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Calibrating Hydraulic Conductivity Estimation Methods for Typical Swedish Tills Used as Core Materials

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Abstract

Empirical hydraulic-conductivity equations are widely used in geotechnical engineering, but none were developed specifically for the low-permeability glacial tills commonly used as core and seepage-control materials. This study evaluates and calibrates empirical conductivity equations for a dataset of 26 till materials from 10 source groups, including one Finnish source group represented by six materials. The main contribution is practical. Based on till-only screening supported by a broader multi-material comparison, the study identifies the Beyer method as the best-performing gradation-based method and the Slichter method as the best-performing porosity-dependent alternative. To adapt further for typical Swedish tills, Beyer showed little benefit from recalibration, whereas Slichter showed clearer improvement from coefficient adjustment. Gradation-based methods implicitly assume a representative packing state, whereas porosity-dependent methods explicitly account for porosity. This study also provides simple practitioner aids for estimating D_{10} and porosity when direct material data are incomplete, for example, when sedimentation analysis or density determinations are missing. The results show that classical equations can support conductivity estimation for typical non-plastic glacial tills, but applicability remains strongly controlled by till domain, data envelope, and method class.

Keywords

Hydraulic Conductivity, Glacial Till, Core Materials, Empirical Equations, Calibration

1. Introduction

Direct determination of hydraulic conductivity is generally preferred in geotechnical engineering, but empirical estimation methods remain widely used in practice, especially at early project stages when grain-size distribution, density-related descriptors, and basic index properties may be available before representative permeability testing has been completed. In dam engineering and earthworks, such equations therefore remain attractive for preliminary material screening, first-pass design judgement, and comparison between alternative borrow or core materials. Fell *et al.* [1] show that permeability remains a central practical issue in dam engineering, while Cedergren [2] emphasized that values derived from empirical relations should be regarded as approximate and that direct test methods are generally preferable when representative determination is required. Wenzel [3] likewise distinguished between indirect and direct permeability methods and noted that the indirect methods are generally less accurate.

Hydraulic conductivity, k, is the proportionality constant in Darcy's law relating discharge velocity to hydraulic gradient under saturated flow conditions. In practice, the term permeability is often used loosely for the same concept, although strictly speaking, hydraulic conductivity also includes the influence of the permeating fluid. The present paper uses hydraulic conductivity in that strict sense while recognizing the looser practical usage found in parts of the engineering literature.

Many empirical equations have been proposed, but they differ substantially in structure, data requirements, and implied field of applicability. Classical examples include Hazen [4], Slichter [5], Kozeny [6], Carman [7], Beyer [8], Shepherd [9], Alyamani and Sen [10], and later reassessments such as Chapuis [11], Odong [12], Wang *et al.* [13], Urumovic *et al.* [14], and Goodarzi *et al.* [15]. Some equations are based mainly on a characteristic grain size, such as D_{10}, whereas others also incorporate grading shape, uniformity, porosity, or void ratio. As a result, the same material can yield substantially different predicted conductivities depending on the equation selected, and comparative studies have repeatedly shown that no single empirical method performs best for all materials [12]-[15].

Most classical hydraulic-conductivity equations were developed for sands or other relatively clean granular materials. Low-permeability glacial tills used as core or seepage-control materials do not fit naturally into that background. They are broad-graded, fines-influenced, and more sensitive to density state than cleaner granular soils, such that changes in compaction can produce marked changes in hydraulic conductivity. They are also hydraulically much tighter than the material domains for which most of the classical equations were originally proposed. This creates a clear practical gap: there is no commonly established family of empirical hydraulic-conductivity equations developed specifically for glacial tills used as impervious or semi-impervious core materials. Practising engineers therefore tend to rely either on direct permeability testing, which is preferred but not always immediately available, or on classical empirical equations whose applicability to till materials is uncertain.

A further complication is that measured hydraulic conductivity is not always a single unique value. Recent long-duration permeability work indicates that a material may reasonably be described by more than one measured conductivity target, including an initial conductivity, k_{in}, a standard interpreted conductivity, k_{std}, and a final conductivity after extended hydraulic conditioning, k_{end} [16]. In parallel, a broader comparison of empirical hydraulic-conductivity equations across several practical material domains showed that tills form a distinct material class and that method performance differs markedly from that observed for filter materials and tailings. In that broader study, Beyer emerged as the strongest gradation-based method for tills, whereas Slichter emerged as the strongest porosity-dependent method [17].

The present study builds directly on those two lines of work. Rönnqvist [16] provides the permeability interpretation framework and the distinction between k_{in}, k_{std}, and k_{end}, whereas Rönnqvist [17] provides the method-screening background across material domains. The purpose here is not to introduce a universal new conductivity equation for tills, but to focus specifically on typical Swedish tills used as core or seepage-control materials and determine which existing classical equations are least unsuitable for that represented till domain, whether limited coefficient-only calibration is worthwhile, how robust the resulting recommendations are, and whether simple practitioner aids can be developed for preliminary use when direct material data are incomplete. More specifically, the study compiles and describes a till dataset suitable for method evaluation, identifies the best-performing gradation-based and porosity-dependent methods for the represented till domain, assesses whether conservative coefficient-only calibration improves performance, evaluates robustness using leave-one-out and leave-one-source-material-out checks, and develops practical estimation aids for porosity and D_{10} when material data are incomplete.

2. Materials and Methods

2.1. Till Dataset

The dataset comprises 26 till materials from 10 source groups, including one Finnish source group represented by six materials (Y1 - Y6), with the remaining materials originating from Swedish sites. Several source materials contributed multiple test specimens. The represented source groups include G/M/S1/S2, R1 - R4, T1 - T3, U1 - U3, X1 - X2, and Y1 - Y6, where the grouped materials originate from the same source but represent different tests. The dataset includes both short-duration tests and longer-duration tests for which multiple conductivity stages are available. The long-duration tests reported in the broader paper by Rönnqvist [17] are C, G, K, and M. In the present expanded till dataset, additional longer-duration materials include S2, V, X1, and Y1 - Y6. Particle size distributions are shown in **Figure 1** and material descriptors in **Table 1**.

