton's supposed universal constant of gravitation, Φ is the gravitational potential, ϱ is the density of matter and ∇^2 is the usual Laplacian operator.

At its inception, our initial thoughts and feelings were that the ASTG-model is but a banal theory of gravitation, extending Sir Isaac Newton's gravitational theory from a central field phenomenon to an azimuthal and polar field phenomenon. For this reason, we noted therein Ref. citenyambuya10a that the ASTG-model is not a new theory of gravitation. However, that view has since changed!

The ASTG-model is a '*seemingly non-relativistic classical theory*' where spin is not only considered but takes center stage, especially when the spin is significantly high. This spin feature is not accounted for in classical theories of gravity, making this development a new theory of gravitation. We would like—*from hereon* to think of it in this way.

As will become clear in the present reading, the ASTG-model is surely a new classical theory of gravitation that makes the *seemingly ambitious hypothesis* that the spin of a gravitating mass has a significant and decisive role in the emergent gravitational field of the spinning mass. The ASTG-model is based[4] on the solutions $\Phi = \Phi(r,\theta)$ of (3), *i.e.*:

[4]This theory can be extended to include the polar solutions $\Phi(r,\theta,\varphi)$. Exploration of these solutions is a task we hope to look into in future readings.

$$\Phi(r,\theta) = -\frac{G\mathcal{M}_{\mathrm{g}}}{r}\left[1+\sum_{\ell=1}^{\infty}\lambda_{\ell}\left(\frac{\mathcal{R}_s}{2r}\right)^{\ell}\mathcal{P}_{\ell}(\cos\theta)\right], \tag{4}$$

where: $\mathcal{P}_{\ell}(\cos\theta) : \ell = 1,2,3,\cdots \text{etc}$, are the usual Legendre polynomials; $\mathcal{R}_s = 2G\mathcal{M}_{\mathrm{g}}/c_0^2$, is generally a parameter with the dimensions of length and has been identified in Ref. [1] with the Schwarzschild radius of the central massive gravitating body in question; $\mathcal{M}_{\mathrm{g}}$ is the gravitational mass of the central gravitating body; c_0 is the speed of Light in *vacuo*, r is the radial distance from this gravitating body, and $\lambda_{\ell} : \ell = 1,2,3,\cdots \text{etc}$, are some dynamic and dimensionless parameters which in the ASTG-model—these are assumed to be related to the gravitating body in question and the explicit dependence of these λ-parameters on the gravitating body's spin. In the present work, we will use the flyby anomalies to fix the issue of the -parameters.

Further—*regarding these* λ *-parameters*—it should be mentioned that the property of them being dynamic parameters related to the spin angular momentum of the gravitating body is the novelty of the ASTG-model. In a way, the dynamism of the λ-parameters makes the ASTG-model a new classical theory of gravitation where the spin of the gravitating mass takes center stage. Moreover, it is assumed that under all conditions of existence, we will have: $\lambda_{\ell} \equiv 0$, whenever spin is dropped (switched off). Clearly, this implies that with the spin switched off, the ASTG-model reduces to the traditional Newtonian gravitational theory that we are familiar with.

