Scientific Research

The Collinear Libration Points in the Elliptic R3BP with a Triaxial Primary and an Oblate Secondary

Jagadish Singh, Aishetu Umar

Department of Mathematics, Faculty of Science, Ahmadu Bello University, Zaria, Nigeria
Email: umaraishetu33@yahoo.com, jgds2004@yahoo.com

Abstract

This paper examines the motion of a dust grain around a triaxial primary and an oblate companion orbiting each other in elliptic orbits about their common barycenter in the neighborhood of collinear libration points. The positions and stability of these points are found to be affected by the triaxiality and oblateness of the primaries, and by the semi-major axis and eccentricity of their orbits. The stability behavior of the collinear points however remains unchanged; they are unstable in the Lyapunov sense.

Keywords

1. Introduction

The famous restricted three-body problem (R3PB) has been receiving considerable attention of scientists and astronomers because of its application in the dynamics of the solar and stellar systems, lunar theory and planetary sciences [1]-[4]. It concerns the motion of an infinitesimal mass under the gravitational influence of two finite masses, called the primaries, which move in circular orbits about their common center of mass on account of their mutual attraction. The solution to this type of problem which has been developed over the centuries from Lagrange [5]-[10] and others, form the basis of the study of the dynamics of celestial bodies, from the computation of the ephemerides to the recent advances in flight dynamics. The original formulation of the circular restricted three-body problem (CR3BP) was based on the approximate circular motion of the planets around the sun and the small masses of the satellites of planets and asteroids compared to the planets' masses. The orbits of most celestial bodies are however elliptic rather than circular, as such, the ER3BP analyses the dynamical sys-

tems more accurately. It possesses five coplanar equilibrium points: three collinear and two triangular, where the gravitational and centrifugal forces just balance each other. The collinear points are generally unstable, while the triangular points are conditionally stable. Their stability occurs in spite of the fact that the potential energy has a maximum rather than a minimum at the latter points.

The ER3BP generalizes the original CR3BP, and improves its applicability, while some outstanding and useful properties of the circular model still hold true or can be adapted to the elliptic case. In particular, possible positions of equilibrium occur when the three bodies form equilateral triangles. An application of this model can be seen in the motion of the Trojan asteroids around the triangular point L_4. The asteroids in this case are only influenced by the gravitational forces of the Sun and Jupiter, and the orbit of Jupiter around the Sun is assumed to be a fixed ellipse. The influence of the eccentricity of the orbits of the primary bodies on the existence of the equilibrium points and their stability has been the subject of a number of communications [11]-[26]. The families of symmetric-periodic orbits in the three-dimensional elliptic problem with a variation of the mass ratio μ and the eccentricity e were studied by Sarris (1989). In the last year, Singh and Umar [25] investigated the effects of the luminosity and oblateness of both primary bodies on the collinear libration points of the binary systems Achird, Luyten 726-8, Kruger 60, Alpha Centauri AB and Xi Bootis moving in elliptic orbits around their common centre of mass. Recently, Singh and Umar [27] have examined the collinear points of the ER3BP with a triaxial bigger primary.

The bodies in the R3BP are strictly spherical in shape, but in nature, celestial bodies are not perfect spheres. They are either oblate or triaxial. The Earth, Jupiter, Saturn, Regulus, Neutron stars and black dwarfs are oblate spheroids [28]-[33]. The Moon, Pluto and its moon Charon are triaxial. The lack of sphericity, triaxiality or oblateness of the celestial bodies causes large perturbations from a two-body orbit. This inspired several researchers [34]-[38] to include non sphericity of the bodies in their studies of the R3BP.

Our aim is to study the effect of the triaxiality of the bigger primary and oblateness of the smaller one on the positions and stability of the collinear libration points. This system can be applied to the Earth-Moon system and to double pulsars. This paper is organized as follows: in Section 2, the governing equations of motion are presented; Section 3 describes the positions of the collinear points, while their linear stability is analyzed in Section 4; finally Section 5 concludes the paper.