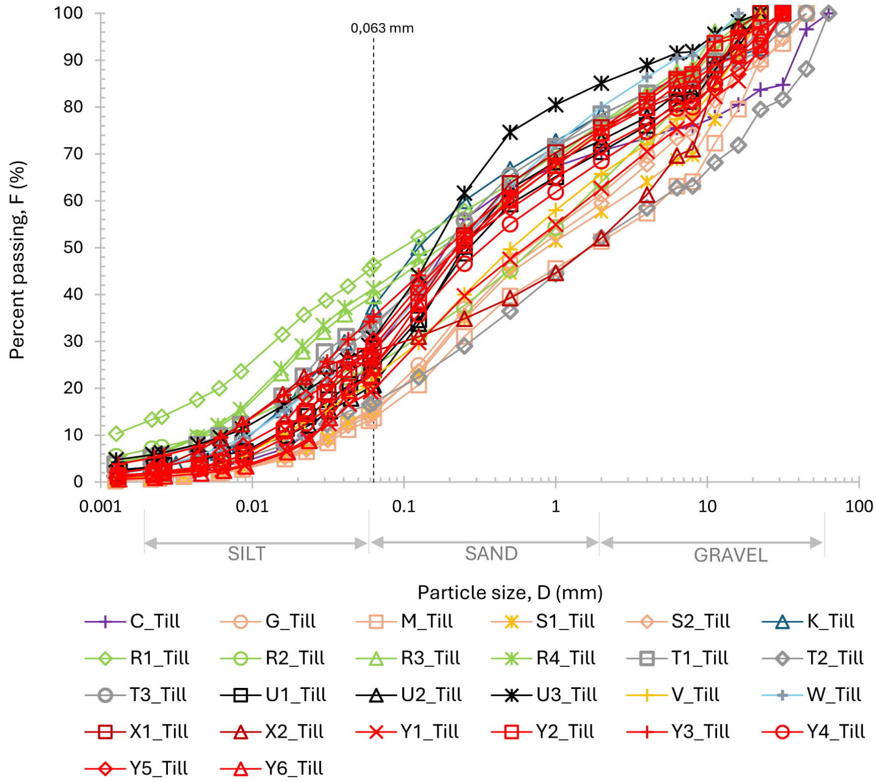


Figure 1. Particle-size distributions for the till dataset. The dataset represents typical Swedish tills, with one material from Finland, and spans a broad range of fines content and grading characteristics.

Hydraulic conductivity was determined from constant-head permeability tests using repeated volume-time measurements. Specimens were prepared at controlled compaction states, generally expressed in terms of relative compaction, and tested under saturated conditions. Flow rate was obtained by collecting outflow over measured time intervals, and conductivity was calculated using Darcy's law. Although the dataset combines results from multiple laboratory programs, the testing approach is consistent with standard permeameter methodology, and the resulting conductivity values are considered comparable for the purposes of the present analysis. The compiled material data include particle density, dry density, maximum dry density, relative compaction, porosity, void ratio, fines content, characteristic grain sizes, and hydraulic conductivity. Particle density was determined according to SS-EN ISO 17892-3 [18] and was used together with dry density to derive porosity and void ratio where relevant. Where modified Proctor data were available, maximum dry density (MDD) was determined from ASTM D1557 [19] and used together with dry density to calculate relative compaction (RC). For the four long-duration anchor materials used to link k_{in}, k_{std}, and k_{end}, the represented till domain spans C_u values from about 16.8 to 129, fines content (<0.063 mm) from about 13.6% to 37.4%, D15 from about 0.015 to 0.07 mm, and measured hydraulic conductivity from about 2×10^{-8} to 7×10^{-6} m/s. Table 1 summarizes the material properties used in the study.

Table 1. Material properties and hydraulic conductivity data for the till dataset.

Material ID	Source	Particle density, ρ_s (Mg/m^3)	Dry density, ρ_d (Mg/m^3)	MDD (Mg/m^3)	Relative compaction, RC (%)	Porosity, n (-)	Void ratio, e (-)	k_{in} (m/s)	k_{std} (m/s)	k_{end} (m/s)	Fines <0.063 mm (%)	C_u (-)	D_{10} (mm)	D_{15} (mm)	D_{20} (mm)	D_{50} (mm)	D_{60} (mm)
C_Till		2.860	1.90	2.11	90.0	0.336	0.506	1.79E−07	3.26E−08	3.18E−08	27.5	16.8	0.022	0.03	0.043	0.193	0.37
G_Till	S	2.765	1.84	2.15	85.4	0.336	0.506	6.98E−06	1.90E−07	4.85E−07	15.8	53.1	0.032	0.057	0.085	0.66	1.7
K_Till		2.794	1.79	2.18	82.1	0.359	0.561	6.64E−07	3.19E−08	2.60E−08	37.4	26.8	0.0095	0.0145	0.021	0.125	0.255
M_Till	S	2.765	1.82	2.15	84.8	0.341	0.517	6.64E−07	5.46E−07	1.76E−07	13.6	128.9	0.038	0.07	0.118	1.75	4.9
R1_Till	R	2.710	2.12	2.22	95.5	0.218	0.278	5.44E−08	-	-	46.3	266.7	0.0012	0.0029	0.006	0.095	0.32
R2_Till	R	2.710	2.03	2.22	91.4	0.251	0.335	3.83E−08	-	-	24.7	300.0	0.005	0.0115	0.026	0.73	1.5
R3_Till	R	2.724	2.05	2.22	92.3	0.247	0.329	1.03E−08	-	-	39.7	97.9	0.0047	0.0085	0.0125	0.176	0.46
R4_Till	R	2.724	2.08	2.22	93.7	0.236	0.310	1.54E−08	-	-	41.3	111.1	0.0045	0.008	0.0115	0.155	0.5
S1_Till	S	2.765	1.98	2.15	92.0	0.285	0.398	3.08E−08	-	-	15.3	77.6	0.033	0.06	0.095	0.88	2.56
S2_Till	S	2.765	1.99	2.15	92.4	0.282	0.392	1.34E−08	9.32E−09	8.48E−09	14.6	60.0	0.035	0.065	0.094	0.76	2.1
T1_Till	T	2.760	1.92	-	-	0.303	0.435	1.11E−08	-	-	33.8	73.3	0.006	0.011	0.018	0.215	0.44
T2_Till	T	2.730	2.00	-	-	0.266	0.362	3.59E−07	-	-	17.0	213.6	0.022	0.041	0.09	1.65	4.7
T3_Till	T	2.720	1.82	-	-	0.331	0.495	1.09E−07	-	-	33.1	35.8	0.0095	0.016	0.023	0.185	0.34
U1_Till	U	2.620	1.97	-	-	0.249	0.332	1.36E−08	-	-	23.9	39.3	0.0135	0.026	0.042	0.27	0.53
U2_Till	U	2.680	2.04	-	-	0.238	0.313	5.27E−08	-	-	20.7	25.9	0.017	0.032	0.06	0.25	0.44
U3_Till	U	2.701	2.10	-	-	0.224	0.289	1.86E−08	-	-	30.6	36.4	0.0066	0.0135	0.024	0.16	0.24
V_Till		2.719	1.99	-	-	0.270	0.369	2.45E−07	2.62E−07	2.33E−07	22.4	75.0	0.016	0.029	0.055	0.52	1.2
W_Till		2.710	2.03	-	-	0.251	0.335	1.18E−07	-	-	28.7	45.7	0.0092	0.016	0.0245	0.223	0.42
X1_Till	X	2.652	1.97	-	-	0.256	0.345	2.96E−07	2.75E−07	3.56E−07	27.9	28.2	0.014	0.022	0.033	0.22	0.395
X2_Till	X	2.661	2.03	-	-	0.236	0.309	3.11E−08	-	-	27.8	560.6	0.0066	0.011	0.0177	1.65	3.7
Y1_Till	Y	2.730	1.96	2.17	90.3	0.282	0.393	6.88E−07	6.12E−07	3.99E−07	19.7	62.8	0.025	0.038	0.065	0.63	1.57
Y2_Till	Y	2.710	1.72	2.17	79.4	0.364	0.573	1.59E−06	1.72E−06	1.84E−06	28.9	33.1	0.0145	0.023	0.0322	0.23	0.48
Y3_Till	Y	2.722	1.63	2.17	75.0	0.402	0.672	5.29E−07	4.74E−07	3.05E−07	35.3	67.7	0.0065	0.011	0.0185	0.195	0.44
Y4_Till	Y	2.739	1.95	2.17	90.0	0.287	0.402	4.56E−07	3.72E−07	1.30E−06	24.6	48.8	0.017	0.029	0.042	0.33	0.83
Y5_Till	Y	2.732	1.97	2.17	90.6	0.280	0.390	6.72E−08	7.21E−08	2.63E−07	30.3	52.2	0.0115	0.0185	0.028	0.23	0.6
Y6_Till	Y	2.722	1.93	2.17	89.1	0.289	0.407	2.26E−07	1.84E−07	1.02E−07	24.3	19.2	0.0255	0.035	0.045	0.245	0.49

