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Mathematical Model for Signal Control and State Monitoring in a Relay Interlocking System

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Abstract

This paper addresses the formalization of entrance and exit signal monitoring information when a relay interlocking system is integrated with the station duty officer's automated workstation (SDO AW). Discrete input signals, state codes, and the update function of a finite-state automaton are defined using the supplied state tables. The 16 entrance signal input combinations are classified into five states, and Boolean expressions are derived for these states. The exit signal table is shown to map eight combinations to only two states and to be incomplete with respect to the other state definitions. A procedure is proposed for processing information quality, timestamps, and the signal identifier together with the state code. Functional dependencies of entrance, exit, and shunting signals, route authorization conditions, the command execution confirmation loop, and verification scenarios are examined.

Keywords

Relay Interlocking, Entrance Signal, Exit Signal, Station Duty Officer's Automated Workstation, Finite-State Automaton, Boolean Function, Monitoring Information

1. Introduction

When relay interlocking equipment is connected to a computer interface, signal states must be identified unambiguously from feedback signals [1]. The indication displayed on SDO AW should represent the monitoring result received from the controlled object rather than merely indicate that a command has been sent. Distinguishing commands from indications forms the signal-related part of the closed command-execution-confirmation-indication loop.

2. Source Data and Modelling Method

For the entrance signal, the source specifies a lamp proving relay (*O*), a calling-on relay (*PS*), and red (*K*) and green (*Ya*) signals. For the exit signal, it specifies a lamp proving relay (*O*), a signal relay (*S*), and a shunting signal (*MS*) [1]-[3].

$$A_i = \left(Q_i, \Sigma_i, \delta_i\right), \ \ i \notin \left\{in, out\right\} \tag{1}$$

Here, Q_i is the set of states, Σ_i is the set of input combinations, and δ_i is the state update function. The model classifies current information as follows:

$$q_i\left(t_{k+1}\right) = \delta_i\left(q_i\left(t_k\right), x_i\left(t_{k+1}\right)\right) = f_i\left(x_i\left(t_{k+1}\right)\right) \tag{2}$$

If the input combination remains unchanged, the state is retained. When the combination changes, the new state is determined from the table. Such a transition represents a change in monitoring information; it does not replace the algorithm that authorizes signal clearance. The interlocking safety logic is retained separately.

3. Mathematical Model of Entrance Signal States

$$x_{in} = \left(O, PS, K, Ya\right) \notin \left\{0,1\right\}^4, \ \ \left|\Sigma_{in}\right| = 2^4 = 16 \tag{3}$$

The combinations are denoted by X_0 - X_{15}. The index is determined from the binary value of the signal vector:

$$j = 8O + 4PS + 2K + Ya \tag{4}$$

Table 1. Entrance signal input combinations and states.

Code	*O*	*PS*	*K*	*Ya*	State
X_0	0	0	0	0	S_4
X_1	0	0	0	1	S_1
X_2	0	0	1	0	S_1
X_3	0	0	1	1	S_1
X_4	0	1	0	0	S_1
X_5	0	1	0	1	S_1
X_6	0	1	1	0	S_1
X_7	0	1	1	1	S_1
X_8	1	0	0	0	S_4
X_9	1	0	0	1	S_2
X_{10}	1	0	1	0	S_1
X_{11}	1	0	1	1	S_5
X_{12}	1	1	0	0	S_3
X_{13}	1	1	0	1	S_5
X_{14}	1	1	1	0	S_3
X_{15}	1	1	1	1	S_5

The states are defined as follows: S_1, red; S_2, green; S_3, red and white; S_4, no information; and S_5, an inadmissible combination. The original designation of S_5 as "non-existent" does not mean that the signal is physically absent. In this paper, it is interpreted as a diagnostic state indicating an input combination not admitted by the model [4]. **Table 1** provides the complete mapping.

Grouping the combinations in **Table 1** yields an indicator function for each state. $C_i = 1$ means that the current combination belongs to state S_i. The symbols $\neg$, $\wedge$, and $\vee$ denote logical NOT, AND, and OR, respectively.

$$C_1 = \left(\neg O \wedge \left(PS \vee K \vee Ya\right)\right) \vee \left(O \wedge \neg PS \wedge K \wedge \neg Ya\right) \tag{5}$$

$$C_2 = O \wedge \neg PS \wedge \neg K \wedge Ya \tag{6}$$

$$C_3 = O \wedge PS \wedge \neg Ya \tag{7}$$

$$C_4 = \neg PS \wedge \neg K \wedge \neg Ya \tag{8}$$

$$C_5 = O \wedge Ya \wedge \left(PS \vee K\right) \tag{9}$$

C_1 covers X_1 - X_7 and X_{10}; C_2 equals one only for X_9; C_3 for X_{12} and X_{14}; C_4 for X_0 and X_8; and C_5 for X_{11}, X_{13}, and X_{15}. Each combination in the table belongs to exactly one state [5] [6]:

$$C_1 \vee C_2 \vee C_3 \vee C_4 \vee C_5 = 1;\ C_i \wedge C_j = 0\ \ (i \neq j) \tag{10}$$

Figure 1 illustrates the entrance signal state identification rule. The next state is determined by the current input group, independently of the previous state. Arrow colours identify the destination states and their associated input groups.