2. Equations of Motion

The equations of motion of a dust grain particle in the ER3BP with a triaxial primary and an oblate secondary in dimensionless-pulsating coordinate system (ξ, η, ζ) are given by

$$\xi'' - 2\eta' = \Omega_\xi \; ; \eta'' + 2\xi' = \Omega_\eta \; ; \zeta'' = \Omega_\zeta \tag{1}$$

with the force function

$$\Omega = \left(1 - e^2\right)^{-\frac{1}{2}} \left[\frac{\xi^2 + \eta^2}{2} + \frac{1}{n^2} \left\{ \frac{(1-\mu)}{r_1} + \frac{\mu}{r_2} + \frac{(1-\mu)(2\sigma_1 - \sigma_2)}{2r_1^3} + \frac{\mu A}{2r_2^3} - \frac{3(1-\mu)(\sigma_1 - \sigma_2)\eta^2}{2r_1^5} \right\} \right] \tag{2}$$

and

$$r_1^2 = (\xi + \mu)^2 + \eta^2 + \zeta^2, r_2^2 = (\xi + \mu - 1)^2 + \eta^2 + \zeta^2 \tag{3}$$

and the mean motion

$$n^2 = \frac{\left(1 + e^2\right)^{\frac{1}{2}} \left(1 + \frac{3A}{2} + \frac{3(2\sigma_1 - \sigma_2)}{2}\right)}{a\left(1 - e^2\right)} \tag{4}$$

$$= \frac{1}{a} \left(1 + \frac{3e^2}{2} + \frac{3A}{2} + \frac{3(2\sigma_1 - \sigma_2)}{2}\right)$$

where the prime represents differentiation w.r.t. the eccentric anomaly e and r_i (i = 1, 2) are the distances between the third body and the primaries; n, a, e, A and σ_i are the mean motion, semi-major axis; eccentricities of the orbits; oblateness and triaxiality factors respectively.

3. Positions of Collinear Points

The collinear points are the solutions of equations $\Omega_\xi = \Omega\eta = \Omega_\zeta = 0$ and $\eta = \zeta = 0$; *i.e.*

$$\Omega_\xi = \left(1-e^2\right)^{-\frac{1}{2}}\left[\xi - \frac{1}{n^2}\left\{\frac{(1-\mu)(\xi+\mu)}{r_1^3} + \frac{\mu(\xi+\mu-1)}{r_2^3} + \frac{3(1-\mu)(\xi+\mu)(2\sigma_1-\sigma_2)}{2r_1^5}\right.\right.$$
$$\left.\left. + \frac{3\mu(\xi+\mu-1)A}{2r_2^5} - \frac{15(1-\mu)(\xi+\mu)(\sigma_1-\sigma_2)\eta^2}{2r_1^7}\right\}\right] = 0$$

$$\Omega_\eta = \left(1-e^2\right)^{-\frac{1}{2}}\eta\left[1 - \frac{1}{n^2}\left\{\frac{(1-\mu)}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3(1-\mu)(4\sigma_1-3\sigma_2)}{2r_1^5} + \frac{3\mu A}{2r_2^5} - \frac{15(1-\mu)(\sigma_1-\sigma_2)\eta^2}{2r_1^7}\right\}\right] = 0$$

$$\Omega_\zeta = \left(1-e^2\right)^{-\frac{1}{2}}\left[-\frac{\zeta}{n^2}\left\{\frac{(1-\mu)}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3(1-\mu)(2\sigma_1-\sigma_2)}{2r_1^5} + \frac{3\mu A}{2r_2^5} - \frac{15(1-\mu)(\sigma_1-\sigma_2)\eta^2}{2r_1^7}\right\}\right] = 0$$

(5)

From Equation (3), with $\eta = \zeta = 0$ and the first of Equation (5), we have

$$\xi n^2 - \frac{(1-\mu)(\xi+\mu)}{|\xi+\mu|^3} - \frac{\mu(\xi+\mu-1)}{|\xi+\mu-1|^3} - \frac{3(1-\mu)(\xi+\mu)(2\sigma_1-\sigma_2)}{2|\xi+\mu|^5} - \frac{3\mu(\xi+\mu-1)A}{2|\xi+\mu-1|^5} = 0$$

(6)

To locate the collinear points on the ξ-axis, we divide the orbital plane into three parts; $\xi < \xi_1$, $\xi_1 < \xi < \xi_2$ and $\xi_2 < \xi$ with respect to the primaries.