2.2. Definition of Till/Core Domain

Glacial till is an unsorted soil material formed by glacial action through erosion, transport, crushing, grinding, and deposition of bedrock and pre-existing surficial materials [20] [21]. The mineralogical composition and grain-size distribution of till are strongly influenced by the local bedrock, and in areas dominated by hard and difficult-to-crush rocks, till is often coarser and more blocky [21]. The till materials in this study come mainly from Sweden, with one material source, Y, from Finland. In Sweden, this is particularly relevant because much of the country is underlain by Precambrian crystalline rocks of the Fennoscandian Shield, whereas younger sedimentary rock cover occurs only in more limited regions [22] [23]. In the represented dataset, these source conditions are associated mainly with low-clay, non-plastic tills. More generally, Swedish tills of the kind considered here are typically non-plastic, although low-plastic behaviour may occasionally occur. Tills derived from softer and more easily comminuted source rocks may be expected to contain higher fines contents and, in some cases, more plastic fines than the tills represented here. The source groups represented here are not drawn from a single geological setting. For example, the G/M/S1/S2 group originates from the southern tip of Sweden, in a region where sedimentary bedrock influence is more common, whereas the R-group originates from the mountainous central-northern part of Sweden within the Caledonian mountain region. These groups also represent opposite fines extremes within the dataset: the R-materials have the highest fines contents and the strongest clay influence, whereas the southern Sweden group has the lowest fines contents and practically no clay. This suggests that part of the observed between-source variation may reflect geological and provenance differences in addition to grading and density state.

The represented materials are therefore typical of a Swedish till/core-material domain characterized by non-plastic to occasionally low-plastic fines, and relatively low hydraulic conductivity. Although the dataset is dominated by Swedish tills, with material source Y from Finland, the results may also have relevance in other formerly glaciated regions where tills were formed under broadly similar bedrock conditions. Such broader use should still be treated cautiously, because local differences in grading, fines character, mineralogy, particle density, and compaction behaviour may affect applicability.

2.3. Available Descriptors

The compiled descriptors include fines content, particle density, dry density, maximum dry density, relative compaction, porosity, void ratio, measured hydraulic conductivity, uniformity coefficient, and characteristic grain sizes including D_{10}, D_{15}, D_{20}, D_{50}, and D_{60}. These descriptors, together with the available conductivity targets, are summarized in **Table 1**.

2.4. Conductivity Targets

Three conductivity targets are distinguished:

- the initial conductivity, k_{in}, is defined as the conductivity derived from the first stable flow measurements, the standard interpreted conductivity, k_{std}, is defined as the average of three consecutive measurements satisfying the 10% stability criterion, $\frac{k_{max} - k_{min}}{k_{mean}} \leq 0.10$, and

- the final conductivity, k_{end}, represents the late-stage conductivity after extended flow duration.

These definitions are consistent with the long-duration permeability framework presented in [16].

The availability of these targets differs between materials. The broadest coverage is currently available for k_{in}, while k_{std} and k_{end} are available for smaller subsets, particularly among the longer-duration tests. The four long-duration anchor tests reported in the broader paper by [17], namely C, G, K, and M, are retained as the original anchor materials linking k_{in}, k_{std}, and k_{end}. In the present expanded till dataset, the longer-duration subset is supplemented by S2, V, X1, and Y1 - Y6, for which at least k_{std} could be established and, in most cases, a k_{end} could be interpreted. Sample size for k_{in} is n = 26 and n = 13 for k_{std} and k_{end}.

3. Methodology

3.1. Method Selection

Figure 2 shows the till-only screening of the empirical hydraulic-conductivity equations considered in [17]. Based on that broader method-comparison study and the present till-only screening, Beyer was retained as the best-performing gradation-based method and Slichter as the best-performing porosity-dependent method. The screening figure shows that Beyer clearly outperformed the other gradation-based equations, whereas Slichter performed best among the porosity-dependent equations. Beyer is therefore retained as the strongest gradation-based method in the represented till domain, and Slichter as the strongest porosity-dependent method. In both cases, the retained equations are examined in original form and with conservative coefficient-only calibration. Predicted versus measured hydraulic conductivity plots for the retained original methods are shown in Figure 3(a) and Figure 3(b).

