Geomaterials, 2026, 16(4), 159-177
https://www.scirp.org/journal/gm
ISSN Online: 2161-7546
ISSN Print: 2161-7538

Hydraulic Conductivity Estimation for Non-Plastic Tailings-Derived Silty Sands

Hans Rönnqvist*

RQV Teknik AB, Hudiksvall, Sweden
Email: *hans.ronnqvist@rqv.se

How to cite this paper: Rönnqvist, H. (2026) Hydraulic Conductivity Estimation for Non-Plastic Tailings-Derived Silty Sands. *Geomaterials*, **16**, 159-177.
https://doi.org/10.4236/gm.2026.164009

Received: June 28, 2026
Accepted: August 31, 2026
Published: September 3, 2026

Abstract

Classical empirical hydraulic-conductivity equations were developed mainly for cleaner granular soils than the fines-influenced materials often encountered in tailings settings. This study evaluates such equations for non-plastic tailings-derived silty sands. Two dataset levels are distinguished: an expanded dataset comprising all 14 tested materials from 6 source groups, and a restricted subset comprising 8 materials from 4 source groups that define the main low- to medium-fines recommendation domain. In this restricted subset, where fines contents are below about 30%, the particle-size distributions resemble ordinary non-plastic silty sands in engineering terms, suggesting broader synergistic effects than merely restricted to the tailings-derived origin. In the expanded dataset, Sherard emerged as the strongest gradation-based method. Within the restricted subset, Slichter emerged as the strongest porosity-dependent method in both original and calibrated form. This distinction is consistent with the underlying formulation of the methods: gradation-based equations implicitly represent a typical medium-dense to dense packing state, whereas porosity-dependent equations explicitly account for densification. Coefficient-only calibration improved both methods at the standard interpreted conductivity target, k_{std}, but Sherard calibration proved more domain-bounded, whereas Slichter transferred more favourably under boundary checking. The results support a domain-bounded recommendation: Sherard as the preferred gradation-based method and Slichter as the preferred porosity-dependent method for low- to medium-fines non-plastic tailings-derived silty sands.

Keywords

Hydraulic Conductivity, Tailings, Silty Sand, Empirical Equations, Calibration, Permeability

1. Introduction

Hydraulic conductivity is ideally measured directly, but empirical equations still play an important role when engineering judgement must be made before representative permeability results are available. This is especially true in dam engineering, mine-water applications, and preliminary filter or tailings assessment, where particle-size distribution and density-related descriptors are often available earlier than reliable laboratory conductivity data.

In this paper, hydraulic conductivity, k, is used in the strict Darcy sense, while recognizing that the term permeability is often used more loosely in engineering practice. A broad family of empirical equations is available, ranging from simple characteristic-size relations to forms that also incorporate porosity or grading shape. Well-known examples include Hazen [1], Slichter [2], Kozeny [3], Carman [4], Beyer [5], Shepherd [6], Alyamani and Sen [7], Sherard [8] and Chapuis [9], together with later reassessments [10]-[13].

Tailings are man-made particulate deposits produced during mineral processing, after the economically valuable fraction has been separated from the ore. Unlike many natural sands, tailings are shaped by crushing, grinding, hydraulic transport, and deposition processes rather than by purely geological weathering and sedimentation. As a result, they often show mineralogically controlled particle densities that differ from those of ordinary quartz-dominated soils. These features make it difficult to assume that classical empirical conductivity equations, many of which were developed for cleaner granular soils, remain directly applicable.

The present dataset represents a restricted subset of such materials: non-plastic tailings-derived silty sands. They are finer and more fines-influenced than conventional clean filter sands, yet they are not clayey tailings or highly plastic fine-grained soils. In engineering terms, they appear close to the fines-rich end of filter behaviour. A useful parallel is provided by Vaughan and Soares [14], who showed that filters designed to retain clay in flocculated form could become finer than those commonly accepted in dam practice, and that their behaviour could be described more generally by permeability than by grading alone. That perspective is relevant here, because the materials considered in the present study sit near this fines-rich transition zone between conventional filter thinking and broader permeability-controlled behaviour.

A further complication is that measured hydraulic conductivity is not always a single unique value. Recent long-duration permeability work indicates that a material may reasonably be described by more than one conductivity target, including an initial conductivity, k_{in}, a standard interpreted conductivity, k_{std}, and a final conductivity after extended hydraulic conditioning, k_{end}. In parallel, the broader comparison of empirical hydraulic-conductivity equations across several practical material domains [15] showed that tailings form a distinct material class and that method performance differs markedly from that observed for filter materials and tills. In that broader study, based on a smaller tailings subset, Hazen emerged as the strongest gradation-based method and Kozeny-Carman as the strongest porosity-dependent method. The present study builds directly on that broader comparison, but focuses specifically on the tailings-derived silty-sand domain, where the expanded dataset and the more restricted recommendation subset lead to a different final recommendation. It also draws on the robustness framework proposed by [16] for interpreting the stability of fitted relationships from limited geotechnical datasets. A parallel study on Swedish tills [17] is being developed separately, but the present manuscript is intentionally framed around a different engineering-material class and a different practical problem.

The purpose is not to introduce a universal new conductivity equation for all tailings, but to determine which existing classical equations are least unsuitable for a restricted domain of non-plastic tailings-derived silty sands, whether conservative coefficient-only calibration is worthwhile, how robust the resulting recommendations are, and whether simple practitioner aids can be developed for preliminary use when direct material data are incomplete. More specifically, the study compiles and describes a tailings-derived silty-sand dataset, identifies the best-performing gradation-based and porosity-dependent methods for the represented domain, assesses whether conservative coefficient-only calibration improves performance, evaluates robustness through leave-one-out and leave-one-source-material-out checks, and develops a bounded fines-based aid for estimating D10 and D15 when the fine tail of the PSD is incomplete.

2. Materials and Dataset

2.1. Tailings-Derived Material Dataset

The compiled dataset comprises 14 tested tailings-derived materials originating from several different mining operations and ore types, including iron ore, gold, copper-bearing ore, and complex sulfide ore. The source structure is important because several tests were performed on the same source material. In the expanded dataset, D, O, and P originate from the same source; L1 and L2 from the same source; Z1 and Z2 from the same source; AA1 and AA2 from the same source; and CC1-CC4 from the same source. The dataset therefore contains repeated tests on a smaller number of distinct source materials, and this distinction is important in both interpretation and robustness assessment.