3.1. Position of L_1 ($\xi < \xi_1$)

Let

$$\xi - \xi_2 = \rho; \xi - \xi_1 = 1 + \rho \Rightarrow \xi = 1 + \rho + \xi_1.$$

Since the distance between the primaries is unity, *i.e.*

$$\xi_2 - \xi_1 = 1 \Rightarrow \xi_1 = -\mu \text{ and } \xi_2 = 1-\mu \text{ then } \xi = 1 + \rho - \mu; r_1 = 1 + \rho; r_2 = \rho$$

(7)

Now, substituting Equation (7) in Equation (6) and clearing the fractions, we obtain

$$2n^2\left(1+\rho-\mu\right)\left(1+\rho\right)^4\rho^4 - 2(1-\mu)(1+\rho)^2\rho^4 - 2\mu(1+\rho)^4\rho^2$$
$$-3(1-\mu)(2\sigma_1-\sigma_2)\rho^4 - 3\mu A(1+\rho)^4 = 0$$

(8)

Solving, we get

$$2n^2\rho^9 + 2n^2(5-\mu)\rho^8 + 2n^2\left(2(5-2\mu)\right)\rho^7 + \left(2n^2(2(5-3\mu)-2\right)\rho^6 + \left(2n^2(5-4\mu)-4(1+\mu)\right)\rho^5$$
$$+\left(2n^2(1-\mu)-2(1+5\mu)-6(1-\mu)\sigma_1+3(1-\mu)\sigma_2-3\mu A\right)\rho^4$$
$$-(8\mu+12\mu A)\rho^3 - (2\mu+18\mu A)\rho^2 - 12\mu\rho A - 3\mu A = 0$$

(9)

3.2. Position of L_2 ($\xi_1 < \xi < \xi_2$)

Now,

$$\xi_2 - \xi = \rho \therefore \xi - \xi_1 = 1-\rho \Rightarrow \xi = 1-\rho-\mu; r_1 = 1-\rho; r_2 = \rho$$

(10)

Equation (10) in (6) yields,

$$2n^2\left(1-\rho-\mu\right)\left(1-\rho\right)^4\rho^4 - 2(1-\mu)(1-\rho)^2\rho^4 + 2\mu(1-\rho)^4\rho^2$$
$$-3(1-\mu)(2\sigma_1-\sigma_2)\rho^4 + 3\mu A(1-\rho)^4 = 0$$

(11)

Solving, we get

$$2n^2\rho^9 - 2n^2(5-\mu)\rho^8 + 2n^2(2(5-2\mu))\rho^7 - (2n^2(2(5-3\mu)-2+4\mu))\rho^6$$
$$+(2n^2(5-4\mu)+4(1-3\mu))\rho^5 - (2n^2(1-\mu)-2(1-7\mu)-6(1-\mu)\sigma_1 + 3(1-\mu)\sigma_2 + 3\mu A)\rho^4 \tag{12}$$
$$+(8\mu+12\mu A)\rho^3 - (2\mu+18\mu A)\rho^2 + 12\mu A\rho - 3\mu A = 0$$

3.3. Position of L_3 ($\xi_2 < \xi$)

Since

$$\xi_2 - \xi_1 = 1 \Rightarrow \xi_1 = 1-\rho;\ \xi - \xi = 2-\rho\ \text{and}\ \xi = \rho-\mu-1\ ; r_1 = 1-\rho; r_2 = 2-\rho \tag{13}$$

Substituting Equation (13) in (6) and solving, we obtain

$$2n^2(\rho-\mu-1)(1-\rho)^4(2-\rho)^4 + 2(1-\mu)(1-\rho)^2(2-\rho)^4 + 2\mu(2-\rho)^2(1-\rho)^4$$
$$+3(1-\mu)(2\sigma_1 - \sigma_2)(2-\rho)^4 + 3\mu A(1-\rho)^4 = 0 \tag{14}$$

Equations (9), (12) and (14) are seventh degree equations and according to Descartes' rule of sign, there exist only one real root each corresponding to the three collinear points $L_{1,2,3}$. Using Equation (11), we show the effect of triaxiality on the position of L_2 for constant oblateness $A = 0.001$; $a = 1.2$, $e = 0.25$, $\mu = 0.35$.

