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Evaluation of Empirical Hydraulic Conductivity Equations Using Standard and Long-Term Conductivity Measurements

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Abstract

Empirical equations for hydraulic conductivity are widely used, but their accuracy is not universal. Gradation-based methods implicitly represent a typical packing state, whereas porosity-dependent methods explicitly account for variations in porosity, which becomes important in density-sensitive materials. This study compares nine empirical methods against three measured conductivity targets: initial measured conductivity (k_in), standard interpreted conductivity (k_std), and final late-stage measured conductivity (k_end), using a laboratory database comprising filter materials, tills, tailings, and one fines-dominated silt. No single equation performed best across all materials and all targets. Most methods agreed most closely with initial measured conductivity (k_in), but clear exceptions were observed. Chapuis agreed most closely with standard interpreted conductivity (k_std), whereas Beyer agreed most closely with final late-stage measured conductivity (k_end). Material domain strongly influenced performance. Filter materials favored porosity-dependent methods, tills favoured Beyer, and tailings favoured Kozeny-Carman. For the till materials represented in this dataset, Beyer was the strongest gradation-based method and aligned most closely with standard interpreted conductivity (k_std). The main finding is that empirical equations do not represent one single hydraulic conductivity value. Instead, they tend to correspond to different measured conductivity states depending on formulation and material domain. The results, therefore, support domain-based method selection rather than a universal recommendation.

Keywords

Hydraulic Conductivity, Empirical Equations, Grain-Size Analysis, Tailings, Till, Filter

1. Introduction

Hydraulic conductivity, k, is the proportionality constant in Darcy's law relating discharge velocity to hydraulic gradient under saturated flow conditions. In practice, the term permeability is often used loosely for the same concept, although strictly speaking, hydraulic conductivity also includes the influence of the permeating fluid. Hydraulic conductivity is commonly estimated from particle-size distribution and density-related parameters using empirical equations. Such methods remain widely used because they are simple, inexpensive, and often available at an early stage of investigation. In dam engineering and earthworks, they are also attractive for preliminary material screening and initial assessment when direct test data are limited. Fell *et al.* [1] show that permeability remains a central practical issue in dam engineering, while Cedergren [2] emphasized that values derived from empirical relations should be regarded as approximate and that direct test methods are generally preferable when representative determination is required. Wenzel [3] likewise distinguished between indirect and direct permeability methods and noted that the indirect methods are generally less accurate.

Many empirical equations have been proposed, but they differ substantially in structure, data requirements, and implied field of applicability. Classic examples include Hazen [4], Slichter [5], Kozeny [6], Carman [7], Beyer [8], Shepherd [9], Alyamani and Sen [10], and later reformulations or reassessments such as Chapuis [11], Odong [12], Wang *et al.* [13], and Urumovic *et al.* [14]. Some equations are based mainly on a characteristic grain size, such as D_{10}, D_{15}, or D_{20}, whereas others also incorporate grading shape, uniformity, porosity, or void ratio. As a result, the same material can yield substantially different predicted conductivities depending on the equation selected. Comparative studies have repeatedly shown that no single empirical method performs best for all materials and that applicability is strongly conditioned by the nature of the dataset against which the equations were originally developed [12]-[15]. Recent laboratory comparison studies continue to identify different best-performing equations for different sand and gravel datasets, reinforcing that empirical ranking remains strongly dataset dependent rather than universal.

Empirical equations are often treated as if they predict one generic hydraulic conductivity, even though both material type and measurement framework matter. Differences in grading, packing, density, fines content, and conditioning can all affect relative equation performance. A further complication is that measured hydraulic conductivity is not always a single unique value. Recent long-duration permeability work indicates that a material may reasonably be described by more than one measured conductivity target, including an initial conductivity, k_in, a standard interpreted conductivity, k_std, and a final conductivity after extended hydraulic conditioning, k_end. Empirical equations are nevertheless usually evaluated against only one measured value.

This study compares nine empirical methods against k_in, k_std, and k_end using a laboratory database comprising filter materials, tills, tailings, and a fines-dominated silt. The aim is not to propose a universal replacement equation, but to evaluate a compact set of widely cited methods and determine which conductivity target each method most closely approximates, and how that relationship changes with material domain.

The main contribution is therefore to evaluate empirical equations against multiple measured conductivity targets across practical engineering domains and thereby provide a basis for domain-based method selection.

2. Materials and Methods

2.1. Materials and Dataset Structure

The study database comprised 17 materials representing four principal engineering domains: filter materials, tills, tailings, and one fines dominated silt. The filter domain included natural filter materials, crushed filter materials, and VSI-derived filter materials. The till domain included low-permeability glacial till materials used, or considered for use, as impervious core soil in embankment dams. The tailings domain included both natural tailings and crushed tailings-related materials. This grouping was adopted because it reflects practical engineering use and because material origin and fabric were expected to influence empirical method performance.

The 17-material database was compiled from closely related laboratory datasets, including the long-duration conductivity dataset used for consistency in material naming and conductivity-target interpretation. The retained materials were those for which the required particle-size distribution data and at least one measured hydraulic conductivity target were available in a sufficiently consistent form for method comparison. The present dataset therefore represents a structured comparison subset rather than all materials from the broader companion dataset.

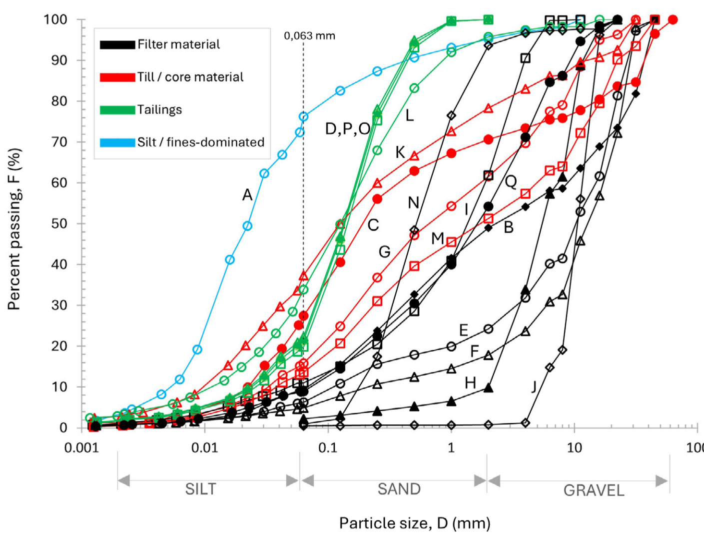


Figure 1. Particle-size distribution curves for the materials included in the empirical comparison, grouped by engineering domain. The figure shows the breadth of the dataset across filter materials, tills, tailings, and the fines dominated silt case.

Table 1. Summary of material descriptors and measured hydraulic conductivity values for the materials included in the study, including material ID, domain assignment, principal gradation descriptors, particle density, dry density, modified Proctor maximum dry density (MDD) where available, and the three measured conductivity targets *k*_in, *k*_std, and *k*_end. Material IDs follow the naming convention used for the companion long-duration conductivity dataset.