4. Energy Balance at Perigee and Asymptotes

The spacecraft data reveals an interesting and impressive energy balance for the energy changes occurring at the perigee, as well as at the incoming and outgoing asymptotes. To clarify, let: $K_{\mathrm{prg}}^{\mathrm{in}} = \frac{1}{2}m_{\mathrm{prg}}^{\mathrm{in}}\left(v_{\mathrm{prg}}^{\mathrm{in}}\right)^2$, and, $K_{\infty}^{\mathrm{in}} = \frac{1}{2}m_{\infty}^{\mathrm{in}}\left(v_{\infty}^{\mathrm{in}}\right)^2$, be the kinetic energies of the spacecraft at the perigee and incoming asymptote, respectively, with: $m_{\mathrm{prg}}^{\mathrm{in}}$, being the mass of the spacecraft at the perigee prior to any speed change and m_{∞}^{in} is the mass of the same spacecraft at incoming asymptote. If—*as every physicist confidently believes*—energy is conserved, then the abrupt and asymptotic energy change, $\Delta K_{\mathrm{prg}} = m_{\mathrm{prg}}v_{\mathrm{prg}}^{\mathrm{in}}\Delta v_{\mathrm{prg}} + \frac{1}{2}\left(\Delta m_{\mathrm{prg}}\right)\left(v_{\mathrm{prg}}^{\mathrm{in}}\right)^2$, occurring at the perigee—this, must persist in the energy balance in the outgoing asymptote, where: $\Delta m_{\mathrm{prg}} = m_{\mathrm{prg}}^{\mathrm{out}} - m_{\mathrm{prg}}^{\mathrm{in}}$, and, $\Delta v_{\mathrm{prg}} = v_{\mathrm{prg}}^{\mathrm{out}} - v_{\mathrm{prg}}^{\mathrm{in}}$, that is to say:

$$\Delta K_{\mathrm{prg}} = \Delta K_{\infty}, \tag{5}$$

where—as usual: $\Delta K_{\infty} = K_{\infty}^{\mathrm{out}} - K_{\infty}^{\mathrm{in}} = m_{\mathrm{in}}v_{\infty}^{\mathrm{in}}\Delta v_{\infty} + \frac{1}{2}\left(\Delta m_{\infty}\right)\left(v_{\infty}^{\mathrm{in}}\right)^2$, with: $\Delta m_{\infty} = m_{\infty}^{\mathrm{out}} - m_{\infty}^{\mathrm{in}}$, where: $m_{\infty}^{\mathrm{out}}$ is the mass of the spacecraft at the outgoing asymptote.

The spacecraft data clearly reveals the following indelible fact of experience:

$$v_{\mathrm{prg}}^{\mathrm{in}}\Delta v_{\mathrm{prg}} = v_{\infty}^{\mathrm{in}}\Delta v_{\infty}. \tag{6}$$

The values of: $v_{\mathrm{prg}}^{\mathrm{in}}\Delta v_{\mathrm{prg}}$ and $v_{\infty}^{\mathrm{in}}\Delta v_{\infty}$, are displayed in **Table 2**, and **Figure 2** illustrates the corresponding graph for these values. From this graph, we obtain an R^2 correlation coefficient value of 99.999%. This result represents a near-perfect fit, making it a most natural and logical conclusion that Equation (6) should be regarded as a *sin-qua-non* reality governing the speed changes of spacecraft in open orbits.

Table 2. Energy balance at perigee and asymptotes.

Spacecraft	$v_{\text{prg}}\Delta v_{\text{prg}}$	$v_{\infty}^{\text{in}}\Delta v_{\infty}$
	(J·kg^{-1})	(J·kg^{-1})
Galileo I	35.20 ± 0.70	35.10 ± 0.70
NEAR	91.80 ± 0.90	92.20 ± 0.90
Cassini	−32.00 ± 17.00	−32.02 ± 16.00
Rosetta I	7.10 ± 0.20	7.00 ± 0.20
MESSENGER	0.08 ± 0.04	0.08 ± 0.04

Clearly, Equation (5) and Equation (6) point to the following undeniable fact:

$$\Delta m_{\text{prg}} = \Delta m_{\infty} = 0 \;\rightarrow\; \left(m_{\text{prg}}^{\text{in}} = m_{\text{prg}}^{\text{out}}\right) \text{ and } \left(m_{\infty}^{\text{in}} = m_{\infty}^{\text{out}}\right), \tag{7}$$

that is to say, the inertia mass of the spacecraft is a constant of motion—it is the same at all points of the orbit—*i.e.*: at the incoming asymptote, at the perigee and as-well as at the outgoing asymptote. This important finding [Equation (7)] points to the fact that the origins of FBAs may very well not be a result of inertial effects as suggested by *e.g.* [20] [21].