The full dataset spans fines contents < 0.063 mm from 9.4% to 59.2%, particle density from 2.79 to 3.96 Mg/m^3, porosity from about 0.40 to 0.57, and measured hydraulic conductivity from about 8×10^{-7} to 3×10^{-5} m/s depending on test stage. Because the finer CC and L materials extend well beyond the main engineering cluster, the main recommendation domain is tied instead to a restricted subset of 8 materials from 4 source groups, with fines contents from 9.4% to 28.3%. The restricted subset was defined based on fines content and overall clustering of the dataset, with inclusion limited to materials representing the main low- to medium-fines domain. This subset was defined prior to detailed method ranking and calibration in order to avoid biasing the selection toward specific model outcomes.

Table 1 summarizes the material data used in the study. The compiled material data include particle density, dry density, maximum dry density (MDD), relative compaction, porosity, void ratio, fines content, characteristic grain sizes, and hydraulic conductivity. Particle density was determined according to SS-EN ISO 17892-3 [18] and was used together with dry density to derive porosity and void ratio where relevant. Where modified Proctor data were available, MDD was determined from ASTM D1557 [19] and used together with dry density to calculate relative compaction (RC). Conductivity targets are represented by k_{in}, k_{std}, and k_{end}, depending on data availability.

Table 1. Material properties and hydraulic conductivity data for the full dataset.

Material ID	Source	Particle density, ρ_s (Mg/m³)	Dry density, ρ_d (Mg/m³)	MDD (Mg/m³)	Relative compaction, RC (%)	Porosity, n (−)	k_{in} (m/s)	k_{std} (m/s)	k_{end} (m/s)	Fines <0.063 mm (%)	Cu (−)	D10 (mm)	D15 (mm)	D20 (mm)	D50 (mm)	D60 (mm)
D_Tailings	DOP	2.834	1.54	1.79	85.8	0.458	1.43E−05	1.05E−05	2.55E−06	21.4	7.2	0.024	0.036	0.057	0.137	0.173
O_Tailings	DOP	2.834	1.59	1.79	89.0	0.438	5.57E−06	6.25E−06	1.49E−06	22.3	7.4	0.023	0.035	0.052	0.135	0.17
P_Tailings	DOP	2.834	1.66	1.79	92.7	0.414	4.56E−06	4.68E−06	2.03E−06	19.9	7.1	0.0255	0.039	0.064	0.145	0.18
L1_Tailings	L	3.130	1.83	2.21	82.6	0.417	1.25E−05	1.21E−05	8.38E−06	33.9	16.5	0.0112	0.0198	0.03	0.125	0.185
L2_Tailings	L	3.130	1.67	2.21	75.7	0.466	7.90E−06	5.72E−06	9.31E−06	33.6	16.4	0.011	0.0205	0.031	0.125	0.18
Z1_Tailings	Z	3.955	1.99	2.32	85.6	0.498	6.38E−06	6.73E−06	1.52E−05	27.3	4.9	0.042	0.052	0.057	0.15	0.206
Z2_Tailings	Z	3.955	1.96	2.32	84.7	0.503	1.34E−05	1.36E−05	7.91E−06	28.3	4.8	0.0405	0.0508	0.0553	0.142	0.193
AA1_Tailings	AA	2.793	1.51	1.83	82.7	0.458	8.85E−06	8.74E−06	2.84E−06	20.3	6.5	0.03	0.041	0.062	0.155	0.195
AA2_Tailings	AA	2.793	1.19	1.83	65.2	0.573	3.26E−05	3.26E−05	2.70E−06	18.5	6.5	0.031	0.045	0.066	0.16	0.2
BB_Tailings	BB	3.060	1.82	2.29	79.7	0.404	1.60E−05	2.54E−05	2.46E−05	9.4	4.3	0.064	0.082	0.104	0.222	0.277
CC1_Tailings	CC	2.960	1.29	–	–	0.566	6.34E−06	–	–	52.5	14.0	0.0055	0.009	0.014	0.058	0.077
CC2_Tailings	CC	2.960	1.36	–	–	0.542	4.24E−06	4.50E−06	3.71E−06	59.2	14.4	0.0045	0.0078	0.0115	0.046	0.065
CC3_Tailings	CC	2.878	1.66	–	–	0.422	7.75E−07	7.76E−07	7.56E−07	38.4	12.5	0.0083	0.0136	0.02	0.083	0.104
CC4_Tailings	CC	2.960	1.62	–	–	0.451	1.11E−06	1.09E−06	1.10E−06	44.7	13.6	0.0066	0.0114	0.0166	0.071	0.09

2.2. The Material Domain

The represented materials are not treated as a single geological family. Instead, the investigated domain is defined primarily in engineering terms as non-plastic tailings-derived silty sands within the gradation, fines-content, porosity, and hydraulic-conductivity range represented by the dataset. The main recommendation domain is tied to the restricted subset, where fines contents range from about 9.4% to 28.3%. This restricted subset is best described as a low- to medium-fines tailings-derived silty-sand domain.

In engineering terms, the restricted subset can also be interpreted tentatively as a fines-influenced extension of the granular filter domain, or as non-plastic silty sands that are close to filter materials but contain too much fines to be treated as conventional clean filters. This interpretation is supported by comparison with the broader permeability paper, where the filter materials showed lower particle densities, approximately 2.69 to 2.86 Mg/m^3, lower fines contents, approximately 0.5% to 11.1%, and porosities ranging from about 0.20 to 0.47. Relative to that filter domain, the present tailings-derived materials extend toward higher particle density, higher fines content, and locally higher porosity.

A useful parallel is provided by [14], who showed that filters designed to retain clay in flocculated form could become finer than those commonly accepted in dam practice, and that their retention behaviour could be described more generally by permeability than by grading alone. The restricted tailings-derived silty-sand domain considered here appears close to this fines-rich end of filter behaviour.

Although Atterberg limits were not determined, the restricted subset showed very low clay-size contents < 0.002 mm, approximately 0.3% to 1.9%, which is consistent with its classification as non-plastic. Same source materials also exhibited pronounced collapsibility behaviour in sand castle tests [20]. Taken together, these observations support treatment of the restricted subset as a coherent density-sensitive engineering class. The designation "non-plastic" is therefore used here as an operational engineering description based on low clay-size content and observed material behaviour, rather than on direct Atterberg limit classification.

2.3. Available Descriptors

The materials are described by fines content, particle density, dry density, maximum dry density, relative compaction, porosity, measured hydraulic conductivity, uniformity coefficient, and characteristic grain sizes including D10, D15, D20, D50, and D60. These descriptors, together with the available conductivity targets, are summarized in **Table 1**.