It is seen from **Table 1** that, the position of L_2 as triaxiality decreases is a shift away from the origin (λ_3). This is shown in **Figure 1** and **Figure 2**, for σ_1, σ_2, λ_1, λ_2, λ_3 and σ_1, σ_2, λ_3 respectively. An interesting result of the numerical computation is the existence of three real roots when $\sigma_1 = \sigma_2 = 0.04$ (*i.e.* oblateness of the primary) and $\sigma_1 = 0.04$; $\sigma_2 = 0.03$.

4. Stability of Collinear Points

In order to study the stability of the collinear points, we consider the characteristic equation of the system [23] given by

$$\lambda^4 - (\Omega^0_{\xi\xi} + \Omega^0_{\eta\eta} - 4)\lambda^2 + \Omega^0_{\xi\xi}\Omega^0_{\eta\eta} - (\Omega^0_{\xi\eta})^2 = 0 \tag{15}$$

The second partial derivatives are:

$$\Omega_{\xi\xi} = (1-e^2)^{-1/2}1 - \frac{1}{n^2}\frac{(1-\mu)}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3(1-\mu)(2\sigma_1-\sigma_2)}{2r_1^5} - \frac{15(1-\mu)(\sigma_1-\sigma_2)\eta^2}{2r_1^7}$$
$$+\left[\frac{1}{n^2}\frac{3(1-\mu)(\xi+\mu)^2}{r_1^5} + \frac{3\mu(\xi+\mu-1)^2}{r_2^5} + \frac{15(1-\mu)(2\sigma_1-\sigma_2)(\xi+\mu)^2}{2r_1^7} - \frac{105(1-\mu)(\sigma_1-\sigma_2)(\xi+\mu)^2\eta^2}{2r_1^9}\right]$$

$$\Omega_{\eta\eta} = (1-e^2)^{-1/2}\left[1 - \frac{1}{n^2}\left(\frac{(1-\mu)}{r_1^3} + \frac{\mu}{r_2^3} + \frac{3(1-\mu)(4\sigma_1-3\sigma_2)}{2r_1^5} + \frac{3\mu A}{2r_2^5} - \frac{105(1-\mu)(\sigma_1-\sigma_2)\eta^2}{2r_1^7}\right)\right.$$
$$\left.+\frac{\eta^2}{n^2}\left(\frac{3(1-\mu)}{r_1^5} + \frac{3\mu}{r_2^5} + \frac{15(1-\mu)(4\sigma_1-3\sigma_2)}{2r_1^7} + \frac{15\mu A}{2r_2^7} - \frac{105(1-\mu)(\sigma_1-\sigma_2)\eta^2}{2r_1^9}\right)\right]$$

$$\Omega_{\xi\eta} = (1-e^2)^{-1/2}\left[\frac{\eta}{n^2}\left(\frac{3(1-\mu)(\xi+\mu)}{r_1^5} + \frac{3\mu(\xi+\mu-1)}{r_2^5} + \frac{15(1-\mu)(\xi+\mu)(2\sigma_1-\sigma_2)}{2r_1^7}\right.\right.$$
$$\left.\left.+\frac{15\mu A(\xi+\mu-1)}{2r_2^7} + \frac{30(1-\mu)(\sigma_1-\sigma_2)(\xi+\mu)}{2r_2^7} - \frac{105(1-\mu)(\sigma_1-\sigma_2)(\xi+\mu)\eta^2}{2r_1^9}\right)\right] \tag{16}$$

Considering the last interval, $\xi_2 < \xi$.
The collinear points exist on the ξ-axis, *i.e.* $\eta = \zeta = 0$, which means

$$r_1 = |\xi+\mu|,\ r_2 = |\xi+\mu-1| \tag{17}$$

Equation (17) in the first of (16) gives

Table 1. Effect of triaxiality on the position of the inner collinear point L_2.