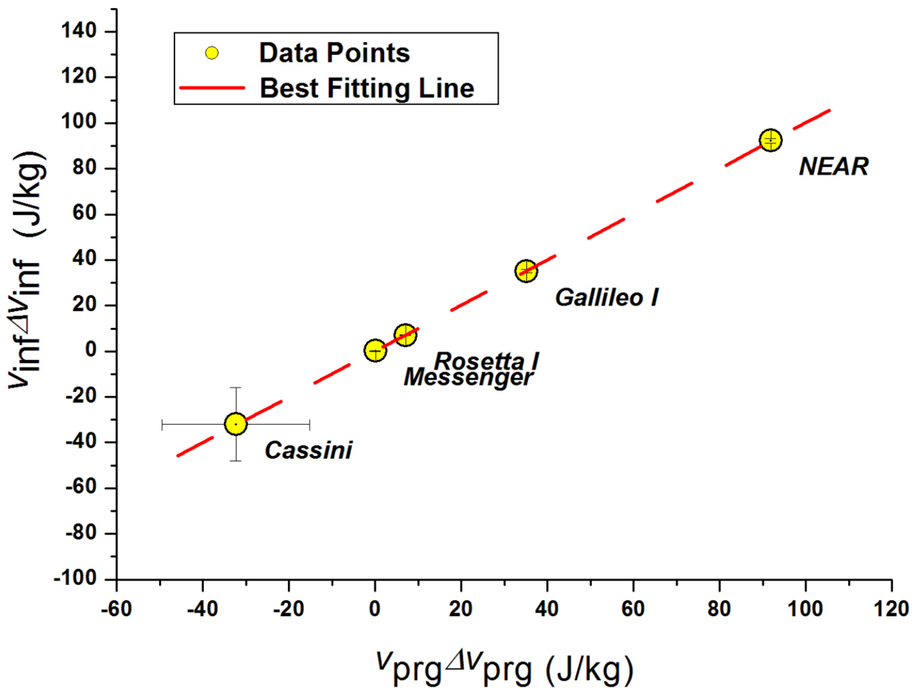

Figure 2. Energy Balance Graph. For the unbiased best fitting line, we obtain: $Y = (1.001 \pm 0.003)X + (0.100 \pm 0.100)$, with an R^2-value of 99.999%. Here: $Y = v_{\text{prg}}\Delta v_{\text{prg}}$ and $X = v_{\infty}^{\text{in}}\Delta v_{\infty}$, and, both (X, Y) are measured in J·kg^{-1}.

This apparent indictment of *McCulloch's Modified Inertia Mass Hypothesis* should logically and naturally extend to the PA [45] as well. We must clarify that we are not ruling out the possibility of changes in the inertia mass of the spacecraft; rather, we are stating that in cases where speed changes have occurred, there appears to be no room for the inertia mass to change.

We will explore, in later readings, the scenarios where no speed changes have occurred. From our preliminary investigations, it appears that in these cases, changes in the inertia mass must have occurred instead of changes in speed. Our aim in making this distinction is to help the reader understand why we are not dismissing the potential for changes in inertia mass altogether.

5. Derivation of the Anderson Formula

We are now going to derive Anderson's empirical formula within the framework of the ASTG-model. For this derivation, it will suffice to consider the ASTG-model only up to first-order approximation. That is to say, we will take the ASTG gravitational potential $\Phi(r,\theta)$, to be:

$$\Phi(r,\theta) = -\frac{G\mathcal{M}_{\rm g}}{r}\left[1+\lambda_1\left(\frac{\mathcal{R}_s}{2r}\right)\cos\theta\right], \tag{8}$$

and from this, it follows that the gravitational force $\boldsymbol{F}_{\rm g}$, acting on a test particle of gravitational mass $m_{\rm g}$, will be:

$$\boldsymbol{F}_{\rm g} = -\frac{G\mathcal{M}_{\rm g}m_{\rm g}}{r^2}\left[1+2\lambda_1\left(\frac{\mathcal{R}_s}{r}\right)\cos\theta\right]\hat{\boldsymbol{r}} - \frac{G\mathcal{M}_{\rm g}m_{\rm g}}{r^2}\left[\lambda_1\left(\frac{\mathcal{R}_s}{2r}\right)\sin\theta\right]\hat{\boldsymbol{\theta}}. \tag{9}$$