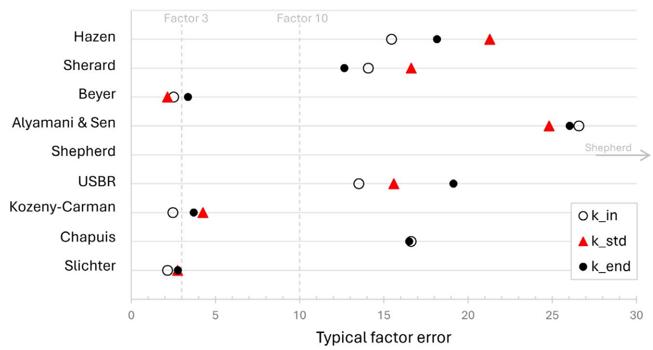


Figure 2. Till-only screening of candidate hydraulic-conductivity equations. Till-only screening of the empirical hydraulic-conductivity equations considered in the broader paper by Rönnqvist (2026b). The figure is included here as a selection figure to show why Beyer and Slichter were retained for deeper evaluation in the present study.

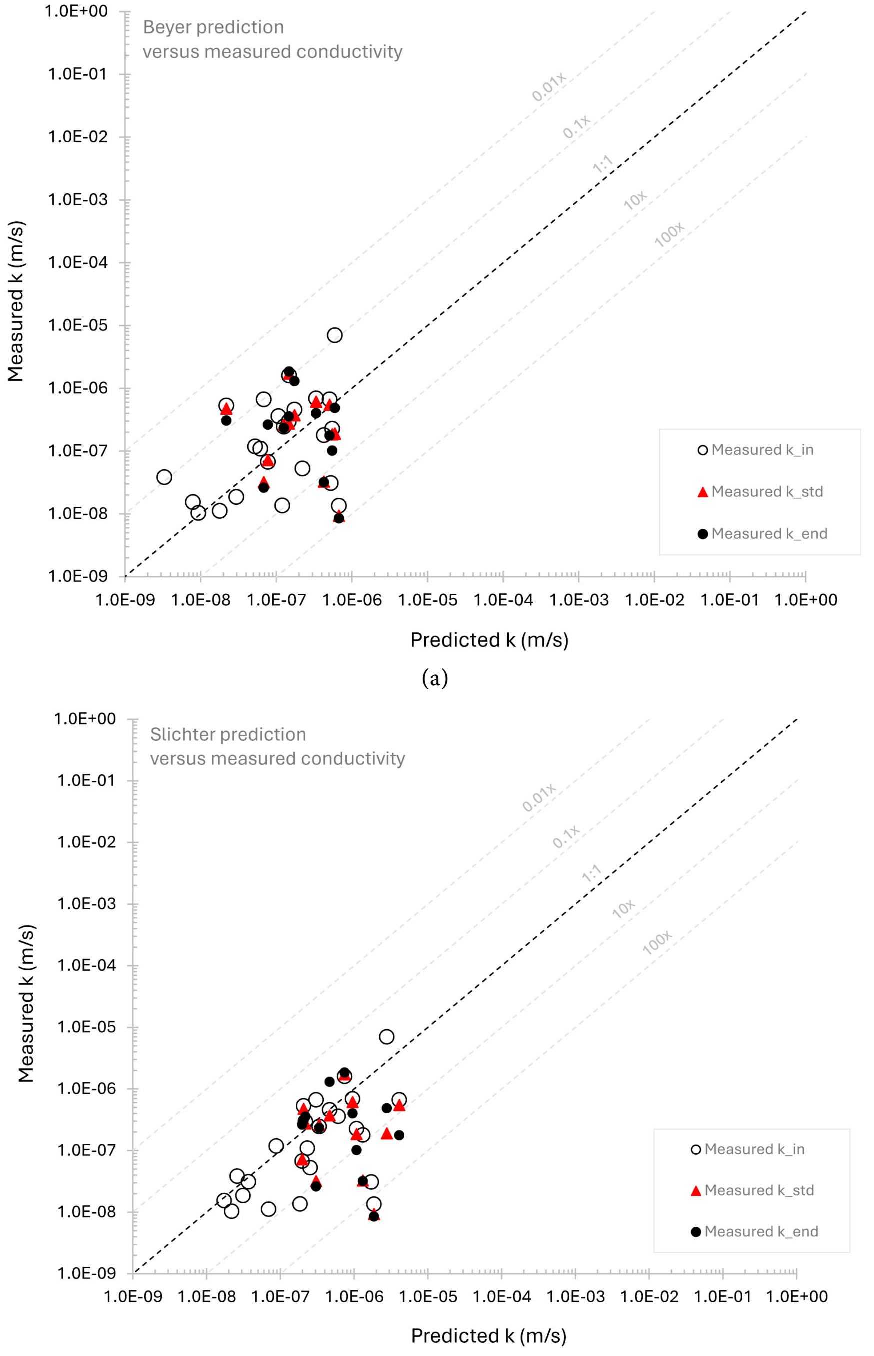


Figure 3. Predicted versus measured hydraulic conductivity for the retained original methods: (a) Beyer and (b) Slichter. The plots show performance relative to the measured conductivity targets and include reference lines indicating 1:1 agreement and order-of-magnitude deviation.

The other screened methods are not described further in the present paper, since the purpose here is not to repeat the full multi-method comparison but to focus on the two retained methods. Readers interested in the broader method set, comparative rankings, and full method descriptions are referred to [17].

3.2. Retained Methods and Implemented Working Forms

The retained methods are summarized in Table 2.

Table 2. Retained empirical hydraulic-conductivity equations and implemented working forms.

Method	Method class	Working form	Key input variables	Original coefficient, C_0	Role in this study	Reference
Beyer	Gradation based	$k = 0.0006\log_{10}\left(\frac{500}{U}\right)d_{10}^2$	D_{10}, U or C_u	0.0006	Best-performing gradation-based method in the till dataset; retained for detailed evaluation and possible coefficient calibration.	Beyer; accessible later source
Slichter	Porosity dependent	$k = 0.0981n^{3.287}d_{10}^2$	D_{10}, n	0.0981	Best-performing porosity-dependent method in the till dataset; retained for detailed evaluation and possible coefficient calibration	Slichter

The Beyer method [8], with historical reference in [24], extends the basic D_{10}-type approach by incorporating uniformity, here expressed as U. It is widely used for natural sediments and is especially relevant for heterogeneous or more broadly graded granular materials within its stated range of applicability. A commonly cited original or generalized form may be written as:

$$k = C\log_{10}\left(\frac{500}{U}\right)d_{10}^2$$

The implemented working form used in this study is:

$$k = 0.0006\log_{10}\left(\frac{500}{U}\right)d_{10}^2$$

Slichter [5] is an early porosity-dependent relation that remains frequently cited in later comparative studies. It is retained here as a classical alternative within the porosity-dependent group. A commonly cited original or generalized form may be written as:

$$k = Cn^{3.287}d_{10}^2$$

The implemented working form used in this study is:

$$k = 0.0981n^{3.287}d_{10}^2$$

3.3. Evaluation Framework

Method performance is assessed by comparing predicted hydraulic conductivity with measured hydraulic conductivity. To retain continuity with the earlier permeability-equations paper, performance is summarized primarily by the typical factor error, E_f. The original value before calibration is denoted $E_{f,0}$.