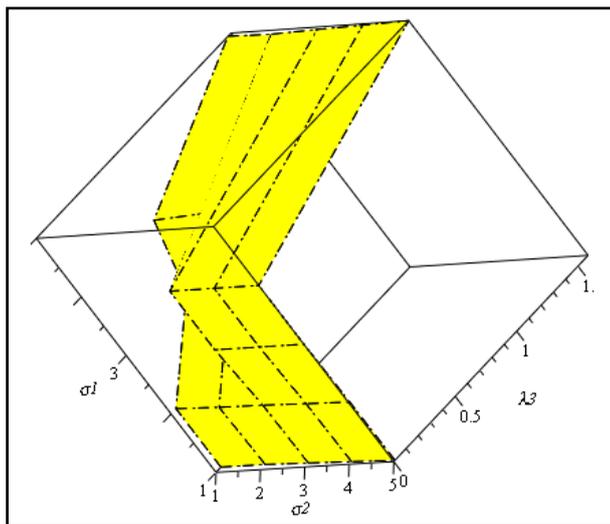| Triaxiality Factors		Characteristics Roots		
σ_1	σ_2	λ_1	λ_2	λ_3
0.04	0.040	0.489138	0.590313	1.55752
0.04	0.030	0.472706	0.617405	1.54967
0.03	0.025	-	-	1.56224
0.02	0.015	-	-	1.57147
0.01	0.075	-	-	1.58266

Figure 1. Effect of triaxiality on the position of the inner collinear point L_2.
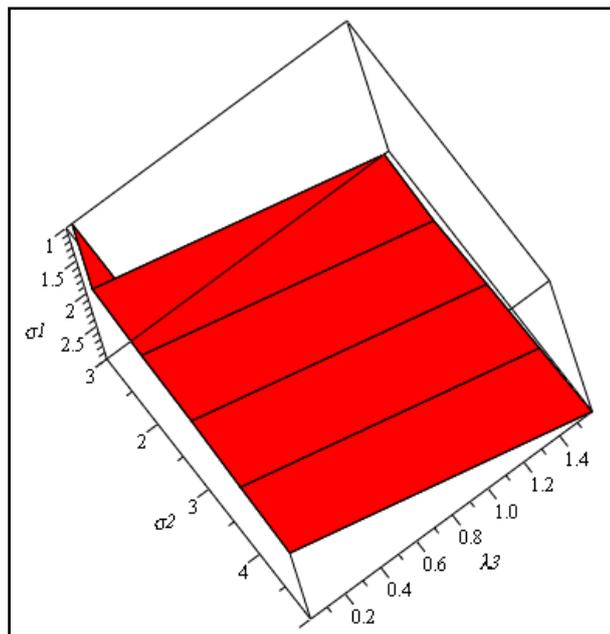

Figure 2. Effect of triaxiality on the position of the inner collinear point L_2.

$$\Omega_{\xi\xi}^{0} = \left(1-e^2\right)^{-1/2}\left[1+\frac{2}{n^2}\left(\frac{(1-\mu)}{|\xi_0+\mu|^{3/2}}+\frac{\mu}{|\xi_0+\mu-1|^{3/2}}+\frac{3(1-\mu)(2\sigma_1-\sigma_2)}{2|\xi_0+\mu|^{5/2}}+\frac{3\mu A(\xi+\mu-1)}{2|\xi_0+\mu-1|^{5/2}}\right)\right] > 0$$

From the second and third of Equation (16) with $\eta = 0$, we get

$$\Omega_{\eta\eta}^{0} = \left(1-e^2\right)^{-1/2}\left[1-\frac{1}{n^2}\left(\frac{(1-\mu)}{|\xi_0+\mu|^{3/2}}+\frac{\mu}{|\xi_0+\mu-1|^{3/2}}+\frac{3(1-\mu)(4\sigma_1-3\sigma_2)}{2|\xi_0+\mu|^{5/2}}+\frac{3\mu A}{2|\xi_0+\mu-1|^{5/2}}\right)\right]$$

and $\Omega_{\xi\eta}^{0} = 0$. The first of Equation (5) with $\eta = 0$, can be written as

$$\frac{(1-\mu)}{r_1^2} = \left(\xi n^2 - \frac{\mu}{r_2^2} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)}{2r_1^4} - \frac{3\mu A}{2_2^4}\right) \tag{18}$$