2.4. Conductivity Targets

There are three conductivity targets: k_{in}, the initial measured conductivity; k_{std}, a standard interpreted conductivity; and k_{end}, the late-stage or end-of-test conductivity. The distinction between these targets and their interpretation follows the broader permeability paper and the related long-duration conductivity work reported by [15]. The present paper retains k_{std} as the primary target for recommendation, since this gives the clearest common basis for comparing original and calibrated performance across the retained methods. The number of observations differs between conductivity targets. The full dataset contains $n = 14$ observations for k_{in}, and $n = 13$ for k_{std}, and k_{end}, respectively, with smaller corresponding subsets for the restricted domain.

3. Retained Empirical Methods

3.1. Method Selection

A broad screening of empirical conductivity equations was first carried out in the companion permeability paper [15]. In that multi-domain study, Hazen was the strongest gradation-based method for the smaller tailings subset, and Kozeny-

Carman the strongest porosity-dependent method. In the expanded tailings-focused dataset used here, Sherard showed the lowest typical factor error among the gradation-based methods, while Slichter outperformed the porosity-dependent alternatives both in original form and after calibration. Hazen and Kozeny-Carman are nevertheless retained as important comparators and continuity links to the broader permeability paper. Alyamani and Sen also performed strongly within the restricted subset, but its ranking was influenced by sensitivity to the D50 value of material P, making it less compelling as the basis for a final practical recommendation. The shift in preferred methods is therefore interpreted as a refinement due to improved domain coverage and increased dataset size, rather than a contradiction between studies. The methods taken forward for detailed analysis are thus Sherard and Hazen on the gradation-based side, and Slichter and Kozeny-Carman on the porosity-dependent side. Table 2 summarizes these methods.

3.2. Retained Methods and Implemented Working Forms

The retained methods are summarized in Table 2.

Table 2. Retained empirical hydraulic-conductivity equations and their role in the study.

Method	Method class	General form	Key input variables	Original coefficient. C_0	Role in this study	Reference
Hazen	Gradation based	$k \propto D10^2$	D10	0.0116	Close-performing gradation-based comparator in the tailings-derived silty-sand dataset; retained for comparison with Sherard and for continuity with the broader permeability paper.	Hazen
Sherard	Gradation based	$k \propto D15^2$	D15	0.0035	Best-performing gradation-based method in the expanded tailings-derived silty-sand dataset; retained as the preferred gradation-based method for the restricted low- to medium-fines domain, with detailed robustness assessment and coefficient calibration.	Sherard
Slichter	Porosity dependent	$k \propto n^{3.287} D10^2$	D10, n	0.0981	Best-performing porosity-dependent method in the restricted subset, in both original and calibrated form; retained as the preferred porosity-dependent method for the restricted low- to medium-fines domain.	Slichter

Continued

Kozeny-Carman	Porosity dependent	$k \propto n^3/(1-n)^2 D10^2$	D10, n	0.0545	Strong porosity-dependent comparator; retained for detailed evaluation, coefficient calibration, and continuity with the broader permeability paper.	Kozeny-Carman

Hazen is a classical D10-based relation that remains widely used for relatively clean granular materials. In the present study it is retained mainly as a close gradation-based comparator and as the method that provided continuity with the smaller tailings subset in the broader permeability paper. In implemented working form,

$$k = 0.0116 D_{10}^2$$

Sherard is a gradation-based relation built around D15. In the present study it emerged as the best-performing gradation-based method in the expanded tailings-derived silty-sand dataset and is retained as the preferred gradation-based method for the restricted low- to medium-fines domain. In implemented working form,

$$k = 0.0035 D_{15}^2$$

Slichter is an early porosity-dependent relation that combines D10 with porosity, n. In the present study it became the strongest porosity-dependent method within the restricted subset, in both original and calibrated form. In implemented working form,

$$k = 0.0981 n^{3.287} D_{10}^2$$

Kozeny-Carman is a classical porosity-dependent relation that combines D10 with a porosity function. It remains useful here as a strong comparator and continuity link to the broader permeability paper. In implemented working form,

$$k = 0.0545 \left[\frac{n^3}{(1-n)^2} \right] d_{10}^2$$

4. Methodology

4.1. Evaluation Framework

Method performance is assessed by comparing predicted hydraulic conductivity with measured hydraulic conductivity. To retain continuity with the broader permeability paper [15], performance is summarized primarily by the typical factor error, E_f. The original value before calibration is denoted $E_{f,0}$.

The typical factor error may be interpreted as the characteristic multiplicative mismatch between predicted and measured conductivity. For example, $E_f = 2$ corresponds to a typical factor-of-two difference. The present paper focuses on practical interpretation of E_f, and intermediate log-ratio statistics are not reported separately except where needed for methodological definition. The typical factor error is defined as

$E_f = 10^{\text{mean}\left(\left|\log_{10}\left(k_{pred}/k_{measured}\right)\right|\right)}$ representing the characteristic multiplicative deviation between predicted and measured conductivity. Calibration was performed by selecting the coefficient C that minimizes E_f. The typical factor error approach and its implementation are described in [15].

4.2. Selection of Best-Performing Methods

Candidate equations were first screened in original form. The expanded-dataset ranking showed that Sherard was the strongest gradation-based method, with Hazen remaining a very close second. For the full dataset, Kozeny-Carman was the strongest porosity-dependent method, whereas in the restricted subset Slichter became the strongest porosity-dependent method.

Because the expanded dataset includes much finer materials than the original subset, performance is interpreted in two layers: the full dataset is retained as a sensitivity and boundary check, while the restricted subset provides the main domain for recommendation.

Given that several specimens originate from the same source material, method screening and calibration were also checked against source-group influence. A comparison based on source-group averaging showed that the method ranking remained consistent, indicating that the results are not dominated by sources with multiple specimens.

4.3. Coefficient-Only Calibration

Calibration, where tested, was restricted to the multiplicative empirical coefficient C while retaining the original functional form of the method. This is the least invasive way to test whether the represented domain requires a different conductivity scale without altering the structure of the original equation.

Calibration was assessed separately against k_{in}, k_{std}, and k_{end}, and performance was compared using typical factor error. The resulting calibration response, including the original coefficient C_0, the best-performing calibrated coefficient C_{cal}, the original typical factor error $E_{f,0}$, the calibrated typical factor error E_f, and the normalized response $\frac{E_f}{E_{f,0}}$, is presented in Section 5.4.

4.4. Robustness Checks

Robustness was evaluated using both leave-one-test-out and leave-one-source-material-out analysis. The purpose was to determine whether the retained recommendations were controlled by individual specimens or by particular source-material groups.