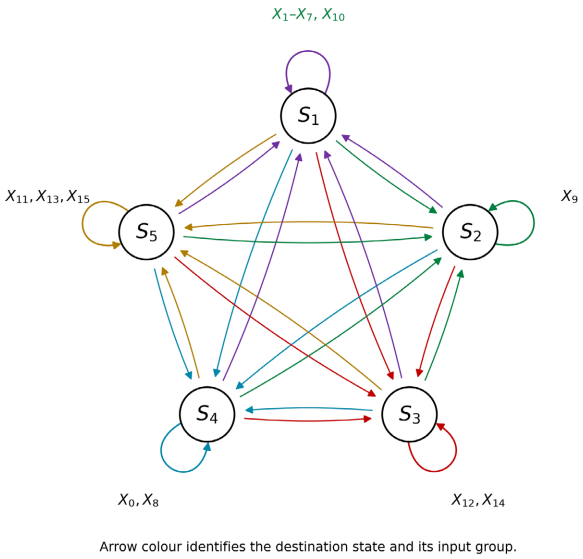


Figure 1. Mapping entrance signal input combinations to states.

For example, receiving X_9 = (1, 0, 0, 1) identifies S_2. If K changes from 0 to 1, X_{11} = (1, 0, 1, 1) is obtained and the transition $S_2 \rightarrow S_5$ is recorded. This represents a transition from the state designated as permissive to a diagnostic state. When Ya changes from 1 to 0, X_{10} = (1, 0, 1, 0) is obtained and the automaton identifies S_1. These examples are table-based calculations rather than physical operational tests.

4. Exit Signal Model and Data Consistency

$$x_{out} = (O, S, MS) \in \{0,1\}^3; \ |\Sigma_{out}| = 8 \tag{11}$$

For the exit signal, the states are defined as S_1, red; S_2, white; S_3, extinguished; S_4, flashing white; and S_5, a non-existent state (Table 2).

Table 2. Mapping actually specified in the exit signal table.

Combinations	Logical condition	State
X_0	$O = 0$, $S = 0$, $MS = 0$	S_4
X_1 - X_7	$O \vee S \vee MS = 1$	S_1
Not specified	No combination specified in the table	S_2, S_3, S_5

$$C_{out,4} = \neg O \wedge \neg S \wedge \neg MS; \quad C_{out,1} = O \vee S \vee MS \tag{12}$$

5. Signal Monitoring on SDO AW

The signal state is associated with an object identifier. A monitoring record may be structured as follows [7] [8]:

$$R_{sig} = \left(\text{Object}_{\text{ID}}, \text{Object}_{\text{Type}}, \text{State}, \text{Timestamp}, \text{Quality}\right) \tag{13}$$

$\text{Object}_{\text{ID}}$ identifies the particular signal; $\text{Object}_{\text{Type}}$ specifies its type; State contains the calculated state; Timestamp records the time of the monitoring information; and Quality indicates data quality. Here, R_{sig} is a monitoring record and is not equivalent to a control authorization in the interlocking system.

Information validity is assessed separately using $v(t)$. It jointly represents communication availability, reception of signals within a consistent time interval, and compliance with the specified data freshness requirement. The freshness threshold must be established from the requirements of the actual system. The indication rule is written as follows:

$$I_{sig}(t) = f_i(x_i(t)) \text{ if } v(t) = 1; \quad \text{unknown if } v(t) = 0 \tag{14}$$

The "unknown" indication denotes a communication or data quality problem. In contrast, S_4 in the entrance model is a table-defined state associated with particular zero-valued input combinations. Keeping these concepts separate prevents valid zero signals from being confused with communication loss. When S_5 is received, the combination, signal identifier, and timestamp are recorded in the diagnostic log [9]-[11].

The monitoring process comprises receiving signal feedback, associating it with the object, checking time and quality, identifying the state using the relevant table, distinguishing diagnostic states, and updating the indication and log. The outcome of a control command is assessed from confirmed feedback received from the signal. Transmission of a command does not establish that it has been executed.