The present manuscript focuses on practical interpretation of E_f, and intermediate log-ratio statistics are not carried forward as separate reported metrics except where necessary for methodological definition. E_f may be understood as the characteristic multiplicative mismatch between predicted and measured conductivity, so that, for example, $E_f = 2$ corresponds to a typical factor-of-two difference. For fuller detail on the typical factor error approach and its implementation, readers are referred to [17].

3.4. Selection of Best-Performing Methods

Uncalibrated candidate equations were first screened on the till dataset. The till-only ranking showed that Beyer was the best-performing gradation-based equation and Slichter the best-performing porosity-dependent equation. These two methods were therefore carried forward for deeper evaluation.

3.5. Coefficient-Only Calibration

Calibration was restricted to adjustment of the multiplicative empirical coefficient (C), while retaining the original functional form of each method. This approach provides a minimal and transparent way to assess whether the represented till domain requires a different conductivity scale.

The optimal coefficient was identified by minimizing the typical factor error E_f. Coefficient values were evaluated over a defined range using discrete increments. Calibration and performance assessment were performed on the same dataset for each conductivity target.

The results of the calibration procedure are presented in Section 4.4.

3.6. Robustness Checks

Robustness checks were performed primarily on the original method forms. The objective was to test whether the practical recommendation was controlled by a single specimen or by a single source-material group. The robustness interpretation follows the approach developed in [25].

Two types of robustness check were used:

- leave-one-out, in which one specimen was omitted at a time and E_f recalculated, and
- leave-one-source-material-out, in which one source-material group was omitted at a time and E_f recalculated.

The results of these checks are presented in Section 4.5.

4. Results

4.1. Till Dataset Characteristics

The represented till materials span a broad range of fines content, density state, and hydraulic conductivity. **Figure 1** shows the particle-size distributions for the till dataset. **Table 1** summarizes the material data used in the study.

4.2. Screening of Candidate Methods and Retained Methods

A till-only screening of the empirical equations is shown in **Figure 2**. The purpose of this figure is not to repeat the full method-comparison exercise, but to show clearly why only two methods were retained for deeper evaluation in the present till-focused study.

The screening confirms Beyer as the strongest gradation-based method and Slichter as the strongest porosity-dependent method for the represented till domain. The other screened methods show substantially larger typical factor errors across the evaluated conductivity targets and were therefore not carried forward.

4.3. Best-Performing Methods in Original Form

Among the gradation-based methods, Beyer gave the lowest typical factor error for the represented tills. Its best original performance occurred at k_{std}, where the typical factor error was about 2.16.

Among the porosity-dependent methods, Slichter gave the lowest typical factor error in original form. Its best original performance occurred at k_{in}, where the typical factor error was about 2.16.

These results support retaining Beyer and Slichter as the two principal methods for the till domain.

4.4. Calibration Response

The two retained methods responded differently to calibration. For Beyer, the original coefficient was already close to optimal. No meaningful improvement was obtained for k_{in} or k_{std}, and only a modest improvement was obtained for k_{end}, where the typical factor error was reduced from about 3.38 to about 3.13, corresponding to $\frac{E_f}{E_{f,0}} = 0.928$.

For Slichter, moderate improvement was obtained for all three conductivity targets. The strongest calibration response occurred at k_{std}, where reducing the coefficient from 0.0981 to about 0.09 lowered the typical factor error from about 2.75 to about 2.53, corresponding to $\frac{E_f}{E_{f,0}} = 0.917$. Smaller improvements were also obtained for k_{in} and k_{end}.

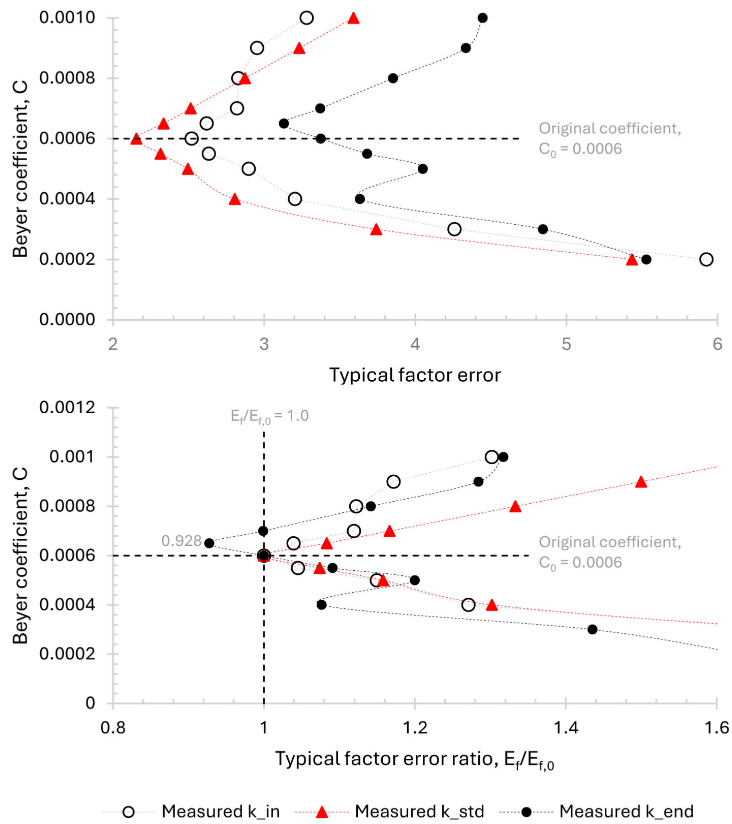
The main calibration results are summarized in Table 3. Figure 4(a) and Figure 4(b) shows the calibration response.

Table 3. Original performance and coefficient-calibration response of the retained methods.