Because several tests originate from the same source material, leave-one-out alone is not sufficient. The preferred hierarchy is therefore: (1) leave-one-test-out, and (2) leave-one-source-material-out, which, in practice, becomes leave-one-

mine-out. This logic is consistent with the broader interpretation framework developed in the parallel robustness paper.

The resulting robustness response, including the sensitivity of the original and calibrated forms to omission of individual tests and source-material groups, is presented in Section 5.5.

5. Results

5.1. Dataset Characteristics

Figure 1 shows the particle-size distributions for the represented tailings-derived silty-sand dataset. Table 1 summarizes the material data.

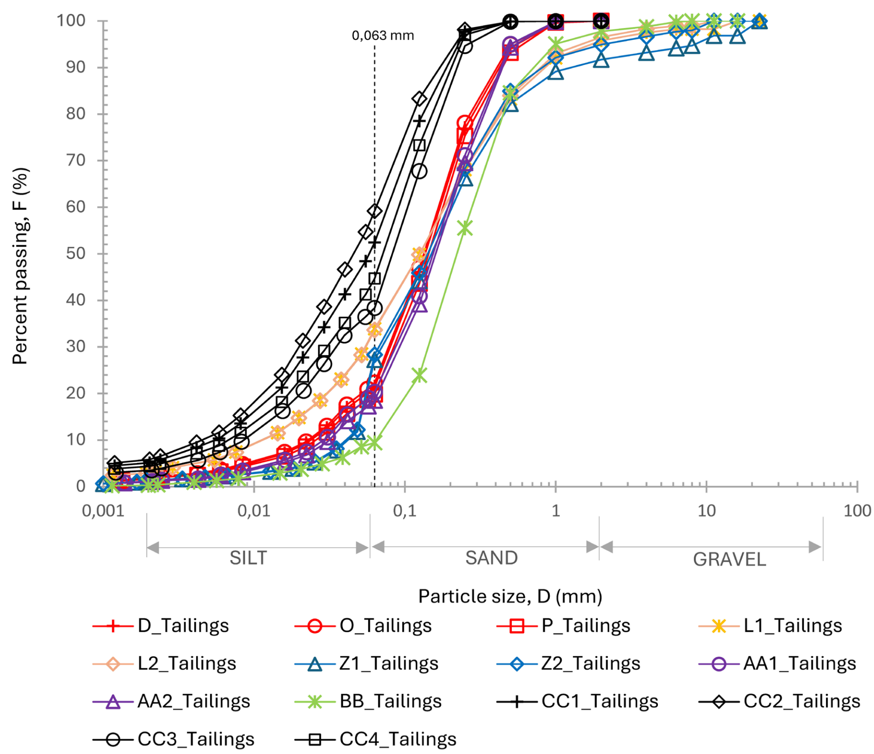

Figure 1. Particle-size distributions for the represented tailings-derived silty-sand dataset.

A distinguishing feature of the represented materials is their, for some materials, comparatively high particle density relative to ordinary sands. In the current dataset, particle density ranges approximately from 2.79 to 3.96 Mg/m^3. This contributes to relatively high porosities even at moderate dry densities, including values up to about 0.57 in the restricted subset. Relative to the filter domain in the broader permeability paper, the present materials therefore combine higher particle density, higher fines content, and locally higher porosity.

5.2. Screening and Retained Methods

Figure 2 summarizes the screening of candidate methods. The figure confirms Sherard as the strongest gradation-based method in the expanded dataset. Within the restricted subset, Alyamani and Sen also performs strongly, ranking ahead of Hazen at the standard interpreted conductivity target, but this response is influenced by sensitivity to the D50 value of material P. Sherard therefore remains the preferred gradation-based method overall.

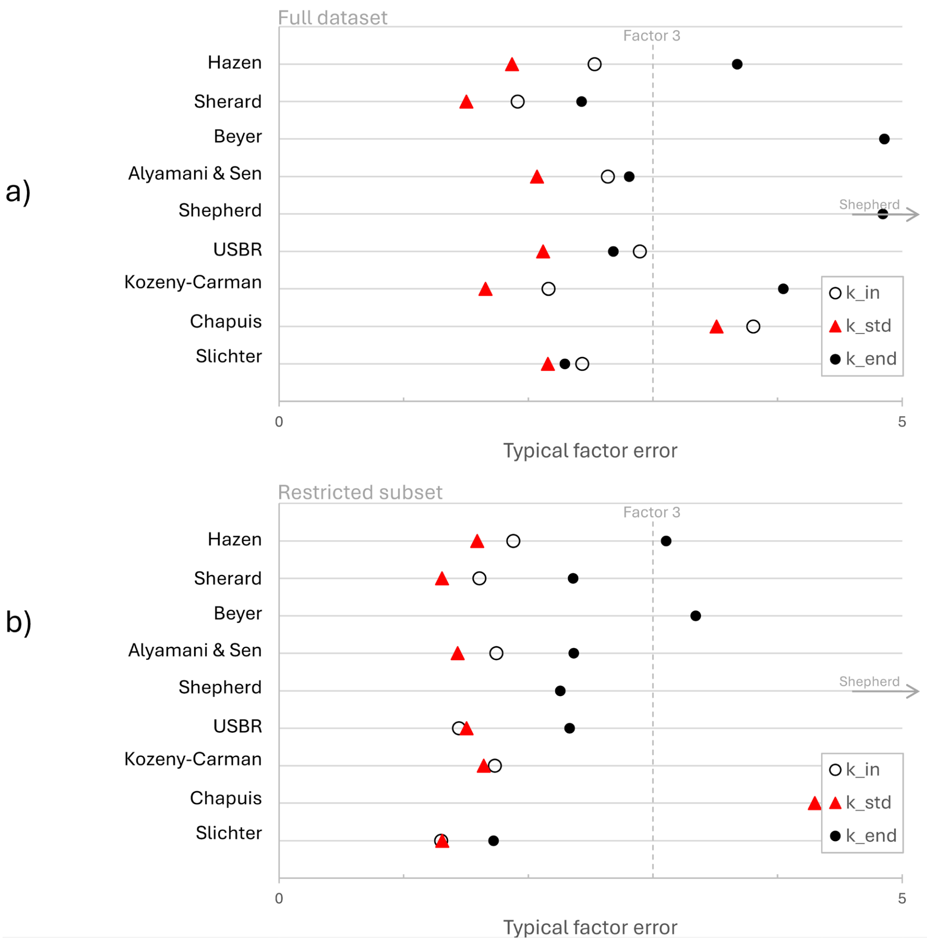

Figure 2. Screening of the empirical hydraulic-conductivity equations considered for the represented tailings-derived silty-sand dataset, in terms of a) full dataset, and b) restricted dataset.