Material ID	Domain	Subtype/ origin	Particle density, ρ_s (Mg/m^3)	Dry density, ρ_d (Mg/m^3)	MDD (Mg/m^3)	Relative compaction, RC (%)	Porosity, n (-)	Void ratio, e (-)	k_{in} (m/s)	k_{std} (m/s)	k_{end} (m/s)	Fines < 0.063 mm (%)	Cu (-)	D10 (mm)	D15 (mm)	D50 (mm)	Included in porosity dependent method
A_Silt	Silt/fines-dominated	Natural soil	2.628	-	-	-	-	-	5.18E−08	4.68E−08	1.35E−08	76.3	5.4	0.005	0.007	0.023	No
B_Filter	Filter materials	Natural filter	2.840	2.14	2.25	95.1	0.25	0.33	8.18E−07	1.30E−07	1.35E−07	9.4	129.4	0.068	0.118	2.2	Yes
C_Till	Till/core materials	Natural till/ core soil	2.860	1.90	2.11	90.0	0.34	0.51	1.79E−07	3.26E−08	3.18E−08	27.5	16.8	0.022	0.030	0.193	Yes
D_Tailings	Tailings	Crushed tailings-derived material	2.834	1.54	1.79	85.8	0.46	0.85	1.43E−05	1.05E−05	2.55E−06	21.4	7.2	0.024	0.036	0.137	Yes
E_Filter	Filter materials	Crushed filter	2.855	1.92	2.36	81.5	0.33	0.48	1.49E−03	9.59E−04	8.46E−04	6.3	135.1	0.111	0.225	10.5	Yes
F_Filter	Filter materials	Crushed filter	2.864	1.97	2.37	83.3	0.31	0.45	4.74E−04	4.51E−04	4.75E−04	4.9	77.3	0.220	1.050	12.700	Yes
G_Till	Till/core materials	Natural till/ core soil	2.765	1.84	2.15	85.4	0.34	0.51	6.98E−06	1.90E−07	4.85E−07	15.8	53.1	0.032	0.057	0.660	Yes
H_Filter	Filter materials	Natural filter	-	1.81	-	-	-	-	8.08E−03	8.92E−03	5.81E−03	2.2	3.6	2.0	2.3	5.4	No
I_Filter	Filter materials	Crushed filter	2.843	1.77	2.22	79.9	0.38	0.60	6.16E−05	3.56E−05	3.95E−05	11.1	40.0	0.047	0.123	1.340	Yes
J_Filter	Filter materials	Crushed filter	-	1.72	-	-	-	-	3.76E−02	3.75E−02	3.57E−02	0.5	2.2	5.4	6.3	10.7	No
K_Till	Till/core materials	Natural till/ core soil	2.794	1.79	2.18	82.1	0.36	0.56	6.64E−07	3.19E−08	2.60E−08	37.4	26.8	0.010	0.015	0.125	Yes
L_Tailings	Tailings	Crushed tailings-derived material	3.130	1.83	2.21	82.6	0.42	0.72	1.25E−05	1.21E−05	8.38E−06	33.9	16.5	0.011	0.020	0.125	Yes
M_Till	Till/core materials	Natural till/ core soil	2.765	1.82	2.15	84.8	0.34	0.52	6.64E−07	5.46E−07	1.76E−07	13.6	128.9	0.038	0.070	1.750	Yes
N_Filter	Filter materials	Natural filter	2.730	1.44	1.75	82.2	0.47	0.90	5.44E−04	5.56E−04	4.91E−04	0.9	3.7	0.180	0.225	0.520	Yes
O_Tailings	Tailings	Tailings	2.834	1.59	1.79	89.0	0.44	0.78	5.57E−06	6.25E−06	1.49E−06	22.3	7.4	0.023	0.035	0.135	Yes
P_Tailings	Tailings	Tailings	2.834	1.66	1.79	92.7	0.41	0.71	4.56E−06	4.68E−06	2.03E−06	19.9	7.1	0.026	0.039	0.15	Yes
Q_Filter	Filter materials	VSI-derived crushed filter	2.692	2.14	2.25	95.3	0.20	0.26	2.19E−06	3.94E−06	1.15E−06	9.0	36.0	0.070	0.130	1.6	Yes

For each material, particle-size distribution data were available together with measured hydraulic conductivity targets and, where applicable, density-related descriptors such as porosity or void ratio. Particle density was determined according to SS-EN ISO 17892-3 [16] and was used together with dry density to derive porosity and void ratio where relevant. Where modified Proctor data were available, maximum dry density (MDD) was determined from ASTM D1557 [17] and used together with dry density to calculate relative compaction (RC). The material IDs follow a consistent naming convention aligned with the companion long-duration conductivity dataset used in this study. Not all methods can be applied to all materials because some equations require porosity-related inputs, whereas others are gradation-based. The comparison, therefore, used the maximum method-by-material coverage allowed by the available data. The particle-size distribution curves of the 17 materials are shown in **Figure 1**, and the principal material descriptors and measured conductivity targets are summarized in **Table 1**.

Operational Definition of Material Domains

The domains used in this study are operational categories defined from engineering function and material origin within this dataset, rather than universal classification boundaries intended for all soils. The proposed domain-based recommendations should therefore be interpreted for materials like those represented here.

Filter materials comprise granular materials intended for filter or transition-zone use. In this dataset, the domain includes eight materials, with Cu ranging from about 2.2 to 135, fines content (<0.063 mm) from about 0.5% to 11.1%, D15 from about 0.12 to 6.3 mm, and measured hydraulic conductivity from about 1.3 $\times 10^{-7}$ to 3.9×10^{-2} m/s considering k_in, k_std, and k_end together.

Tills comprise low-permeability glacial till materials used, or considered for use, as impervious core soil in embankment dams and other seepage-control zones. In this dataset, the domain includes four materials, with Cu ranging from about 16.8 to 129, fines content (<0.063 mm) from about 13.6% to 37.4%, D15 from about 0.015 to 0.07 mm, and measured hydraulic conductivity from about 2.6×10^{-8} to 7.0×10^{-6} m/s.

Tailings comprise granular mine waste materials produced during mineral processing, together with tailings-derived granular materials. In this dataset, the domain includes four materials, with Cu ranging from about 7.1 to 16.5, fines content (<0.063 mm) from about 19.9% to 33.9%, D15 from about 0.02 to 0.04 mm, and measured hydraulic conductivity from about 1.5×10^{-6} to 1.4×10^{-5} m/s.

The fines-dominated silt is treated separately as an illustrative outlier rather than as a domain with sufficient coverage for general recommendation. It comprises one material with fines content (<0.063 mm) of about 76.3%, D15 of about 0.007 mm, Cu of about 5.4, and measured hydraulic conductivity from about 1.4 $\times 10^{-8}$ to 5.2×10^{-8} m/s.

These ranges are dataset descriptors rather than prescriptive classification boundaries. The full material-level descriptors and measured conductivity targets are summarized in **Table 1**.