Using Equation (18),

$$\Omega_{\eta\eta}^{0} = \left(1-e^2\right)^{-1/2}\left[1-\frac{1}{n^2}\left(\frac{1}{r_1}\left\{\xi n^2 - \frac{\mu}{r_2^2} - \frac{3(1-\mu)(2\sigma_1-\sigma_2)}{2r_1^4} - \frac{3\mu A}{2_2^4}\right\}+\frac{\mu}{r_2^3}+\frac{3(1-\mu)(4\sigma_1-3\sigma_2)}{2r_1^5}+\frac{3\mu A}{2_2^5}\right)\right] \tag{19}$$

but, $r_1 = \xi+\mu \Rightarrow \xi = r_1 - \mu$, which on substitution in Equation (19), yields

$$\Omega_{\eta\eta}^{0} = \left(1-e^2\right)^{-1/2}\left[1-\frac{1}{n^2}\left(\left\{\frac{(r_1-\mu)n^2}{r_1}-\frac{\mu}{r_1 r_2^2}-\frac{3(1-\mu)(2\sigma_1-\sigma_2)}{2r_1^5}-\frac{3\mu A}{2r_1 r_2^4}\right\}+\frac{\mu}{r_2^3}+\frac{3(1-\mu)(4\sigma_1-3\sigma_2)}{2r_1^5}+\frac{3\mu A}{2r_2^5}\right)\right]$$

Neglecting higher order terms in e^2, a, A and σ_i ($i = 1, 2$), we get

$$\Omega_{\eta\eta}^{0} = \frac{\mu}{r_1}+\frac{1}{n^2}\left(\left\{\frac{\mu}{r_2^2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)-\frac{3(1-\mu)}{r_1^5}(\sigma_1-\sigma_2)+\frac{3\mu A}{2r_2^4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\right\}\right)$$

Since $\mu < \dfrac{1}{2}$ and $A, \sigma_i, e^2 \ll 1$, $r_1 > 1$, $r_2 < 1$,

$$\frac{\mu}{r_1}+\frac{1}{n^2}\left(\left\{\frac{\mu}{r_2^2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)-\frac{3(1-\mu)}{r_1^5}(\sigma_1-\sigma_2)+\frac{3\mu A}{2r_2^4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\right\}\right) < 0 \Rightarrow \Omega_{\eta\eta}^{0} < 0$$

Also, the last of Equation (16) by virtue of $\eta = 0$ gives $\Omega_{\xi\eta}^{0} = 0$.
Similarly, for the collinear points lying in $(\xi_1 < \xi < \xi_2)$ and $(\xi < \xi_1)$,

$$\Omega_{\xi\xi}^{0} > 0,\ \Omega_{\eta\eta}^{0} < 0\ \text{and}\ \Omega_{\xi\eta}^{0} = 0$$

Since $\Omega_{\xi\xi}^{0}\Omega_{\eta\eta}^{0}-\left(\Omega_{\xi\eta}^{0}\right)^2 < 0$ the discriminant of Equation (15) is positive and its roots can expressed as $\lambda_{1,2} = \pm b$ and $\lambda_{3,4} = \pm ic$, where b and c are real numbers, thus, the solution is unstable. We may therefore conclude that the stability of the collinear points does not change despite of perturbations on account of triaxiality and oblateness introduced.

6. Conclusion

The positions of the collinear equilibrium points when the primary is a triaxial rigid body and the secondary is an oblate spheroid have been obtained (Equations (9), (12), (14) and **Figure 1** and **Figure 2**). They are seen to be affected by the shape of the participating bodies and their orbital geometries. These results agree with [27] in the absence of oblateness and with [37] in the circular case. The collinear points also remain unstable due to the presence of real roots and despite the introduction of the triaxiality and oblateness parameters.

References

[1] Topputo, F., Vasile, M. and Bernelli-Zazzara, F. (2005) Earth-to-Moon Low Energy Transfers Targeting L1 Hyperbolic Transit Orbits. *Annals of the New York Academy of Sciences*, **1065**, 55-76.

http://dx.doi.org/10.1196/annals.1370.025

[2] Belbruno, E., Topputo, F. and Gidea, M. (2008) Resonance Transitions Associated to Weak Capture in the Restricted Three-Body Problem. *Advances in Space Research*, **42**, 1330-1351. http://dx.doi.org/10.1016/j.asr.2008.01.018

[3] Romagnoli, D. and Circi, C. (2009) Earth-Moon Weak Stability Boundaries in the Restricted Three and Four Body Problem. *Celestial Mechanics and Dynamical Astronomy*, **103**, 79-103. http://dx.doi.org/10.1007/s10569-008-9169-y

[4] Bazso, A. (2012) 43rd Lunar and Planetary Science Conference.