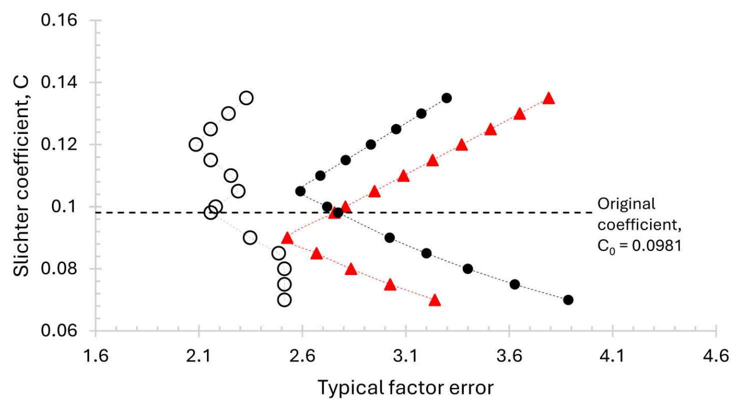
Method	Best-performing target	Calibration response, target conductivity	Original coefficient, C_0	Typical factor error, original ($E_{f,0}$)	Calibrated coefficient, C_{cal}	Typical factor error, calibrated (E_f)	Relative change, ($E_f/E_{f,0}$)	Interpretation
Beyer	k_{std}	k_{in}	0.0006	2.52	No response	-	-	No improvement
		k_{std}		2.16	No response	-	-	Best original performance, already at optimum

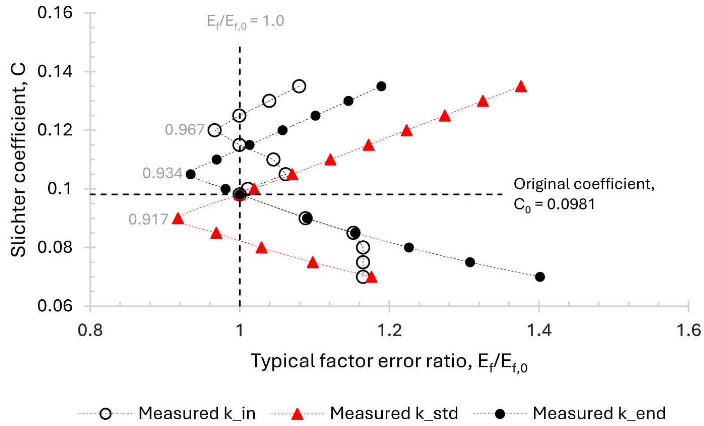
Continued

		k_{end}		3.38	0.00065	3.13	0.928	Moderate improvement, only for k_{end}
		k_{in}		2.16	0.12	2.09	0.967	Small improvement, best original performance
Slichter	k_{in}	k_{std}	0.0981	2.75	0.09	2.53	0.917	Strongest calibration response
		k_{end}		2.77	0.105	2.59	0.934	Moderate improvement



(a)





(b)

Figure 4. Calibration response of the retained methods: (a) Beyer and (b) Slichter. The upper panels show typical factor error as a function of the method coefficient, and the lower panels show normalized factor error relative to the original coefficient.

4.5. Robustness of the Retained Methods

For Slichter, leave-one-out analysis gave $\frac{E_f}{E_{f,0}}$ values between about 0.995 and 1.005, indicating extremely small sensitivity to removal of a single specimen. Leave-one-source-material-out analysis gave $\frac{E_f}{E_{f,0}}$ values between about 0.990 and 1.080, indicating good source-material-level robustness and no evidence that the overall result was controlled by a single group.

For Beyer, leave-one-out analysis gave $\frac{E_f}{E_{f,0}}$ values between about 0.969 and 1.032, indicating that the observed performance was not controlled by any single data point. Leave-one-source-material-out analysis gave $\frac{E_f}{E_{f,0}}$ values between about 0.959 and 1.122, indicating that the result was somewhat more influenced by source-material grouping than Slichter, with the Y-group exerting the largest influence, but still without undermining the overall recommendation.

4.6. Practical Estimation Aids

Two practitioner-oriented aids were developed as part of the practical contribution of the study.

First, a porosity aid based on relative compaction was established for typical Swedish tills using the fitted relationship:

$$n = 1.06 - 0.0086\text{RC}$$

Trend-based porosity estimates were computed at the midpoint of three RC bins and paired with the observed range in each bin. Because porosity must itself be estimated when direct density information is unavailable, application of porosity-dependent methods introduces an additional source of uncertainty beyond the

D_{10} estimate alone. The estimation aid for porosity is shown in Figure 5(a) and tabulated in Table 4.

Second, a fines-based estimation aid was developed for D_{10} using fitted fines-content trends. Suggested D_{10} values were taken from the fitted trend at the midpoint of each fines-content bin, with the observed interval shown in parentheses. The estimation aid for D_{10} is shown in Figure 5(b) and Table 5. The RC-porosity trend was fitted to 16 datapoints and gave an R^2 value of 0.87, whereas the fines-D_{10} trend was fitted to 26 datapoints and gave an R^2 value of 0.78. As a simple sensitivity check, both aids were also examined by leave-one-out recalculation of R^2. In this context, an increase in R^2 indicates that the omitted observation was relatively poorly aligned with the fitted trend, whereas a decrease indicates that the omitted observation contributed positively to the stability of the fit. Neither effect by itself implies that the point is erroneous or should be excluded. For the fines-D_{10} aid, the relative range in R^2/R_0^2 was 0.934 to 1.086, with the largest increase obtained by omitting R^2, for which R^2 increased from 0.78 to 0.84. For the RC-porosity aid, the corresponding range was 0.953 to 1.075, with the largest increase obtained by omitting C and the largest reduction by omitting Y3.

Table 4. Practitioner-oriented estimation aids for RC-porosity.

Compaction class	Relative compaction, RC (%)	Suggested porosity, n (-), and typical range	Reliability
Loose	70 - 80	0.415 (0.402 - 0.364)	Low to moderate
Medium dense	80 - 90	0.329 (0.359 - 0.336)	Moderate
Dense to very dense	90 - 100	0.243 (0.282 - 0.218)	Moderate to good

Using the fitted relationship:

$$n = -0.00860\ RC + 1.06014$$

Suggested porosity values are taken from the fitted RC-porosity trend at the midpoint of each RC bin and are intended as practical estimates only for typical Swedish tills when direct porosity data are unavailable. Typical range of porosity range within each bin is shown in parentheses. Reliability is qualitative and reflects the expected usefulness of the estimate within the represented till domain. Direct measurement remains preferable whenever available. Note: the porosity ranges represent typical intervals based on clustering of the dataset and do not correspond to strict minimum-maximum values within each compaction range.

Table 5. Practitioner-oriented estimation aids for particle size D_{10}.

Fines content, <0.063 mm (%)	Suggested D_{10} (mm)	Reliability
10 - 20	0.043 (0.022 - 0.038)	Low to moderate
20 - 30	0.015 (0.005 - 0.0255)	Moderate
30 - 40	0.005 (0.006 - 0.0115)	Moderate
40 - 50	0.002 (0.0012 - 0.0045)	Low

Suggested D_{10} values are taken from the fitted fines-content trend at the midpoint of each fines-content bin. The observed interval within each bin is shown in parentheses. The tabulated values are intended as practical estimates only and should not replace direct determination when the fine tail of the PSD is available. Bins with very few materials should be interpreted cautiously.