2.2. Measured Hydraulic Conductivity Targets

In the present paper, k is used consistently to denote hydraulic conductivity. The term permeability is used only in a broader historical or conventional sense, for example, when referring to permeability tests or to terminology used in older references.

Three measured hydraulic conductivity targets were considered: *k*_in, *k*_std, and *k*_end. The underlying reference hydraulic-conductivity tests were performed according to ASTM and ISO laboratory permeability procedures, as reflected in the cited standards. Here, *k*_in denotes the initial measured conductivity, *k*_std denotes the standard interpreted conductivity used as the principal reporting value in the long-duration test framework, and *k*_end denotes the final measured conductivity at the end of the test. As shown in Table 1, the spread between *k*_in, *k*_std, and *k*_end is material dependent and can be substantial within the filter, till, and tailings domains. In some cases, the values differ by more than one order of magnitude, indicating that the choice of conductivity target can make a real difference.

This distinction was adopted because hydraulic conductivity may evolve during testing. An empirical equation may therefore align more closely with an initial, intermediate, or later conductivity state depending on both its formulation and the material considered. Treating measured conductivity as a single fixed target would therefore conceal part of the practical meaning of the comparison.

2.3. Retained Empirical Equations

The retained quantitative method set comprises six gradation-based methods and three porosity dependent methods. The retained methods were selected to represent a compact set of established empirical equations that remain widely cited and practically used in geotechnical and hydrogeological work. Together, they span the main formulation types encountered in the literature, namely gradation based and porosity dependent approaches, and could be applied consistently to the available dataset.

In the equations below, hydraulic conductivity is denoted by *k* and is expressed in m/s. Particle sizes are expressed in mm, and porosity- or void-ratio terms are included where required by the method. Historical method papers are cited where relevant for attribution of the original equations. However, several of the older sources were not readily accessible in full, and the implemented equations were therefore taken from accessible later sources that reproduce, summarize, or reassess the original formulations. Where necessary, empirical coefficients were used in unit-consistent forms corresponding to the spreadsheet implementation adopted in the present study. The equations presented below should accordingly be understood as the working forms used in the present comparison rather than as verbatim reproductions of the original publications. A compact overview of the retained methods, required inputs, and applicability notes is given in Table 2.

Table 2. Summary of the retained empirical hydraulic conductivity equations, including method family, required input parameters, and general applicability notes.

Method	Method family	Required inputs	General applicability note
Hazen	Gradation based	D_{10}	Best suited to relatively clean, fairly uniform sands; should be used cautiously for broadly graded, fines-influenced, or low-permeability materials outside the original clean-sand context
Sherard	Gradation based	D_{15}	Primarily intended for granular filter materials in dam engineering; most relevant where filter gradation is already a central design descriptor
Beyer	Gradation based	D_{10}, U or Cu	Intended for non-uniform sands and granular soils where uniformity is expected to influence conductivity; especially useful for heterogeneous granular materials within the represented gradation range
Alyamani and Sen	Gradation based	D_{10}, D_{50}, I_0	Applicable where a fuller representation of grading-curve shape is available; more sensitive than simple D_{10}-type methods to the fine side of the particle-size distribution
Shepherd	Gradation based	D_{50}, C, m	Regression-style grain-size estimate based mainly on median grain size; best regarded as a broad empirical comparator rather than a strongly domain-specific design equation
USBR	Gradation based	D_{20}	Historically used in hydrogeological and dam-related practice for granular soils; should be used cautiously for materials outside its intended calibration range, especially fines-influenced or low-conductivity soils
Kozeny-Carman	Porosity dependent	D_{10}, n	Best suited to granular soils where porosity data are available and pore-geometry effects are expected to matter; less suitable where density-related input data are absent
Chapuis	Porosity dependent	e, D_R	Practical porosity-sensitive alternative for soils where void ratio and an effective grain-size descriptor are available; particularly useful when density effects are expected to influence conductivity
Slichter	Porosity dependent	D_{10}, n	Classical porosity-dependent relation mainly associated with sands and other granular soils; most relevant where porosity data are available and the material is not dominated by plastic fines

2.3.1. Gradation-Based Methods

Hazen

Hazen [4] is a classical d_{10}-based relation developed for relatively uniform sands. It is retained here as a simple and widely recognized gradation-based estimator.

Commonly cited original or generalized form:

$$k = Cd_{10}^2$$

Implemented working form used in this study:

$$k = 0.0116d_{10}^2$$

Sherard

Sherard's relation [18] is commonly used in the filter literature and is particularly relevant in geotechnical and dam-engineering applications where filter gradation is already central to interpretation.

Commonly cited original or generalized form:

$$k = Cd_{15}^{2}$$

Implemented working form used in this study:

$$k = 0.0035d_{15}^{2}$$

Beyer

The Beyer method [8], and supporting historical reference in [19], extends the basic d_{10}-type approach by incorporating uniformity (here expressed as U).

It is widely used for natural sediments and is especially relevant for heterogeneous or more broadly graded granular materials within its stated range of applicability.

Commonly cited original or generalized form:

$$k = C\log_{10}\left(\frac{500}{U}\right)d_{10}^{2}$$

Implemented working form used in this study:

$$k = 0.0006\log_{10}\left(\frac{500}{U}\right)d_{10}^{2}$$

Alyamani and Sen

The Alyamani and Sen method [10] uses both grading-curve shape and characteristic grain sizes. It is therefore more sensitive than the simpler d_{10}-type formulas to how the finer portion of the grading curve is represented. The I_0 parameter is the grading-curve intercept obtained from the straight-line segment between d_{10} and d_{50} on the semi-logarithmic particle-size distribution curve and taken as the corresponding intercept on the particle-size axis. In practice, this is equivalent to graphically extending the line through d_{10} and d_{50} to its intercept or calculating the same intercept analytically from the two characteristic sizes.

Commonly cited original or generalized form:

$$k = C\left[I_0 + 0.025\left(d_{50} - d_{10}\right)\right]^{2}$$

Implemented working form used in this study:

$$k = 0.01505\left[I_0 + 0.025\left(d_{50} - d_{10}\right)\right]^{2}$$

Shepherd

Shepherd [9] proposed a regression-style grain-size relation based on published datasets of unconsolidated sediments. The parameters C and m are empirical coefficients. In this study, it is retained as a widely cited gradation-based comparator.

Commonly cited original or generalized form:

$$k = Cd_{50}^{m}$$

Implemented working form used in this study:

$$k = 0.0001 d_{50}^{2.3}$$

USBR

The USBR method is a classical d20-based relation derived from dam-engineering practice [14]. Modern reassessments indicate that its commonly used analytical form can yield systematically low values if applied uncritically outside its intended range, but it remains a useful and widely recognized gradation-based estimator.