5.1. Functional Signal States and Operational Dependencies

The functional signal model relates the current state to the operator command,

route, points, track sections, and interlocking conditions [12] [13]. This approach supplements the monitoring decoder with a control model: the decoder identifies the monitoring information received, whereas the functional model determines which signal state may be established under the specified conditions [14] [15]. These two tasks should be represented by separate mathematical functions.

$$S_{sig}(t+1) = F_{sig}\left[S_{sig}(t), U_{sig}(t), R(t), J(t), W(t), B(t)\right] \quad (15)$$

Here, S_{sig} is the signal aspect, U_{sig} is the control command, R indicates route selection, J represents the required and proved point positions, W describes the relevant track section states, and B represents safety and interlocking conditions. F_{sig} expresses the operational logic of the existing interlocking system. It must not be equated with a function that changes graphical symbols on the workstation.

The functional states of entrance, exit, and shunting signals are summarized in Table 3. The entrance decoder codes S_1 - S_5 are not identical to the set of functional signal aspects: for example, the decoder includes diagnostic and no-information states, while the functional model distinguishes yellow aspects. Their complete correspondence is established using the signalling and monitoring circuitry [5].

Table 3. Comparison of the functional signal models in the article.

Object	Functional states	Dependencies checked
Entrance signal	Red; yellow; green; two yellow lights; lunar white in a special calling-on mode	Reception route, track vacancy, point proving and locking, conflicting movements
Exit signal	Red; yellow; green; lunar white in shunting mode	Departure route, block conditions, point and track monitoring
Shunting signal	Blue; lunar white	Shunting route, track and point monitoring, interlocking

An entrance signal may display the relevant permissive aspect when the train reception route is ready. Selection of yellow or green is associated with the conditions for further movement, while two yellow lights relate to a separate reception direction. The lunar-white calling-on signal does not replace the permissive aspect of an ordinary reception route; its conditions are treated as a special mode. This distinction must also be maintained in the command type and outcome indication on the workstation.

For an exit signal, departure from a station track to the block section is assessed jointly with route readiness and block information. Lunar white belongs to a separate shunting mode and must not be confused with the green or yellow aspect of a train departure route. For a shunting signal, lunar white authorizes shunting movement and blue prohibits it [3].

The combined vector represents signals with different functions within one information structure without making their aspect meanings identical. The restrictive aspect of entrance and exit signals is red, whereas that of a shunting signal is blue. A permissive aspect is not established when the operational conditions are

unconfirmed. If communication is lost, the workstation does not assume that the physical signal is red; instead, the interface displays "monitoring state unknown".

5.2. Closed Loop for Confirming Signal Command Execution

The operator request, adaptation of the command to the interlocking interface, execution, feedback, and workstation indication are treated as separate stages of one process. A signal request is represented as follows [8] [9]:

$$C_j = \left(ID_j, O_j, A_j, t_{0,j}\right);\ U_j = T_k\left(C_j\right) \tag{16}$$

ID_j is the request identifier, O_j is the signal or its associated route, A_j is the required action, and $t_{0,j}$ is the request time. T_k is the adaptation function for an interlocking interface of type k. If the relay interface does not transmit the identifier, ID_j is not interpreted as an internal interlocking signal; it links log records in the workstation and integration facility.

$$Z_i = H_k\left(Y_i, t_{\gamma,i}, \kappa_i\right) = \left(O_i, \hat{S}_i, t_{\gamma,i}, \kappa_i\right) \tag{17}$$

Y_i is the raw feedback signal, H_k converts it into an object state, $\hat{S}_i$ is the identified state, $t_{\gamma,i}$ is the recording time, and κ_i is information quality. The quality indicator κ_i is distinguished from the automaton state $q_i(t)$ in Equation (2). Separate State and Quality fields are used in the software implementation. Figure 2 shows the command and feedback paths.

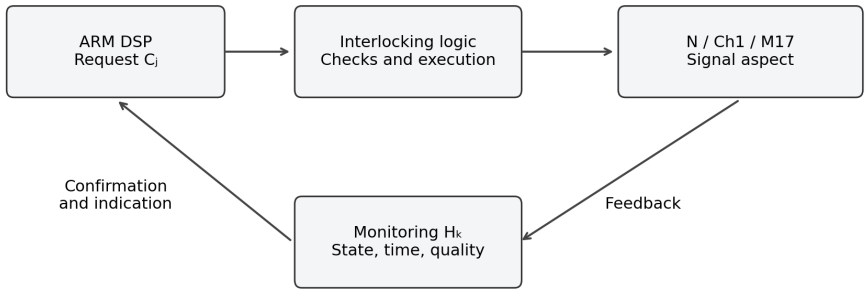


Figure 2. Closed loop for signal commands and state monitoring.

Table 4. Indication of signal control request outcomes.