On the porosity-dependent side, Slichter gave the lowest factor error in the restricted subset and remained the strongest porosity-dependent method after calibration. Kozeny-Carman remains an important comparator and continuity link to the broader permeability paper, but the restricted-domain evidence no longer favours it over Slichter.

The restricted subset, with fines contents up to about 30%, can therefore be interpreted as a low- to medium-fines tailings-derived silty-sand domain that is not far from ordinary non-plastic silty sands in engineering terms, even though the materials remain tailings-derived in origin. In that sense, the results may also be relevant to evaluation of similar fines-influenced silty sands beyond the immediate tailings context, provided the represented gradation, density, and non-plasticity conditions are reasonably matched.

5.3. Original Performance

Figure 3 shows the predicted-versus-measured conductivity relationships for the retained original methods. Among the gradation-based methods, Sherard gave the lowest typical factor error in the expanded dataset. Alyamani and Sen also performed strongly within the restricted subset, but Sherard remained the preferred gradation-based method because it combined the strongest overall performance with a simpler and more transparent practical form, and because the Alyamani and Sen result was affected by unproportional sensitivity to the D50 value of material P. Hazen remains useful mainly as a comparator and continuity link to the broader permeability paper. Among the porosity-dependent methods, Slichter gave the lowest typical factor error in the restricted subset in original form. Kozeny-Carman remained competitive, especially when the full dataset was considered, but no longer dominated the restricted-domain ranking.

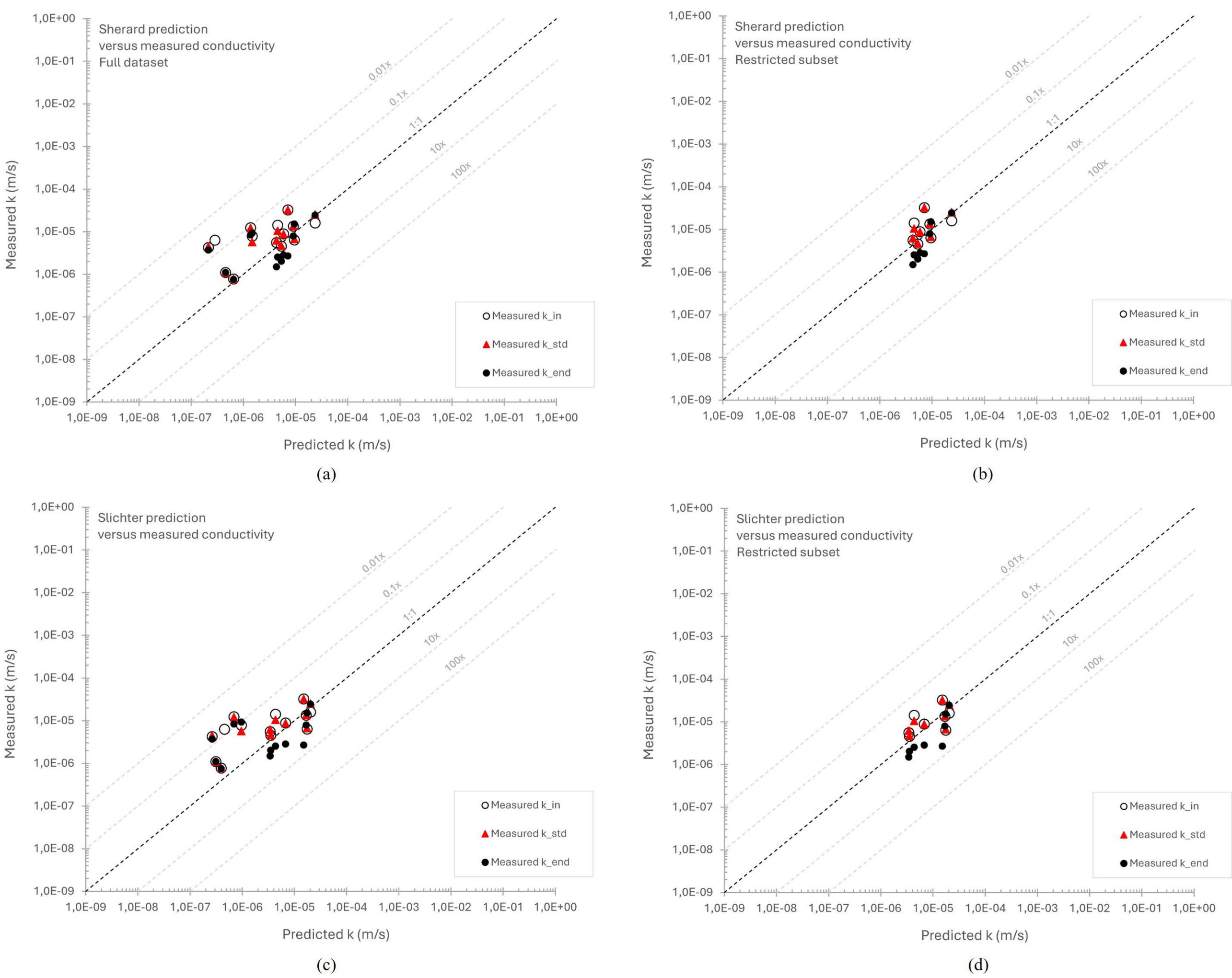

Figure 3. Predicted versus measured hydraulic conductivity for the retained original methods: (a) Sherard, full dataset; (b) Sherard, restricted subset; (c) Slichter, full dataset; and (d) Slichter, restricted subset. The plots show performance relative to the measured conductivity targets and include reference lines indicating 1:1 agreement and order-of-magnitude deviation.

5.4. Calibration Response

The retained methods responded differently to coefficient-only calibration, as shown in Figure 4 and summarized in Table 3. To make the calibration response easier to interpret, both the original coefficient C_0, the best-performing calibrated coefficient C_{cal}, the original typical factor error $E_{f,0}$, the calibrated typical factor error E_f, and the normalized response $\frac{E_f}{E_{f,0}}$ should be reported explicitly for each conductivity target.