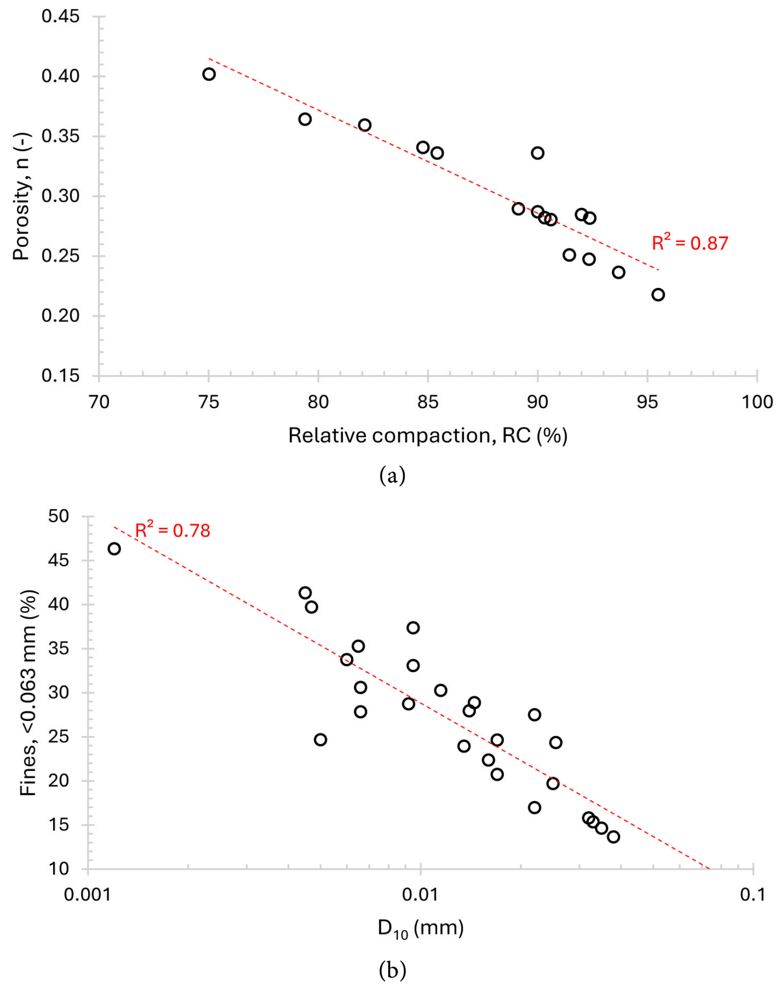


Figure 5. Practitioner-oriented estimation aids for the till dataset: (a) RC (relative MDD from modified Proctor) versus porosity and (b) fines content versus D_{10}. The plotted trends provide the basis for the practitioner-aid discussion and worked examples presented in the manuscript.

Third, a simple particle-density aid was established for typical Swedish tills to support porosity estimation when direct particle-density measurements are unavailable. For the represented till dataset, the mean particle density was 2.735 Mg/m^3 with a sample standard deviation of 0.060 Mg/m^3, giving an approximate ±1 SD range of 2.675 to 2.795 Mg/m^3. This indicates that particle density is sufficiently concentrated for a representative default value to be useful in preliminary work, although direct measurement remains preferable when available.

These practitioner aids are explicitly based on the parameter range and material spread represented by the present dataset, rather than on a broader universal soil domain. The D_{10} estimates also become less reliable with increasing fines content, particularly toward the fines-rich end of the represented till domain. Earlier exploratory work also considered D_{15} and D_{20}, but because the retained recommended methods in the present manuscript require only D_{10} on the gradation side, the final practitioner aid is limited to D_{10}.

4.7. Worked Examples Using the Practitioner Framework

To illustrate how the framework may be used in practice, when the fine tail of the particle-size distribution is unavailable, two worked examples were examined: one fines-rich case based on R1 and one lower-fines case based on Y1. The results are summarised in Table 6.

Table 6. Worked example results.

Case	Known inputs	Estimated aid inputs	Method	Coefficient used	Estimated k (m/s)	Measured reference	Interpretation
R1-type fines-rich case	Fines = 46.3%; D60 = 0.32 mm; RC = 95%	D10 = 0.002 mm; C_u = 160; n = 0.243	**Beyer**	Original	1.19E−09	k_{in} = 5.44E−08	Strong underprediction (x0.02)
				Calibrated	1.29E−09		Strong underprediction (x0.02)
			Slichter	Original	3.75E−09		Underprediction, but closer than Beyer (x0.07)
				Calibrated to k_in	4.59E−09		Underprediction, but closer than Beyer (x0.08)
Y1-type lower-fines case	Fines = 19.7%; D60 = 1.57 mm; RC = 90%	D10 = 0.043 mm; C_u = 36.5; n = 0.286	**Beyer**	Original	1.26E−06	k_{in} = 6.88E−07	Moderate overprediction (x1.8)
				Calibrated	1.37E−06		Moderate overprediction (x2.0)
			Slichter	Original	2.97E−06		Clear overprediction (x4.3)
				Calibrated to k_in	3.63E−06		Clear overprediction (x5.3)

4.7.1. R1-Type Fines-Rich Case

A fines-rich case was defined using the same general characteristics as R1. The assumed known inputs were fines content of 46.3%, D_{60} = 0.32 mm, and construction to code at about 95% relative compaction. Because the PSD below 0.063 mm was assumed unavailable, D_{10} was estimated from the fines-based aid. From the 40% - 50% fines bin, the suggested D_{10} was taken as 0.002 mm. Estimation of D_{10} in this fines bracket is known to have relatively low reliability and should therefore be treated as particularly uncertain in this example. This gave Cu = D_{60}/D_{10} = 0.32/0.002 = 160. Porosity was estimated from the RC-based aid as n ≈ 0.243.

Applying the practitioner framework gave estimated hydraulic conductivities ranging from about 1.2×10^{-9} to 4.6×10^{-9} m/s depending on method and coefficient choice. In this case, use of Slichter requires two estimated inputs, D_{10} and porosity, whereas Beyer requires only the D_{10} estimate together with D_{60} to derive C_u. The porosity-dependent route therefore carries an additional source of potential error.

For comparison, the actual R1 values were D_{10} = 0.0012 mm, porosity n = 0.218, and measured k_{in} = 5.44 × 10^{-8} m/s. Using the actual D_{10} and porosity values rather than the practitioner-aid estimates reduced the predicted conductivity even further, showing that the discrepancy in this fines-rich case is not caused only by uncertainty in the practitioner aid, but also by limited applicability of the empirical equations themselves at the very fine end.