Outcome	Assessment condition
Confirmed	The required state is proved by valid feedback and matches the current request
Rejected	A rejection response is received from the interlocking for the specific request
Pending	The request is accepted; confirmation has not yet arrived within the specified interval
Unconfirmed	The time limit has expired, or feedback is absent, stale, contradictory, or inconsistent with the request

Table 4 presents the command outcome separately from the physical signal aspect. For example, "pending" is a state of the operator request rather than a new

signal colour. This distinction prevents command acceptance from being confused with command execution in the interface.

$$P_{conf,j} = \left[\hat{S}_i = S_{req,i}\left(C_j\right)\right] \wedge Q_{valid,i} \wedge M\left(O_i, C_j\right) \tag{18}$$

Here, $\hat{S}_i$ is the identified feedback state of object i, $S_{req,i}(C_j)$ is the state required for that object by request C_j, and $Q_{valid,i}$ indicates that the feedback is valid and current.

The matching condition M checks the object address, required state, and assessment interval of the current request. If the relay interface does not return a command ID, temporal proximity alone is insufficient to associate execution with that command. The sequence of outstanding requests for one object must be monitored, and an old message must not be used to confirm a new request. Loss of feedback alone is not classified as "rejected"; without a rejection response, the outcome is "unconfirmed".

5.3. Model Verification Scenarios and Timing Criteria

Under the modelling methodology, signal monitoring is checked through route selection, identification of operational conditions, recording of initial states, submission of the request, observation of execution and feedback, and comparison of indications [5] [8] [9]. The verification scenarios and expected monitoring outcomes are summarized in Table 5.

Table 5. Verification scenarios for the signal model.

Scenario	Input situation	Expected monitoring outcome
Normal reception	Route conditions confirmed; valid feedback	The N signal state is displayed from feedback; the request is confirmed
Occupied section	A relevant section is occupied or its monitoring is faulty	Normal reception authorization is not established; readiness is not confirmed
Point proving lost	The required position is not proved	The condition for a permissive aspect is recorded as unmet
Conflicting route	A conflicting opposing movement exists	A new ordinary movement authorization is not established
Delayed confirmation	Feedback has not arrived within the specified interval	Initially pending; unconfirmed after the time limit
Communication loss	Invalid monitoring information	State indicated as unknown; the last colour is not treated as current confirmation
Contradictory combination	X_{11}, X_{13}, or X_{15} in the entrance decoder	S_5 diagnostic state and log entry
Shunting mode	Separate shunting conditions for a shunting or exit signal	Lunar-white proving is not equated with train movement authorization

$$T_j = t_{ind,j} - t_{0,j} \tag{19}$$

$$T_j = T_{adapt\text{-}transmit,j} + T_{IL\text{-}feedback,j} + T_{indication,j} \tag{20}$$

$t_{ind,j}$ is the time at which the confirmed outcome is displayed on the workstation. The components are calculated from sequential, non-overlapping log intervals. The operator's decision time before the request is not automatically included in this sum. The outcome type is recorded separately for rejected or unconfirmed requests; these requests are not included in the statistics of confirmed execution times.

6. Logical Verification Results and Discussion

An exhaustive combinational check was performed for the entrance signal: each of the 16 binary vectors was evaluated using Equations (5)-(9) and compared with Table 1. The results are summarized in Table 6. This calculation establishes internal model consistency; it does not measure the reliability or safety level of the actual equipment [8] [9].

Table 6. Distribution of combinations in the entrance model.

State	Combinations	Count
S_1	X_1 - X_7, X_{10}	8
S_2	X_9	1
S_3	X_{12}, X_{14}	2
S_4	X_0, X_8	2
S_5	X_{11}, X_{13}, X_{15}	3
Total	X_0 - X_{15}	16

The calculated combinations are mutually exclusive and cover the entire input domain. The distinction between functional aspects and monitoring codes is important: diagnostic state S_5 must not be interpreted as a fifth physical signal aspect. The functional model represents operational dependencies, whereas the decoder interprets feedback information. In the corrected entrance signal graph (Figure 1), the input group associated with S1 is X1 - X7 and X10; X12 belongs to S3.

The exit model table contains all eight combinations but covers only states S_1 and S_4. Thus, completeness of the combinations does not imply completeness of the state definitions. The practical outcome of the logical verification is a consistent rule for the entrance signal decoder and identification of the exit signal dependencies that require clarification.

7. Conclusions

Information relating to entrance, exit, and shunting signals was extracted, and separate mathematical expressions were formulated for entrance and exit signals. For the entrance signal, 16 combinations were mapped unambiguously to five states; completeness and mutual exclusivity of the state indicators were verified by combinational calculation.

The exit-table incompleteness is a substantive limitation of the supplied data

and remains honestly stated. Resolving it requires the author-approved exit signal relay truth table; no unsupported new states or experimental claims were added.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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