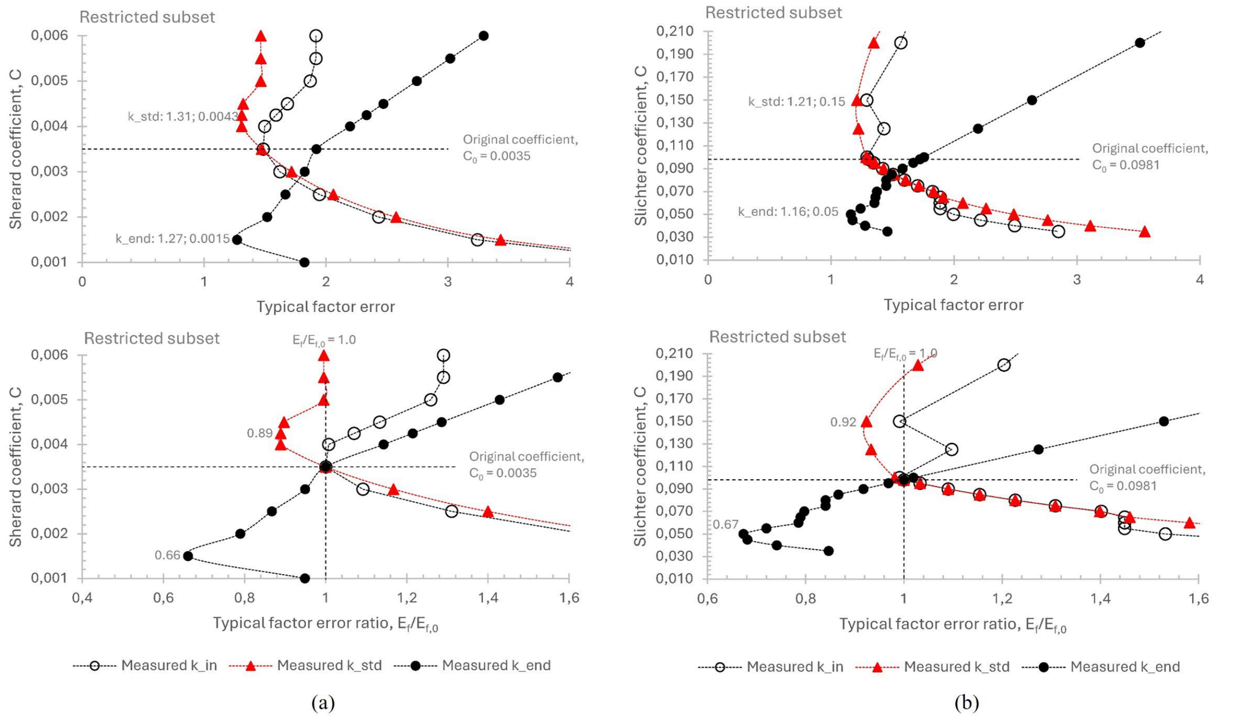

Figure 4. Calibration response of the retained best methods: (a) Sherard and (b) Slichter. The upper panels show typical factor error as a function of the method coefficient, and the lower panels show normalized factor error relative to the original coefficient.

Table 3. Calibration response of the retained best methods for the restricted subset.

Method	Best-performing target	Calibration response, target conductivity	Original coefficient, C_0	Typical factor error, original ($E_{f.0}$)	Calibrated coefficient, C_{cal}	Typical factor error, calibrated (E_f)	Relative change, ($E_f/E_{f.0}$)	Improvement (1 – Ef/Ef,0) x 100	Interpretation
Sherard	k_{std}	k_{in}	0.0035	1.49	No response	-	-	-	No improvement
		k_{std}		1.47	0.0043	1.31	0.89	10.9%	Moderate improvement
		k_{end}		1.92	0.0015	1.27	0.66	33.9%	Strong calibration response

Continued

Slichter	k_{in}	k_{in}	0.0981	1.30	No response	-	-	-	No improvement
		k_{std}		1.31	0.15	1.21	0.92	7.6%	Moderate improvement
		k_{end}		1.72	0.05	1.16	0.67	32.6%	Strong calibration response

For Sherard, the original coefficient is $C_0 = 0.0035$. Calibration on the restricted subset showed essentially no worthwhile gain at k_{in}, moderate improvement at k_{std}, and stronger improvement at k_{end}. At k_{std}, the best response occurred near $C_{cal} = 0.0043$, where $E_{f,0}$ decreased from about 1.47 to about 1.31, corresponding to $\frac{E_f}{E_{f,0}} \approx$ 0.89. At k_{end}, the strongest response occurred near $C_{cal} = 0.0015$, where $E_{f,0}$ decreased from about 1.92 to about 1.27, corresponding to $\frac{E_f}{E_{f,0}} \approx$ 0.66. This indicates that Sherard can be improved by calibration, but also that the preferred coefficient depends noticeably on which conductivity target is used.

For Slichter, the original coefficient is $C_0 = 0.0981$. Calibration on the restricted subset showed moderate improvement at k_{std} and strong improvement at k_{end}. At k_{std}, the best response occurred near $C_{cal} = 0.15$, where $E_{f,0}$ decreased from about 1.31 to about 1.21, corresponding to $\frac{E_f}{E_{f,0}} \approx$ 0.92. At k_{end}, the best response occurred near $C_{cal} = 0.05$, where $E_{f,0}$ decreased from about 1.72 to about 1.16, corresponding to $\frac{E_f}{E_{f,0}} \approx$ 0.67. Relative to Sherard, the Slichter response appears slightly more stable around the k_{std} optimum, while still allowing substantial improvement at k_{end}.

A useful summary statistic in Table 3 is therefore the percentage improvement relative to the original form, computed as $\left(1-\frac{E_f}{E_{f,0}}\right)\times 100$. This would make the practical meaning of calibration much easier to compare across methods and conductivity targets.

Overall, the calibration results suggest three distinct behaviours. First, Sherard is already reasonably close to earlier-stage conductivity in original form. Second, Slichter is the most attractive porosity-dependent candidate because it combines comparatively low original error with useful but not excessive calibration response.

5.5. Robustness

Overall, the retained methods performed well in the robustness checks. The original forms of both Sherard and Slichter showed only very small sensitivity to omission of individual tests or source-material groups, confirming that the main recommendations are not controlled by single specimens.

The most noteworthy result concerns the calibrated forms. Calibrated Sherard improved fit within the restricted subset, but the improvement was less stable than for the original form, with $\frac{E_f}{E_{f,0}}$ ranging from about 0.936 to 1.068 in the leave-one-source-material-out analysis. By contrast, calibrated Slichter remained both improved and highly stable, with leave-one-source-material-out $\frac{E_f}{E_{f,0}}$ values between about 0.982 and 1.016.