4.7.2. Y1-Type Lower-Fines Case

A lower-fines case was defined using the same general characteristics as Y1. The assumed known inputs were fines content of 19.7%, D_{60} = 1.57 mm, and construction at the lower end of code, taken as RC = 90%. From the 10% - 20% fines bin, the suggested D_{10} was taken as 0.043 mm. This gave $C_u = D_{60}/D_{10}$ = 1.57/0.043 ≈ 36.5. Porosity was estimated from the fitted RC-porosity trend as n = 1.06 − 0.0086(90) ≈ 0.286.

Applying the practitioner framework gave estimated hydraulic conductivities ranging from about 1.3 × 10^{-6} to 3.6 × 10^{-6} m/s depending on method and coefficient choice. Here again, use of Slichter requires both an estimated D_{10} and an estimated porosity, whereas Beyer requires only the D_{10} estimate together with D_{60} to derive C_u. This should be expected to increase the potential uncertainty of the porosity-dependent estimate.

The actual measured Y1 values were k_{in} = 6.88 × 10^{-7} m/s, k_{std} = 6.12 × 10^{-7} m/s, and k_{end} = 4.0 × 10^{-7} m/s. In this more moderate-fines case, the framework still overpredicted, but Beyer remained within roughly a factor of two of the measured k_{in}, whereas Slichter overpredicted more strongly.

These worked examples show that the practitioner framework can produce preliminary hydraulic-conductivity estimates from incomplete material information, but that the reliability depends strongly on fines regime and on the applicability limits of the retained empirical equations. In the fines-rich R1-type case, the framework still underpredicted strongly, indicating that the equation form itself becomes a major limitation. In the more moderate-fines Y1-type case, the framework gave more realistic estimates, with Beyer giving a conductivity within about a factor of two of the measured k_{in}.

5. Discussion

5.1. Why Tills Differ from Cleaner Granular Materials

The represented tills differ from clean sands and many filter materials in three important respects: they contain higher fines contents, they are more broadly graded, and their hydraulic behaviour is more strongly affected by density state. These characteristics help explain why method performance differs from that seen in broader granular-soil datasets.

Gradation-based methods implicitly represent a typical medium-dense packing state (approximately RC 90% - 100%), whereas porosity-dependent methods explicitly account for variations in densification. This distinction is important in density-sensitive materials, where porosity-dependent formulations provide a more physically representative description of hydraulic behaviour.

5.2. Interpretation of the Retained Methods

Beyer performed best among the gradation-based methods, suggesting that the combination of characteristic particle size and grading non-uniformity captures an important part of the hydraulic behaviour of the represented tills. At the same time, the lack of meaningful calibration response for k_{in} and k_{std} suggests that the original coefficient is already close to appropriate for this material domain.

Slichter performed best among the porosity-dependent methods and showed somewhat greater sensitivity to coefficient adjustment. This suggests that porosity does contain useful predictive information for these tills, but also that the original coefficient may be slightly less well aligned to this domain than in the case of Beyer.

5.3. Practical Use and Limits

The practitioner aids are intentionally simple to support preliminary engineering judgement for typical Swedish tills when detailed material data are missing. One aid provides estimated D_{10} from fines content, and the other estimated porosity from compaction state. Both are fitted directly to the parameter range represented by the present dataset and should therefore be interpreted as dataset-based tools rather than universal predictive relationships. When both estimated D_{10} and estimated porosity are used together, as in Slichter, the potential error is greater than when only D_{10} must be estimated, as in Beyer. Both should be treated cautiously, particularly at the margins of the represented data range.

The results are most applicable to low-clay, predominantly non-plastic tills derived mainly from crystalline shield-rock environments similar to those represented in the dataset. Occasional low-plastic behaviour may occur, but the represented till group is generally non-plastic. This includes not only the Swedish and Finnish materials represented here, but potentially also tills from other formerly glaciated areas where similar crystalline or metamorphic bedrock conditions dominate. More generally, use outside the exact represented till group may still be reasonable where the material remains within, or close to, the overall spread of fines content, gradation, density state, and hydraulic conductivity covered by the present method dataset. By contrast, application far outside that envelope should be treated as substantially more uncertain and therefore risky. They should not be assumed to apply directly to more clay-rich or plastic tills developed from softer and more easily comminuted source materials.

5.4. Limits of Calibration

The study was intentionally restricted to coefficient-only calibration. This provides a conservative test of whether the original methods require domain adaptation. In the present case, the results show that such adaptation is limited for Beyer and modest for Slichter. This remains useful even where calibration gains are small, because it helps identify when original equations are already close to optimal and when they are not.

6. Conclusions

- Typical Swedish tills form a distinct hydraulic-conductivity estimation domain relative to cleaner granular materials.
- Beyer was the best-performing gradation-based method for the represented till dataset, and Slichter was the best-performing porosity-dependent method.
- Beyer gave its best original performance at k_{std} and showed little or no benefit from coefficient recalibration, with only modest improvement for k_{end}.
- Slichter gave its best original performance at k_{in} and showed moderate but consistent improvement from coefficient-only calibration, with the strongest calibration response at k_{std}.
- Robustness checks showed that both retained methods were reasonably stable, although Beyer was somewhat more sensitive than Slichter to source-material grouping.
- Practical estimation aids for porosity and fines-based D_{10} can support preliminary engineering estimates for typical Swedish tills when direct material data are incomplete but should not replace direct determination when such data are available. The potential error is expected to increase when both D_{10} and porosity must be estimated simultaneously. Use outside the overall data envelope represented by the present till dataset should be treated as substantially more uncertain and therefore risky.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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List of Notations

k	hydraulic conductivity, m/s
k_{pred}	predicted hydraulic conductivity, m/s
k_{measured}	measured hydraulic conductivity, m/s
k_{in}	initial measured hydraulic conductivity, m/s
k_{std}	standard interpreted hydraulic conductivity, m/s
k_{end}	final measured hydraulic conductivity, m/s
D_x	particle size corresponding to x% passing, for example D_{10}, mm
C_u	coefficient of uniformity
n	porosity
e	void ratio
U	uniformity coefficient used in the Beyer equation
C_0	original empirical coefficient
C_{cal}	calibrated empirical coefficient
E_f	typical factor error
$E_{f,0}$	typical factor error for the original method form
$\frac{E_f}{E_{f,0}}$	typical factor error ratio
R^2	coefficient of determination
R_0^2	coefficient of determination for the original fitted trend
RC	relative compaction (%), defined as ρ_d/MDD × 100
ρ_s	particle density, Mg/m³
ρ_d	dry density, Mg/m³
MDD	maximum dry density using modified Proctor, Mg/m³