Commonly cited original or generalized form:

$$k = C d_{20}^{2.3}$$

Implemented working form used in this study:

$$k = 0.0036 d_{20}^{2.3}$$

2.3.2. Porosity Dependent Methods

Kozeny-Carman

Kozeny-Carman [6] [7] is the most established physically motivated permeability relation in the retained set. It explicitly includes the influence of pore structure through porosity and is therefore well suited as a benchmark porosity dependent method.

Here, C_{KC} is an empirical coefficient reflecting the chosen unit convention and the idealized assumptions embedded in the equation.

Commonly cited original or generalized form:

$$k = C_{KC} \left[\frac{n^3}{(1-n)^2} \right] d_{10}^2$$

Implemented working form used in this study:

$$k = 0.0545 \left[\frac{n^3}{(1-n)^2} \right] d_{10}^2$$

Chapuis

Chapuis [11] reformulated permeability prediction in a compact porosity-sensitive form using void ratio and an effective grain-size descriptor. The equation is presented here in the implemented working form used in the spreadsheet calculations It is retained here as a practical porosity dependent alternative to Kozeny–Carman. Here, D_R is the effective grain-size descriptor used in the method, and in the present implementation, D_R was taken as the effective diameter d_{10}, consistent with the commonly used working form of the Chapuis equation.

Commonly cited original or generalized form:

$$k = C \left[\left(\frac{e^3}{1+e} \right) D_R^2 \right]^m$$

Implemented working form used in this study:

$$k = 0.024622 \left[\left(\frac{e^3}{1+e} \right) D_R^2 \right]^{0.7825}$$

Slichter

Slichter [5] is an early porosity dependent relation that remains frequently cited in later comparative studies. It is retained as a classical alternative within the porosity dependent group.

Commonly cited original or generalized form:

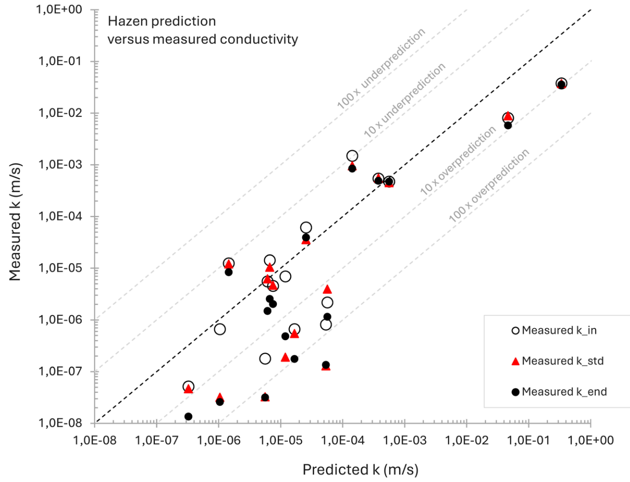
$$k = Cn^{3.287} d_{10}^2$$

Implemented working form used in this study:

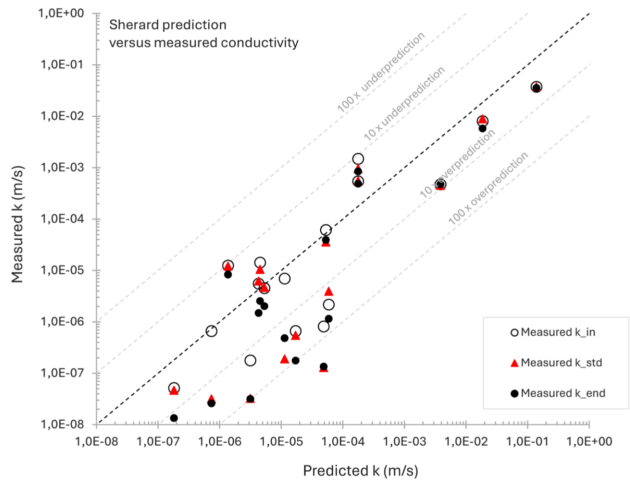
$$k = 0.0981 n^{3.287} d_{10}^2$$

2.4. Performance Evaluation

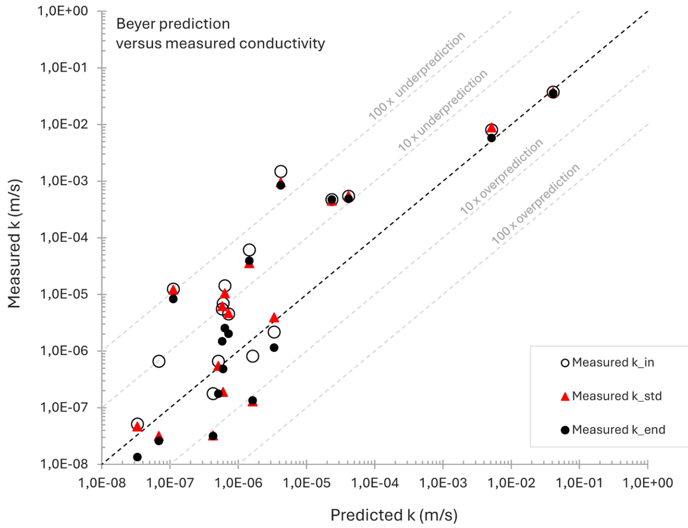
The method-by-method predicted-versus-measured comparisons are presented in Figure 2 panels. Method performance was evaluated by comparing predicted and measured conductivity through the ratio *k*_pred/*k*_measured. A value of 1 indicates exact agreement, values greater than 1 indicate overprediction, and values less than 1 indicate underprediction.



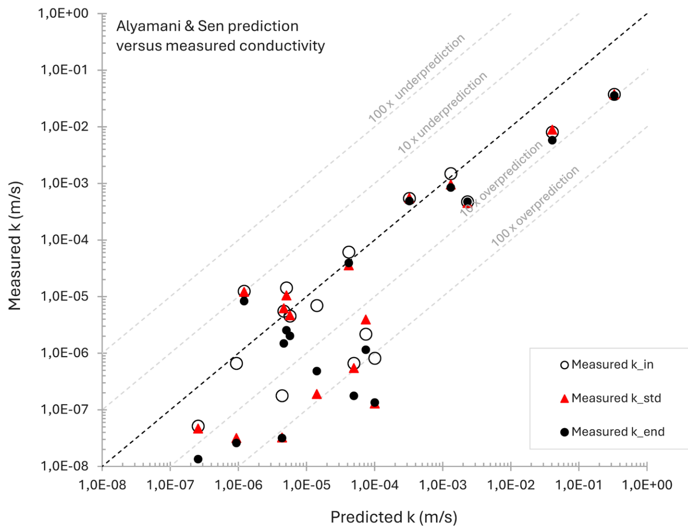
(a)