[5] Lagrange (1772) Collected Works Paris. VI, 229.

[6] Laplace and Delaunay (1867) Memoire sur la theorie de la Lune, Mem. *De l' des Science*, 28 and 29.

[7] Poincare (1892) Les Methodes Nouevelles de la Mechanique ce'leste Guthier villars, Paris, Chap. V, 250.

[8] Birkhoff (1927) Dynamical System. American Mathematical Society, New York.

[9] Szebehely, V.G. (1967) Theory of Orbits. Academic Press, New York.

[10] Danby, J.M.A. (1988) Fundamentals of Celestial Mechanics. 2nd Edition, Willmann-Bell, Inc., Virginia.

[11] Kumar, V. and Choudry, R.K. (1990) Nonlinear Stability of the Triangular Libration Points for the Photo Gravitational Elliptic Restricted Problem of Three Bodies. *Celestial Mechanics and Dynamical Astronomy*, **48**, 299-317. http://dx.doi.org/10.1007/BF00049387

[12] Markellos, V.V., Perdios, E. and Labropoulou, P. (1992) Linear Stability of the Triangular Equilibrium Points in the Photogravitational Elliptic Restricted Problem I. *Astrophysics and Space Science*, **194**, 207-213. http://dx.doi.org/10.1007/BF00643991

[13] Sahoo, S.K. and Ishwar, B. (2000) Stability of Collinear Equilibrium Points in the Generalized Photogravitational Elliptic Restricted Three-Body Problem. *Bulletin of Astronic Society of India*, **28**, 579.

[14] Kunitsyn, A.L. (2001) The Stability of Collinear Libration Points in the Photogravitational Three-Body Problem. *Journal of Applied Mathematics and Mechanics*, **65**, 703-706. http://dx.doi.org/10.1016/S0021-8928(01)00075-2

[15] Zimovshchikov, A.S. and Tkhai, V.N. (2004) Instability of Libration Points and Resonance Phenomena in the Photogravitational Elliptic Restricted Three-Body Problem. *Solar System Research*, **38**, 155-164. http://dx.doi.org/10.1023/B:SOLS.0000022826.31475.a7

[16] Szenkovits, F. and Mako, Z. (2005) Publications of the Astronomy Department of Eotvos Lorand University, Budapest, 15, 221.

[17] Ammar, M.K. (2008) The Effect of Solar Radiation Pressure on the Lagrangian Points in the Elliptic Restricted Three-Body Problem. *Astrophysics and Space Science*, **313**, 393-408. http://dx.doi.org/10.1007/s10509-007-9709-z

[18] Kumar, S. and Ishwar, B. (2009) Solutions of Generalized Photogravitational Elliptic Restricted Three-Body Problem, *AIP Conference Proceedings*, **1146**, 456. http://dx.doi.org/10.1063/1.3183564

[19] Kumar, S. and Ishwar, B. (2011) Locations of Collinear Equilibrium Points in the Generalized Elliptic Restricted Three-Body Problem. *International Journal of Engineering*, *Science and Technology*, **3**, 157-162.

[20] Narayan, A. and Ramesh, C. (2011) Effects of Photogravitational and Oblantensss on the Triangular Lagrangian Points in the Elliptical Restricted Three Body Problem. *International Journal of Pure and Applied Mathematics*, **68**, 201.

[21] Narayan, A. and Ramesh, C. (2011) Stability of Triangular Equilibrium Points In Elliptical Restricted Three Body Problem under the Effects of Photogravitational and Oblateness of Primaries. *International Journal of Pure and Applied Mathematics*, **70**, 735.