We know that the kinetic energy: $\frac{1}{2}m_i v^2$, of this test particle is such that:

$$\frac{1}{2}m_i v^2 = \int_{(r,\theta)}^{(\infty,\theta)} \boldsymbol{F}_{\rm g}\cdot \mathrm{d}\boldsymbol{r}, \tag{10}$$

where: m_i, is the inertial mass of this test particle.

$$\frac{1}{2}m_i v^2 = \frac{G\mathcal{M}_{\rm g}m_{\rm g}}{r}\left[1+\lambda_1\left(\frac{\mathcal{R}_s}{r}\right)\cos\theta\right], \tag{11}$$

hence:

$$v^2 = \frac{2G\mathcal{M}_{\rm g}}{r}\left[1+\lambda_1\left(\frac{\mathcal{R}_s}{r}\right)\cos\theta\right] = v^2(r,\theta), \tag{12}$$

where in arriving at Equation (12), we have assumed the complete equity and equivalence of the gravitational and inertial mass—*i.e.*: $m_{\rm g} = m_i$.

We know that: $v = v(r,\theta)$. Because of this—by way of taking a differential element of Equation (12), we can compare the speeds of the spacecrafts at the same radial distance on either arms of the orbit—*i.e.*, the incoming ($r = r_{\rm in}$, $\theta = \theta_{\rm in}$) and outgoing ($r = r_{\rm out}$, $\theta = \theta_{\rm out}$) asymptotes but at different azimuthal angles—*i.e.*: $r = r_{\rm in} = r_{\rm out}$; $\theta_{\rm in} \neq \theta_{\rm in}$. So doing, and thereafter dividing the resultant expression by $2v^2$, we obtain:

$$\frac{\Delta v}{v} = \lambda_1\left(\frac{G\mathcal{M}_{\rm g}\mathcal{R}_s}{(rv)^2}\right)\left(\cos\theta_{\rm out} - \cos\theta_{\rm in}\right) = \lambda_1\left(\frac{G\mathcal{M}_{\rm g}\mathcal{R}_s}{(rv)^2}\right)\Delta(\cos\theta), \tag{13}$$

where as defined in the introduction section [on p.(5)]: $\Delta(\cos\theta) = \cos\theta_{\rm out} - \cos\theta_{\rm in}$, and, l, is the semi-lactus rectum of the orbit of the spacecraft in question. We know that the specific orbital angular momentum: $J_\varphi = rv$, therefore:

$$\frac{\Delta v}{v} = \lambda_1 \left(\frac{G\mathcal{M}_{\rm g}\mathcal{R}_s}{J_\varphi^2} \right) \Delta(\cos\theta). \tag{14}$$

For nearly Newtonian orbits such as those of the spacecrafts executing Earth flyby manoeuvres, we known that for nearly Newtonian orbits: $J_\varphi \sim G\mathcal{M}_{\rm g} l$, hence:

$$\frac{\Delta v}{v} = \lambda_1 \left(\frac{\mathcal{R}_s}{l} \right) \Delta(\cos\theta). \tag{15}$$

Hence, at the far distance along the incoming and outgoing asymptotes evaluated as the same radial distance: $r = r_{\rm in} = r_{\rm out}$, we will have:

$$\frac{\Delta v_\infty}{v_\infty^{\rm in}} = -\lambda_1 \left(\frac{\mathcal{R}_s}{l} \right) \left(\cos\theta_{\rm in} - \cos\theta_{\rm out} \right). \tag{16}$$