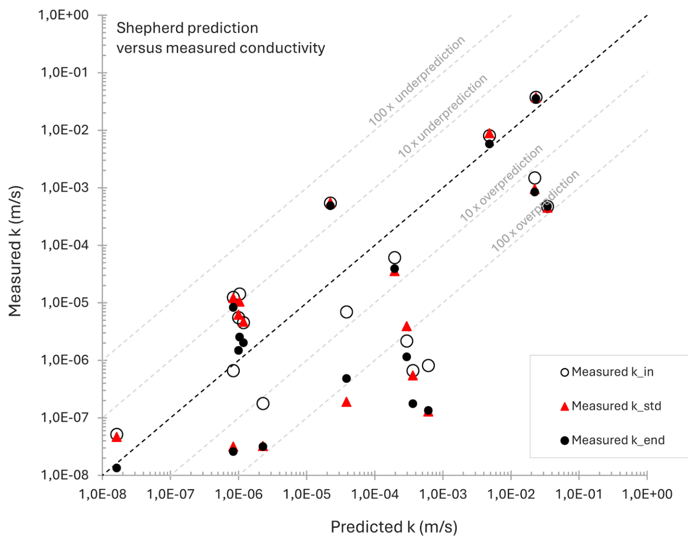
(b)


Beyer prediction
versus measured conductivity
100 x underprediction
10 x underprediction
10 x overprediction
100 x overprediction
Measured k_in
Measured k_std
Measured k_end
Measured k (m/s)
Predicted k (m/s)

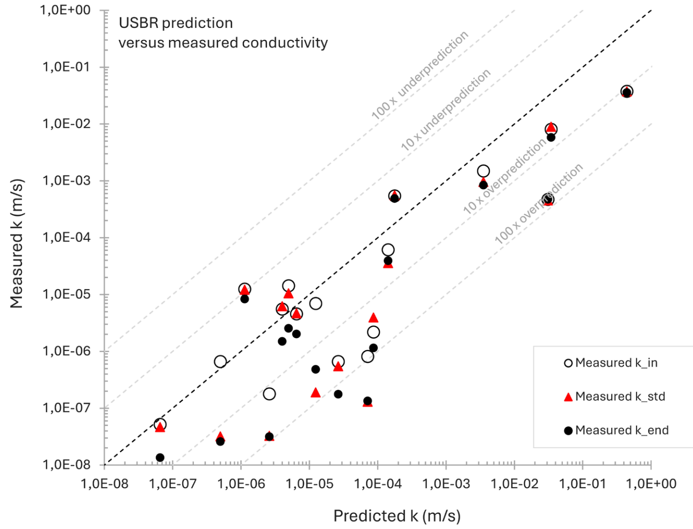
(c)


Alyamani & Sen prediction
versus measured conductivity
100 x underprediction
10 x underprediction
10 x overprediction
100 x overprediction
Measured k_in
Measured k_std
Measured k_end
Measured k (m/s)
Predicted k (m/s)

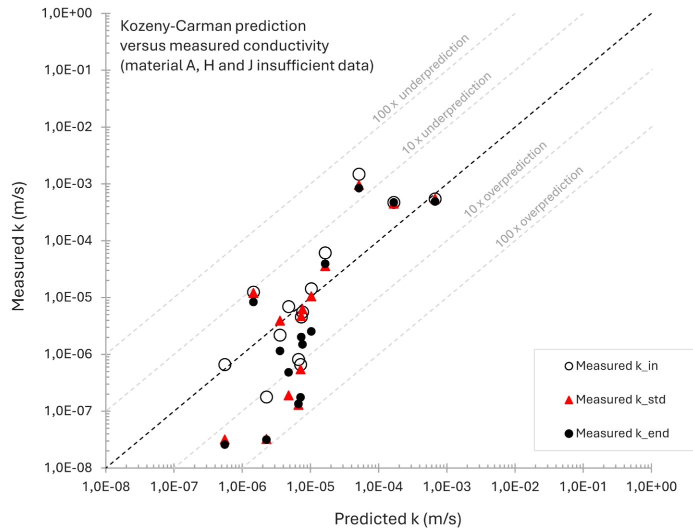
(d)


Shepherd prediction
versus measured conductivity
100 x underprediction
10 x underprediction
10 x overprediction
Measured k_in
Measured k_std
Measured k_end
Measured k (m/s)
Predicted k (m/s)

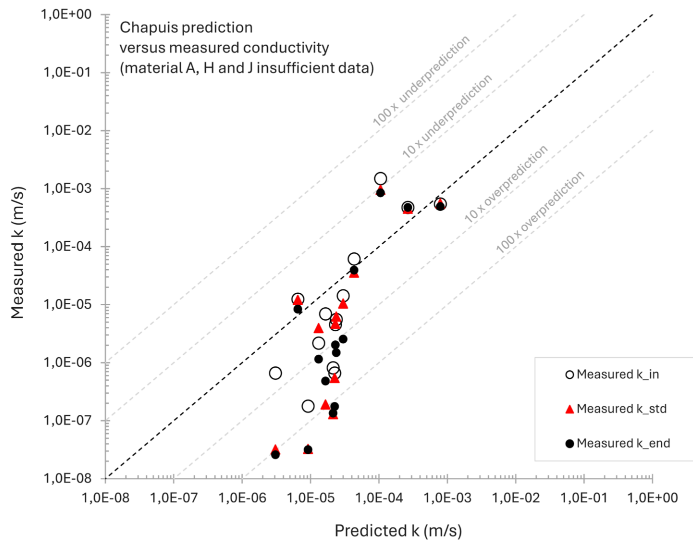
(e)


USBR prediction
versus measured conductivity
100 x underprediction
10 x underprediction
10 x overprediction
100 x overprediction
Measured k (m/s)
Predicted k (m/s)
1,0E+00
1,0E-01
1,0E-02
1,0E-03
1,0E-04
1,0E-05
1,0E-06
1,0E-07
1,0E-08
1,0E-08
1,0E-07
1,0E-06
1,0E-05
1,0E-04
1,0E-03
1,0E-02
1,0E-01
1,0E+00
Measured k_in
Measured k_std
Measured k_end

(f)


Kozeny-Carman prediction
versus measured conductivity
(material A, H and J insufficient data)
100 x underprediction
10 x underprediction
10 x overprediction
100 x overprediction
Measured k (m/s)
Predicted k (m/s)
1,0E+00
1,0E-01
1,0E-02
1,0E-03
1,0E-04
1,0E-05
1,0E-06
1,0E-07
1,0E-08
1,0E-08
1,0E-07
1,0E-06
1,0E-05
1,0E-04
1,0E-03
1,0E-02
1,0E-01
1,0E+00
Measured k_in
Measured k_std
Measured k_end

(g)