[22] Singh, J. and Umar, A. (2012) Motion in the Photogravitational Elliptic Restricted Three-Body Problem under an Oblate Primary. *The Astronomical Journal*, **143**, 109. http://dx.doi.org/10.1088/0004-6256/143/5/109

[23] Singh, J. and Umar, A. (2012) On the Stability of Triangular Equilibrium Points in the Elliptic R3BP under Radiating and Oblate Primaries. *Astrophysics and Space Science*, **341**, 349-358. http://dx.doi.org/10.1007/s10509-012-1109-3

[24] Singh, J. and Umar, A. (2013) On "out of Plane" Equilibrium Points in the Elliptic Restricted Three-Body Problem with Radiating and Oblate Primaries. *Astrophysics and Space Science*, **344**, 13-19. http://dx.doi.org/10.1007/s10509-012-1292-2

[25] Singh, J. and Umar, A. (2013) Collinear Equilibrium Points in the Elliptic R3BP with Oblateness and Radiation. *Advances in Space Research*, **52**, 1489-1496. http://dx.doi.org/10.1016/j.asr.2013.07.027

[26] Singh, J. and Umar, A. (2013) Application of Binary Pulsars to Axisymmetric Bodies in the Elliptic R3BP. *Astrophysics and Space Science*, **348**, 393-402. http://dx.doi.org/10.1007/s10509-013-1585-0

[27] Singh, J. and Umar, A. (2014) On Motion around the Collinear Libration Points in the Elliptic Restricted Three-Body Problem with a Bigger Triaxial Primary. *New Astronomy*, **29**, 36-41. http://dx.doi.org/10.1016/j.newast.2013.11.003

[28] Arutyunyan, G.G., Sedrakyan, D.M. and Chubaryan, E.V. (1971) Rotating White Dwarfs in the General Relativity Theory. *Astrophysics*, **7**, 274.

[29] Papoyan, V.V., Sedrakyan, D.M. and Chubaryan, E.V. (1971) Newtonian Theory of Rapidly Rotating White Dwarfs. *Astrophysics*, **7**, 55.

[30] Laarakkers, W.G. (1999) Quadrupole Moments of Rotating Neutron Stars. *The Astrophysical Journal*, **512**, 282. http://dx.doi.org/10.1086/306732

[31] Shibata, M. (1998) Effects of the Quadrupole Moment of Rapidly Rotating Neutron Stars on the Motion of the Accretion Disks. *Progress of Theoretical Physics*, **99**, 69-78. http://dx.doi.org/10.1143/PTP.99.69

[32] Boshkayev, K., Quevedo, H. and Ruffini, R. (2012) Gravitational Field of Compact Objects in General Relativity. *Physical Review D*, **86**, Article ID: 064043.

[33] Heyl, J.S. (2000) Gravitational Radiation from Strongly Magnetized White Dwarfs. *Monthly Notices of the Royal Astronomical Society*, **317**, 310-314.

[34] Sharma, R.K. and Rao, P.V.S. (1976) Stationary Solutions and Their Characteristic Exponents in the Restricted Three-Body Problem When the More Massive Primary Is an Oblate Spheroid. *Celestial Mechanics*, **13**, 137-149. http://dx.doi.org/10.1007/BF01232721

[35] Elipe, A. and Ferrer, S. (1985) On the Equilibrium Solutions in the Circular Planar Restricted Three Rigid Bodies Problem. *Celestial Mechanics*, **37**, 59-70. http://dx.doi.org/10.1007/BF01230341

[36] Sharma, R.K., Taqvi, Z.A. and Bhatnagar, K.B. (2001) Existence and Stability of Libration Points in the Restricted Three-Body Problem When the Primaries Are Triaxial Rigid Bodies. *Celestial Mechanics and Dynamical Astronomy*, **79**, 119-133. http://dx.doi.org/10.1023/A:1011168605411

[37] Singh, J. and Begha, J.M. (2011) Stability of Equilibrium Points in the Generalized Perturbed Restricted Three-Body Problem. *Astrophysics and Space Science*, **331**, 511-519. http://dx.doi.org/10.1007/s10509-010-0464-1

[38] Singh, J. (2012) Motion around the Out-of-Plane Equilibrium Points of the Perturbed Restricted Three-Body Problem. *Astrophysics and Space Science*, **342**, 303-308. http://dx.doi.org/10.1007/s10509-012-1187-2