From the configuration of the (r, θ, φ) spherical coordinate system and how it relates to the the (RA, DEC)-system of coordinates for the the given set-up, we know that:

$$\theta = \delta, \tag{17}$$

where δ is the declination angle, and inserting this Equation (17) into Equation (16), it follows that:

$$\frac{\Delta v_\infty}{v_\infty^{\rm in}} = -\lambda_1 \left(\frac{\mathcal{R}_s}{l} \right) \left(\cos\delta_{\rm in} - \cos\delta_{\rm out} \right), \tag{18}$$

hence, a comparison of this Equation (18) with the [2]'s empirical formula given in Equation (1), this ceremoniously leads us to:

$$\lambda_1 \left(\frac{\mathcal{R}_s}{l} \right) = -\frac{2R_\oplus \omega_\oplus}{c_0} = -\kappa_{\rm A}^\oplus. \tag{19}$$

In this way, we have derived [9]'s empirical formula from the domains of the ASTG-model.

Of this derivation [*i.e.*, Equation (18) and Equation (19)], we must hasten to say that—as per our long-held suspicion first expressed in the founding paper of the ASTG model [1]; [2]'s formula suggests an intimate connection between the -parameters and the spin properties of the central gravitating body in question. Without any doubt whatsoever, such a fundamental and pristine derivation of a previously unexplained empirically derived formula not only lends impetus to the theory from which this empirical formula has been derived (in this case, the ASTG model), but also provides a glimmer of hope that this theory may indeed contain elements of truth.

Therefore, to sum up everything: in general, for an arbitrary body whose gravitational mass is $\mathcal{M}_{\rm g}$, has radius R and spin angular speed ω; to first order approximation, according to the foregoing, the gravitational potential of such a massive gravitating body experienced by an arbitrary orbiting test body in an orbit whose semi-lactus rectum is l, is according ASTG-model given by:

$$\Phi(r,\theta) = -\frac{G\mathcal{M}_{\mathrm{g}}}{r}\left[1 - \frac{\kappa_{\mathrm{A}}}{2}\left(\frac{l}{r}\right)\cos\theta + \sum_{\ell=2}^{\infty}\lambda_{\ell}\left(\frac{\mathcal{R}_s}{2r}\right)^{\ell}\mathcal{P}_{\ell}(\cos\theta)\right]. \tag{20}$$

This expression captures most of the essential gravitational influences acting on the test body in the vicinity of a spinning gravitating mass. This result of Equation (20) is our second major result of the present reading with Equation (18) being the first. Hence—moving forward as we develop the ASTG-model further, we shall take this result [Equation (20)] and carry it up until such a time that a disagreement with empirical evidence is found. The next task is to figure out the rest of the parameters—*i.e.*: λ_{ℓ} for $\ell \geq 2$. We shall make an attempt at this [*i.e.*, figuring out the λ_{ℓ} ‘s for $\ell \geq 2$] in the next reading.

6. General Discussion

The analysis of FBAs as conducted herein have revealed intriguing insights into the gravitational interactions experienced by spacecraft during their journeys near massive celestial bodies. As indicated by previous research, including the empirical findings of Anderson *at al.* [2], the relationship between incoming and outgoing velocities of spacecraft suggests a nuanced interplay between gravitational forces and inertial effects. Our analysis seems to rule out inertial effects and points to angular effects that arise from an azimuthally symmetric gravitational potential. We have been able to derive the Anderson *at al.* [2] empirical formula from the ASTG-model and we consider this a significant development in the ASTG-model.

According to Shiga [69], the relationship proposed by Anderson *at al.* [2] in Equation (1) emerged serendipitously after realizing that the *MESSENGER* spacecraft approached and departed the Earth symmetrically about the equator (*i.e.*, it approached at a latitude of 31 degrees North and departed at 32 degrees South). This symmetry suggested a strong link between the anomaly and the Earth’s rotation, as well as the incoming and outgoing velocity vectors. As previously demonstrated, this insight led [8] to successfully establish an empirical relationship involving the incoming and outgoing declination angles of the spacecraft’s orbit. Despite numerous efforts to derive this formula from fundamental physics (*e.g.*, Refs. [10]-[12]), the empirical relationship proposed by [2] suffers from the chronic setback of lacking a real physical explanation. In the present work, we have shown that this formula can be derived from the fundamental premises of the ASTG-model, thereby framing the FBA phenomenon as a purely gravitational phenomenon.