Chapuis prediction
versus measured conductivity
(material A, H and J insufficient data)
100 x underprediction
10 x underprediction
10 x overprediction
100 x overprediction
Measured k (m/s)
Predicted k (m/s)
1,0E+00
1,0E-01
1,0E-02
1,0E-03
1,0E-04
1,0E-05
1,0E-06
1,0E-07
1,0E-08
1,0E-08
1,0E-07
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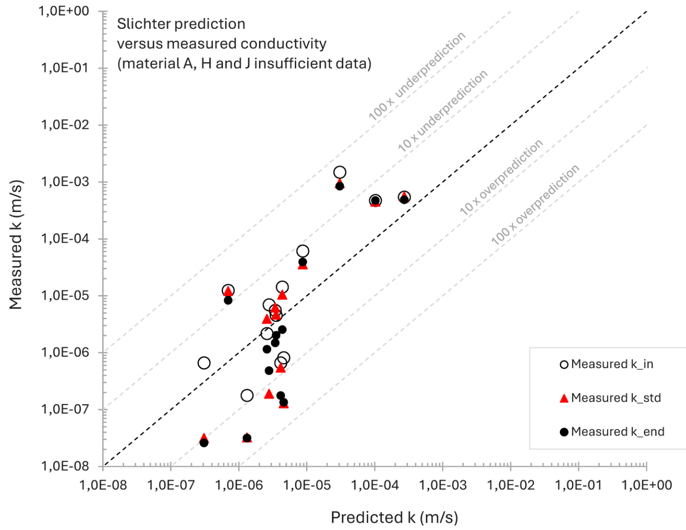
1,0E-05
1,0E-04
1,0E-03
1,0E-02
1,0E-01
1,0E+00
Measured k_in
Measured k_std
Measured k_end

(h)



(i)

Figure 2. Measured-versus-predicted hydraulic conductivity for the retained empirical equations relative to k_in, k_std, and k_end. The panels show where each method plots in conductivity space and whether it aligns more closely with the initial, standard interpreted, or final measured conductivity target. Porosity dependent panels include only materials for which sufficient density-related input data were available. Panel set: (a) Hazen; (b) Sherard; (c) Beyer; (d) Alyamani and Sen; (e) Shepherd; (f) USBR; (g) Kozeny-Carman; (h) Chapuis; (i) Slichter.

Hydraulic conductivity often varies by several orders of magnitude. For that reason, prediction error is better described by factor difference, that is, by how many times a prediction differs from the measured value, rather than by an absolute difference. The comparison was therefore carried out on a logarithmic basis so that equal overprediction and underprediction by the same factor were treated consistently.

For each method and conductivity target, the ratio k_pred/k_measured was transformed using log10 (k_pred/k_measured), for which zero indicates exact agreement, positive values indicate overprediction, and negative values indicate underprediction.

The signed median of this quantity was used as a supplementary bias indicator, that is, to show whether a method tends overall to overpredict or underpredict, while the median absolute log-ratio was used as the principal error metric. For practical interpretation, the corresponding typical factor error was derived from the absolute median log-ratio and expresses the typical factor by which predicted and measured conductivity differ. For example, a typical factor error of 2 means that predictions typically differ from measurements by about a factor of 2. The fractions of predictions within factors of 3 and 10 were considered as supplementary interpretive indicators, but the comparison presented in the main tables and figures is based primarily on typical factor error.

The stated applicability ranges of the empirical equations were taken from the original or later authoritative sources where available. Several materials in the present dataset lie partly outside those stated ranges with respect to grain size, grading, fines content, or data requirements. The equations were nevertheless evaluated across the full dataset to examine both nominal applicability and out-of-domain behaviour. The results should therefore be interpreted as comparative empirical performance rather than as strict validation within each method's originally intended range. The main domain definitions used for this interpretation are summarized in Table 3.

Table 3. Operational domain summary and best performing methods by domain. For each domain, the table gives the number of materials, observed descriptor ranges, the best gradation based and porosity dependent methods, the conductivity target most closely approximated, and the corresponding typical factor error.

Domain	n materials	Cu range	Fines < 0.063 mm (%)	D15 range (mm)	Measured k range (m/s)	Best gradation based method	Closest target	Typical factor error	Best porosity dependent method	Closest target	Typical factor error
Filter materials	8	2.2 - 135	0.5 - 11.1	0.118 - 6.3	1.30E−07 - 3.91E−02	Sherard	k_end	4.31	Chapuis	k_std	2.38
Till and core materials	4	16.8 - 129	13.6 - 37.4	0.0145 - 0.070	2.60E−08 - 6.98E−06	Beyer	k_std	2.61	Slichter	k_in	3.94
Tailings	4	7.1 - 16.5	19.9 - 33.9	0.0198 - 0.039	1.49E−06 - 1.43E−05	Hazen	k_std	1.59	Kozeny-Carman	k_std	1.38
Silt/fines dominated	1	5.4	76.3	0.0072	1.35E−08 - 5.18E−08	Shepherd	k_end	1.2	-	-	-

Because method input requirements differed, the number of valid predictions was method-specific. The gradation-based methods were evaluated on all 17 materials, whereas the porosity-dependent methods were evaluated only for the subset of materials with sufficient density-related input data. In the present dataset, this corresponds to 17 valid predictions for the gradation-based methods and 14 valid predictions for the porosity-dependent methods in the global comparison.

2.5. Material Domains and In-Domain Assessment

The primary domain grouping used in the study was:

- Filter materials
- Tills
- Tailings
- Silt/fines dominated

Sub-domains were used where relevant to distinguish natural filters, crushed filters, VSI filters, till materials used as impervious core soil, tailings, and crushed tailings-related materials. This domain structure formed the basis for the in-domain performance summaries and the practical recommendations derived from the comparison.

Because the recommendations are domain based, the defining properties of each domain are made explicit using the number of materials and the observed ranges of the most relevant descriptors, particularly Cu, fines content, D15, and measured hydraulic conductivity. These values are intended to show what materials are actually covered by the present recommendations, not to define universal classification boundaries.

3. Results

3.1. Dataset Coverage and Predicted-versus-Measured Behaviour

The dataset spans a broad range of gradations and conductivity levels across filter materials, tills, tailings, and one fines dominated silt. Figure 1 shows the particle-size distributions grouped by domain and illustrates both the breadth of the database and the overlap between some engineering classes.

Predicted-versus-measured conductivity plots for the retained empirical equations are presented in Figure 2. These show that method behaviour in conductivity space is strongly method-dependent. Some equations plot relatively close to the 1:1 line for one conductivity target but not for the others, whereas other methods show broader scatter or consistent over- or underprediction across the full range.

3.2. Global Comparison across *k*_in, *k*_std, and *k*_end

Across the entire dataset, most empirical methods aligned most closely with *k*_in. This was particularly evident for Hazen, Sherard, Alyamani and Sen, Shepherd, Kozeny-Carman, and Slichter in the global comparison. Chapuis was the clearest exception, aligning most closely with *k*_std, whereas Beyer aligned most closely with *k*_end.

Figure 3 summarizes the global comparison using typical factor error for each method against *k*_in, *k*_std, and *k*_end. The figure shows that method ranking depends not only on the selected equation but also on the conductivity target used for comparison. No single equation performed best across all three targets. A compact summary of the retained methods themselves is given in Table 3.