Boundary checking against the full dataset further separated the two recommended methods. When the restricted-subset calibrated Sherard coefficient was applied to the full dataset at k_{std}, E_f increased from about 1.504 to about 1.73, indicating that the calibration is domain-bounded. In contrast, the restricted-subset calibrated Slichter coefficient improved from about 2.157 to about 1.578 under the same check, corresponding to $\frac{E_f}{E_{f,0}} \approx$ 0.73. This supports Slichter as the more transferable calibrated method, while Sherard remains useful primarily within the restricted recommendation domain.

5.6. Practical Estimation Aid for D10 and D15

A practitioner-oriented aid was developed for estimating D10 and D15 from fines content within the restricted subset. A key observation from the PSDs is that the Z source shows an unusual fines-tail shape, with a comparatively steep transition in the approximate 0.04 to 0.06 mm range. This suggests that Z does not follow the same fines-to-D10 and fines-to-D15 relationship as the main restricted-subset trend, even though its total fines content falls within the core domain. When Z is excluded, the restricted-subset fines-D10 relationship becomes very strong, with R^2 about 0.97, and the fines-D15 equally with R^2 about 0.99. When Z is included, the corresponding R_2 values deteriorate sharply, to about 0.11 and 0.22. This strongly suggests that atypical fine-tail shape, not only total fines content, can control the relationship. This supports the use of a restricted subset not only in terms of fines content but also in terms of fines-tail shape consistency.

The resulting practitioner aid is therefore presented as a bounded lookup table for the restricted subset excluding Z. **Table 4** provides suggested D10 and D15 values for fines-content bands from 10% to 30%, together with approximate ranges and qualitative reliability labels. The middle fines intervals are judged more reliable because they are better supported by the main trend, whereas the edge intervals should be treated more cautiously.

Although no formal porosity- or RC-based practitioner table is recommended, the restricted subset excluding Z shows a tendency toward a narrower compaction-porosity range for the main cluster of materials. Typical values fall approximately within RC ≈ 83% - 93% and porosity n ≈ 0.41 - 0.46, although individual materials outside this central cluster extend beyond these ranges. These values should therefore be interpreted as indicative rather than strict bounds.

Table 4. Practitioner D10/D15 estimation aid for the restricted subset excluding Z, with extrapolated edge intervals.

Fines content, <0.063 mm (%)	Suggested D10 (mm)	Suggested D15 (mm)	Reliability
10 - 15	0.051 (0.04 - 0.06)	0.067 (0.06 - 0.08)	low
15 - 20	0.034 (0.03 - 0.04)	0.048 (0.04 - 0.06)	moderate
20 - 25	0.022 (0.02 - 0.03)	0.034 (0.03 - 0.04)	moderate
25 - 30	0.015 (0.01 - 0.02)	0.024 (0.02 - 0.03)	low

Note: The 10% - 15% and 25% - 30% fines intervals are weakly supported or extrapolated and should be interpreted with caution.

6. Discussion

6.1. Why This Material Domain Behaves Differently from Clean Sands

The represented materials differ from clean sands and many filter materials in several important respects: they contain higher fines contents, they are more fines-influenced in the silty-sand range, they show comparatively high particle densities, and their hydraulic behaviour is more strongly affected by density-sensitive structure. These characteristics help explain why method performance differs from that seen in broader granular-soil datasets.

6.2. Interpretation of the Retained Methods

A useful interpretation is to distinguish between gradation-based and porosity-dependent formulations by how they treat density. Gradation-based methods implicitly assume a representative packing equivalent to medium-dense sands, roughly RC ≈ 90% - 100% (often about 85% - 100% depending on compaction definition). Porosity-dependent methods, in contrast, include porosity (or void ratio) explicitly, allowing conductivity to respond directly to changes in densification and fabric.

In the present tailings-derived silty-sand domain, this distinction is important. The materials exhibit comparatively high particle densities and porosities, and their hydraulic behaviour is sensitive to density and structure. As a result, the implicit packing assumption embedded in gradation-based methods is not fully representative, whereas porosity-dependent methods are better able to capture the observed variation in conductivity. This helps explain the strong performance of Slichter within the restricted subset, and the continued usefulness, but more domain-bounded behaviour, of Sherard.

Sherard performed best among the gradation-based methods, suggesting that

D15 captures the relevant controlling grain-size scale more effectively than D10 once the tailings-derived domain broadens toward finer non-plastic silty materials. Hazen nevertheless remained very competitive, especially as a continuity link to the smaller tailings subset in the broader paper.

Slichter performed best among the porosity-dependent methods within the restricted subset, in both original and calibrated form. This suggests that the porosity-D10 combination in Slichter captures the restricted-domain hydraulic behaviour more closely than Kozeny-Carman. Kozeny-Carman remains useful as a comparator and continuity reference, but the combined restricted-subset and boundary-check evidence now favours Slichter as the final porosity-dependent recommendation. In that sense, the present tailings paper complements rather than mirrors the parallel till paper: both studies evaluate classical conductivity equations within a restricted engineering domain, but they arrive at different preferred methods because the represented material classes and controlling descriptors differ.

This refinement relative to the broader permeability study reflects the increased resolution obtained by focusing on a more narrowly defined material domain. In the earlier multi-domain comparison, method selection necessarily reflected a smaller tailings subset and broader variability, whereas the present study isolates a more homogeneous engineering class. The resulting shift in preferred methods should therefore be understood as a domain-specific refinement rather than a contradiction. For broader preliminary assessment outside such restricted domains, comparison across multiple well-performing methods may still be informative.

6.3. Practical Use and Limits

The final recommendations are intentionally domain-bounded. The results apply primarily to the represented low- to medium-fines non-plastic tailings-derived silty-sand domain, with fines contents up to about 30%. The finer materials in the full dataset are retained mainly as a sensitivity and boundary check. The paper therefore does not propose a universal tailings method.

The practitioner aid for D10 and D15 is likewise bounded. It is based on a small restricted subset and it should therefore be treated as a preliminary engineering aid rather than a general predictive model.

6.4. Limits of Calibration

The study was intentionally restricted to coefficient-only calibration. This provides a conservative test of whether the original methods require domain adaptation. In the present case, the results show that such adaptation is moderate but worthwhile for Sherard and Slichter at k_{std}, and stronger for late-stage conductivity. Sherard calibration appears more strongly tied to the restricted domain, whereas Slichter calibration transfers more favourably under moderate domain broadening.

7. Conclusions

1) The represented tailings-derived materials can be described in engineering terms as non-plastic low- to medium-fines silty sands, with the main recommendation domain limited to fines contents up to about 30%.