One significant observation is that FBAs appear to exclusively affect hyperbolic (open) orbits, leading to their absence in missions like Gravity Probe B, which operates within closed low Earth orbit. This raises important questions regarding the applicability of the *Equivalence Principle*, as the lack of detected anomalies in such missions implies a potential breakdown of this fundamental concept under the specific conditions.

The law of conservation of energy, as highlighted through Equation (6), strongly supports the notion that these anomalies must be reconciled with established physical laws. This conservation implies that any inertial-related phenomena proposed as explanations for FBAs should be critically evaluated. Our findings suggest that these anomalies are not simply artifacts of inertial effects, but rather phenomena that may emerge only in the context of open orbits around rotating gravitating masses.

This leads to the conclusion that FBAs are intricately linked to the specific dynamics of spacecraft as they navigate the gravitational fields of large bodies. Further investigations are warranted to explore the underlying mechanisms that give rise to these anomalies, as they may provide deeper insights into gravitational interactions and the nature of spacetime itself. In summary, the present study of flyby anomalies not only challenges our understanding of gravitational physics but also invites a re-evaluation of established principles, underscoring the need for continued exploration in this fascinating area of research.

While terrestrial effects, such as atmospheric drag and solar radiation pressure, have been proposed as potential contributors to FBAs, this study finds that they are unlikely to fully account for the observed anomalies. The absence of anomalies in specific spacecraft missions, like Gravity Probe B, further emphasizes the need to isolate the gravitational dynamics from environmental influences. Future studies should focus on refining measurements and controlling for terrestrial effects to clarify their roles in spacecraft navigation.

The findings from this study open several avenues for further research. Investigating the nature of the parameters within the ASTG-model could lead to a more comprehensive understanding of gravitational forces in varying orbital contexts. Additionally, exploring the implications of FBAs on the broader framework of gravitational theories, including potential extensions to General Relativity, could yield valuable insights into fundamental physics. Moreover, the relationship between gravitational anomalies and the spinning nature of celestial bodies warrants deeper exploration. Future missions designed to study gravitational interactions during flybys could provide critical data to test the predictions of the ASTG-model and refine our understanding of gravitational phenomena.

7. Conclusions

Assuming the correctness (*i.e.*, acceptability) of what has been presented herein, we reach the following conclusion regarding the origins and nature of flyby anomalies:

1. Gravitational Phenomenon: The FBA can be fundamentally understood as a gravitational phenomenon, as derived from the ASTG-model.

2. Law of Conservation of Energy: As every physicist would confidently expect, from Equation (6), it strongly appears that the sacrosanct Law of Conservation of Energy is upheld during Earth flyby gravity assist maneuvers. This supports the notion that, despite the complexities involved in such interactions, the fundamental principles of physics remain intact, ensuring that energy is conserved throughout the process.

3. Origins of FBA from Inertia Related Effects: The implied conservation of energy (*via* Equation 6) strongly suggests the possibility of inertia-related phenomena being the cause of flyby anomalies (FBAs). Consequently, we out-rightly rule out all attempts that rely on inertial effects as a possible cause of FBAs. This conclusion has nothing to do with the ASTG-model or any model for that matter.

4. FBA Orbit Dependence: The anomaly appears to affect only hyperbolic orbits in the vicinity of a spinning gravitating mass, which may explain its absence in low Earth orbit missions like Gravity Probe B. This specificity suggests that the dynamics of such interactions are crucial for the manifestation of the anomalies.

5. Further Investigation: Further studies are necessary to explore the underlying mechanisms of the FBA and its implications for our understanding of gravitational interactions in different orbital regimes.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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