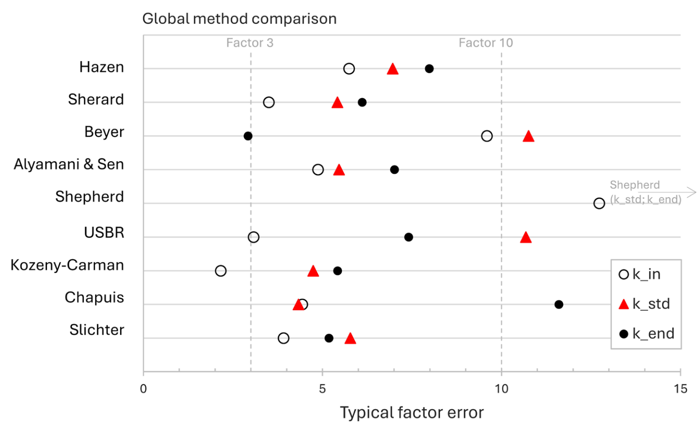


Figure 3. Typical factor error for the retained empirical hydraulic conductivity methods relative to *k*_in, *k*_std, and *k*_end for the entire dataset. Lower values indicate closer agreement between predicted and measured conductivity. Vertical reference lines indicate factors of 3 and 10.

3.3. Domain Dependence

Performance varied strongly by material domain. Figure 4 summarizes the best performing gradation based and porosity dependent methods for each main domain, and Table 3 gives the corresponding domain ranges, closest targets, and typical factor errors in tabulated form. Filter materials favoured porosity dependent methods, tills favoured Beyer, and tailings favoured Kozeny-Carman, while several gradation-based methods also performed well in the tailings domain.

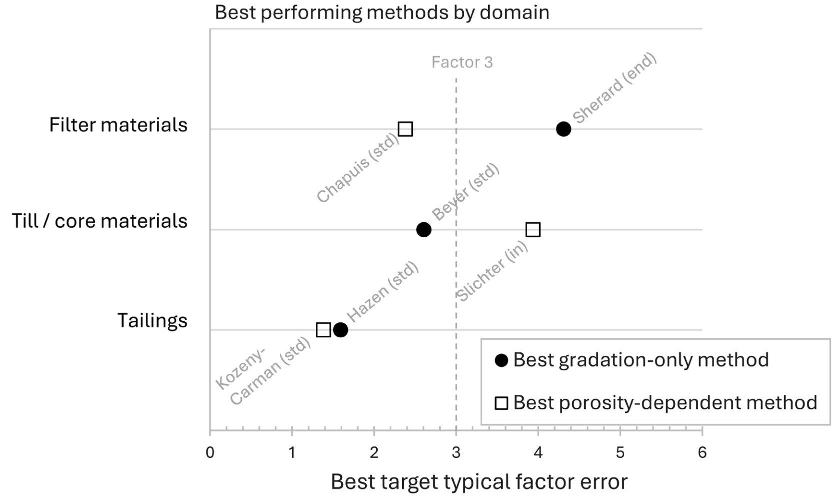


Figure 4. Best-performing methods by domain, showing the strongest gradation based and porosity dependent methods for the three multi-material domains using typical factor error at the conductivity target they most closely approximate. Lower values indicate closer agreement. The parenthetical term in each label identifies the conductivity target most closely approximated.

Within the filter domain, the porosity dependent methods gave substantially lower typical factor errors than the gradation-based methods. Within the till domain, Beyer was the strongest gradation-based method and compared favourably with the retained porosity dependent alternatives. Within the tailings domain, Kozeny-Carman performed best overall, but Hazen and USBR also gave comparatively strong results among the gradation-based equations.

4. Discussion

4.1. No Universal Best Method

The results show that no single empirical equation performed best across all materials and all conductivity targets. This is consistent with the broader empirical-equation literature, where multiple formulations continue to coexist and no universally applicable PSD-based method has emerged. This comparison reinforces that the apparent success of a given equation depends on both the measured conductivity target and the material class to which it is applied.

The domain-specific rankings should also be interpreted with appropriate caution because two of the domains contain only four materials and the fines dominated silt domain contains only a single material. Accordingly, the "best method" labels reported here should be understood as conditional on the present dataset rather than as universal recommendations. In particular, the single-material silt case is included as an illustrative outlier and is not sufficient to support a broader domain-level recommendation.

4.2. Why Target Conductivity Matters

The distinction between *k*_in, *k*_std, and *k*_end proved important. If only one measured conductivity had been used as the comparison target, several method-specific relationships would have been obscured. The global comparison showed that most methods aligned most closely with *k*_in, but this was not universal. Chapuis aligned more closely with *k*_std, whereas Beyer aligned more closely with *k*_end. This indicates that empirical equations should not be interpreted simply as predictors of hydraulic conductivity in the abstract. Rather, they appear to approximate different stages or states of measured conductivity evolution. More broadly, the comparison highlights the need for transparent guidance on when a hydraulic conductivity test should be considered complete or sufficiently stabilized for reporting, because different stopping points may produce materially different conductivity values.

4.3. Why Domain Matters

The domain-based comparison showed clear differences between filter materials, tills, and tailings. Part of this variation is likely related to application outside the original data domain of some equations. This is particularly relevant for broadly graded, fines-influenced, or low-conductivity materials, for which several classical formulas were not originally developed.

The filter domain strongly favoured porosity dependent methods, which is consistent with the importance of packing and pore geometry in relatively clean granular materials. In contrast, tills favoured Beyer, suggesting that incorporation of grain-size heterogeneity through uniformity may be particularly useful in lower-conductivity granular soils. Tailings showed strong performance from Kozeny–Carman, but also comparatively good performance from several gradation-based methods, indicating that this material class cannot be reduced to a single empirical behaviour. It should be noted that the tailings subset evaluated in the present study is limited in size. Subsequent analysis using an expanded dataset of non-plastic tailings-derived silty sands indictes that Sherard may provide a more robust gradation-based relation than Hazen, and that Slichter may outperform Kozeny-Carman among porosity-dependent methods within that restricted domain. These refinements do not contradict the present findings but rather reflect reflect improved resolution obtained from a larger and more domain-focused dataset. For preliminary assessment, the combined use of Hazen, Sherard, Slichter, and Kozeny-Carman may therefore be used to establish a bounded estimation range for tailings-derived materials. A related pattern was observed by the author in earlier infiltrometer and laboratory permeability comparisons on Swedish dam-related soils, where Hazen's equation with the commonly used C = 0.01 consistently overestimated hydraulic conductivity and better agreement was obtained with substantially lower effective C-values, approximately 0.0024 to 0.0043 [20]. In that small dataset, this corresponds approximately to reducing a factor error of about 3.8 to about 1.25. One of those low-conductivity soils belonged to the same source material family as material A in this study, although it was tested separately and at higher fines content.

4.4. Practical Implications

From a practical standpoint, the results suggest that empirical equations should be selected by material domain rather than applied universally. Where porosity or void-ratio data are available, porosity dependent methods may provide substantial improvement in some domains, particularly for filter materials. Where only grain-size data are available, the results support continued use of gradation-based methods, provided that the material domain and the likely conductivity target are considered explicitly. This is not only a statistical issue. Errors of one order of magnitude or more in estimated hydraulic conductivity may have real practical implications for seepage assessment, material screening, and preliminary design decisions.