2) Sherard is the preferred gradation-based method for the restricted domain, while Hazen remains an important comparator and continuity link to the broader permeability paper.

3) Slichter is the preferred porosity-dependent method for the restricted domain, in both original and calibrated form, while Kozeny-Carman remains an important comparator and continuity reference.

4) Coefficient-only calibration improves both Sherard and Slichter at k_{std}, but Sherard calibration is more domain-bounded, whereas Slichter calibration transfers favourably to the broader dataset.

5) Robustness checks show that the retained original methods are highly stable and that the calibrated retained methods remain stable enough to support practical use within the restricted domain.

A bounded fines-based practitioner aid for estimating D10 and D15 can be proposed for the restricted subset excluding the Z source, but reliability remains limited by the small number of datapoints and sensitivity to atypical fines-tail shape.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Hazen, A. (1892) Some Physical Properties of Sands and Gravels, with Special Reference to Their Use in Filtration. 24th annual report of the Massachusetts State Board of Health, Public Document No. 34, 539-556.

[2] Slichter, C.S. (1899) Theoretical Investigation of the Motion of Ground Waters. Nineteenth Annual Report of the United States Geological Survey to the Secretary of the Interior, 1897-1898, Part II—Papers Chiefly of a Theoretic Nature, U.S. Government Printing Office, 295-384.

[3] Kozeny, J. (1927) Über kapillare Leitung des Wassers im Boden. *Sitzungsberichte der Akademie der Wissenschaften in Wien*, **136**, 271-306.

[4] Carman, P.C. (1937) Fluid Flow through Granular Beds. *Transactions of the Institution of Chemical Engineers*, **15**, 150-166.

[5] Beyer, W. (1964) Zur Bestimmung der Wasserdurchlässigkeit von Kiesen und Sanden aus der Kornverteilungskurve. *Wasserwirtschaft-Wassertechnik*, **14**, 165-169.

[6] Shepherd, R.G. (1989) Correlations of Permeability and Grain Size. *Groundwater*, **27**, 633-638. https://doi.org/10.1111/j.1745-6584.1989.tb00476.x

[7] Alyamani, M.S. and Şen, Z. (1993) Determination of Hydraulic Conductivity from Complete Grain-Size Distribution Curves. *Groundwater*, **31**, 551-555. https://doi.org/10.1111/j.1745-6584.1993.tb00587.x

[8] Sherard, J.L., Dunnigan, L.P. and Talbot, J.R. (1984) Basic Properties of Sand and Gravel Filters. *Journal of Geotechnical Engineering*, **110**, 684-700.

https://doi.org/10.1061/(asce)0733-9410(1984)110:6(684)

[9] Chapuis, R.P. (2012) Predicting the Saturated Hydraulic Conductivity of Soils: A Review. *Bulletin of Engineering Geology and the Environment*, **71**, 401-434.

[10] Odong, J. (2008) Evaluation of Empirical Formulae for Determination of Hydraulic Conductivity Based on Grain-Size Analysis. *The Journal of American Science*, **4**, 1-6.

[11] Wang, J., François, B. and Lambert, P. (2017) Equations for Hydraulic Conductivity Estimation from Particle Size Distribution: A Dimensional Analysis. *Water Resources Research*, **53**, 8127-8134. https://doi.org/10.1002/2017wr020888

[12] Urumović, K., Borović, S., Urumović, K. and Navratil, D. (2020) Validity Range and Reliability of the United States Bureau of Reclamation (USBR) Method in Hydrogeological Investigations. *Hydrogeology Journal*, **28**, 625-636. https://doi.org/10.1007/s10040-019-02080-2

[13] Goodarzi, M.R., Vazirian, M. and Niazkar, M. (2024) Hydraulic Conductivity Estimation: Comparison of Empirical Formulas Based on New Laboratory Experiments. *Water*, **16**, Article 1854. https://doi.org/10.3390/w16131854

[14] Vaughan, P.H. and Soares, H.F. (1982) Design of Filters for Clay Cores of Dams. *Journal of the Geotechnical Engineering Division*, **108**, 17-31. https://doi.org/10.1061/ajgeb6.0001233

[15] Rönnqvist, H. (2026) Evaluation of Empirical Hydraulic Conductivity Equations Using Standard and Long-Term Conductivity Measurements. *Geomaterials*, **16**, 67-87. https://doi.org/10.4236/gm.2026.163005

[16] Rönnqvist, H. (2026) Robustness of Fitted Behavioural Relationships from Limited Geotechnical Datasets: Application to Sand Castle Test Data. *Open Journal of Statistics*, **16**, 247-258. https://doi.org/10.4236/ojs.2026.164011

[17] Rönnqvist, H. (2026) Calibrating Hydraulic Conductivity Estimation Methods for Typical Swedish Tills Used as Core Materials. *Geomaterials*, **16**, 106-126. https://doi.org/10.4236/gm.2026.163007

[18] SS-EN ISO 17892-3 (2015) Geotechnical Investigation and Testing—Laboratory Testing of Soil—Part 3: Determination of Particle Density. Swedish Institute for Standards.

[19] ASTM International (2012) ASTM D1557-12e1: Standard Test Methods for Laboratory Compaction Characteristics of Soil Using Modified Effort (56,000 ft-lbf/ft^3 (2,700 kN-m/m^3)). ASTM International.

[20] Rönnqvist, H. (2026) Crack-Stopping in Natural and Crushed Filter Materials Assessed Using the Sand Castle Test. *Géotechnique Letters*. https://doi.org/10.1680/jgele.26.00030

List of Notations

K	hydraulic conductivity, m/s
k_{pred}	predicted hydraulic conductivity, m/s
$k_{measured}$	measured hydraulic conductivity, m/s
k_{in}	initial measured hydraulic conductivity, m/s
k_{std}	standard interpreted hydraulic conductivity, m/s
k_{end}	final measured hydraulic conductivity, m/s
D_x, d_x	particle size corresponding to x% passing, for example D10, d10, mm
C_u	coefficient of uniformity
n	porosity
e	void ratio
	uniformity coefficient used in the Beyer equation
C_0	original empirical coefficient
C_{cal}	calibrated empirical coefficient
E_f	typical factor error
$E_{f,0}$	typical factor error for the original method form
$\frac{E_f}{E_{f,0}}$	typical factor error ratio
R^2	coefficient of determination
R^2_0	coefficient of determination for the original fitted trend
RC	relative compaction (%), defined as ρ_d/MDD × 100
ρ_s	particle density, Mg/m^3
ρ_d	dry density, Mg/m^3
MDD	maximum dry density using modified Proctor, Mg/m^3