For the till materials represented in this dataset, Beyer provided the most accurate overall performance and aligned most closely with standard interpreted conductivity (k_std), making it the preferred gradation-based method within this domain.

4.5. Limits for Fines Dominated Soils

The fines dominated silt case showed that some gradation-based methods can, in individual cases, give close agreement with measured conductivity. More broadly, the present comparison intentionally includes both within-domain and out-of-domain applications, because in practice these equations are often used beyond their original development range. However, only one such material was represented, and porosity dependent methods could not be evaluated for that case. No general recommendation can therefore be made for fines dominated soils based on the present dataset.

5. Conclusions

- Empirical performance depended on both the selected equation and the measured conductivity target used for comparison.
- No single equation performed best across all materials and conductivity targets. Most methods aligned most closely with initial measured conductivity (k_in), but clear exceptions were observed.
- Material domain was strongly influential. Filter materials favoured porosity dependent methods, tills favoured Beyer, and tailings showed strongest agree-

ment with Kozeny-Carman within the present dataset. For tailings-derived materials, these results should be interpreted as preliminary, pending confirmation from larger domain-specific datasets.

- In the till subset, Beyer was the clearest gradation-based choice and aligned most closely with standard interpreted conductivity (k_std).
- The results support domain-based method selection rather than a universal recommendation. Where density-related data are available, porosity dependent methods should generally be preferred, especially for filter materials.
- Overall, empirical equations do not predict one single abstract hydraulic conductivity value. Instead, they tend to align with different measured conductivity states depending on formulation and material domain.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Fell, R., MacGregor, P., Stapledon, D., Bell, G. and Foster, M. (2015) Geotechnical Engineering of Dams. 2nd Edition, CRC Press.

[2] Cedergren, H.R. (1989) Seepage, Drainage, and Flow Nets. 3rd Edition, Wiley.

[3] Wenzel, L.K. (1942) Methods for Determining Permeability of Water-Bearing Materials, with Special Reference to Discharging-Well Methods. U.S. Geological Survey Water-Supply Paper 887. U.S. Government Printing Office, Washington.

[4] Hazen, A. (1892) Some Physical Properties of Sands and Gravels, with Special Reference to Their Use in Filtration. *Annual Report of the Massachusetts State Board of Health*, **24**, 539-556.

[5] Slichter, C.S. (1899) Theoretical Investigation of the Motion of Ground Waters. In: *U.S. Geological Survey*, 19*th Annual Report*, *Part II*, Government Printing Office, 295-384.

[6] Kozeny, J. (1927) Über kapillare Leitung des Wassers im Boden. *Sitzungsberichte der Akademie der Wissenschaften in Wien*, **136**, 271-306.

[7] Carman, P.C. (1937) Fluid Flow through Granular Beds. *Transactions of the Institution of Chemical Engineers*, **15**, 150-166.

[8] Beyer, W. (1964) Zur Bestimmung der Wasserdurchlässigkeit von Kiesen und Sanden aus der Kornverteilungskurve. *Wasserwirtschaft Wassertechnik*, **14**, 165-168.

[9] Shepherd, R.G. (1989) Correlations of Permeability and Grain Size. *Groundwater*, **27**, 633-638. https://doi.org/10.1111/j.1745-6584.1989.tb00476.x

[10] Alyamani, M.S. and Şen, Z. (1993) Determination of Hydraulic Conductivity from Complete Grain-size Distribution Curves. *Groundwater*, **31**, 551-555. https://doi.org/10.1111/j.1745-6584.1993.tb00587.x

[11] Chapuis, R.P. (2004) Predicting the Saturated Hydraulic Conductivity of Soils: A Review. *Bulletin of Engineering Geology and the Environment*, **63**, 291-298.

[12] Odong, J. (2007) Evaluation of Empirical Formulae for Determination of Hydraulic Conductivity Based on Grain-Size Analysis. *Journal of American Science*, **3**, 54-60.

[13] Wang, J., François, B. and Lambert, P. (2017) Equations for Hydraulic Conductivity Estimation from Particle Size Distribution: A Dimensional Analysis. *Water Resources*

Research, **53**, 8127-8134. https://doi.org/10.1002/2017wr020888

[14] Urumović, K., Borović, S., Urumović, K. and Navratil, D. (2019) Validity Range and Reliability of the United States Bureau of Reclamation (USBR) Method in Hydrogeological Investigations. *Hydrogeology Journal*, **28**, 625-636. https://doi.org/10.1007/s10040-019-02080-2

[15] Goodarzi, M.R., Vazirian, M. and Niazkar, M. (2024) Hydraulic Conductivity Estimation: Comparison of Empirical Formulas Based on New Laboratory Experiments. *Water*, **16**, Article 1854. https://doi.org/10.3390/w16131854

[16] Swedish Standards Institute (SIS) (2016) SS-EN ISO 17892-3:2016. Geotechnical Investigation and Testing—Laboratory Testing of Soil—Part 3: Determination of Particle Density. SIS, Stockholm.

[17] ASTM International (2024) ASTM D1557-24: Standard Test Methods for Laboratory Compaction Characteristics of Soil Using Modified Effort (56,000 ft-lbf/ft^3 [2,700 kN-m/m^3]). ASTM International, West Conshohocken, PA.

[18] Sherard, J.L., Dunnigan, L.P. and Talbot, J.R. (1984) Basic Properties of Sand and Gravel Filters. *Journal of Geotechnical Engineering*, **110**, 684-700. https://doi.org/10.1061/(asce)0733-9410(1984)110:6(684)

[19] Grischek, T. and Beyer, K.D. (2019) Wolfgang Beyer: A Groundwater Scientist from Dresden. *Groundwater*, **57**, 980-983. https://doi.org/10.1111/gwat.12944

[20] Rönnqvist, H. (2018) Double-Ring Infiltrometer for In-Situ Permeability Determination of Dam Material. *Engineering*, **10**, 320-328. https://doi.org/10.4236/eng.2018.106022

List of Notations

k	hydraulic conductivity, m/s
k_pred	predicted hydraulic conductivity, m/s
k_measured	measured hydraulic conductivity, m/s
k_in	initial measured hydraulic conductivity, m/s
k_std	standard interpreted hydraulic conductivity, m/s
k_end	final measured hydraulic conductivity, m/s
Dx	particle size corresponding to x% passing, for example D10, D15, D17, D20, D30, D50, and D60, mm
Cu	coefficient of uniformity
n	porosity
e	void ratio
U	uniformity coefficient used in the Beyer equation
I_0	intercept parameter derived from the grading curve in the Alyamani and Sen equation
C and m	empirical coefficients in the Shepherd equation
C_{KC}	empirical coefficient in the adopted Kozeny-Carman working form
D_R	effective grain-size descriptor used in the Chapuis equation
RC	relative compaction (%), defined as ρ_d/MDD x 100
MDD	Maximum dry density obtained from Modified Proctor compaction testing (Mg/m^3), determined in accordance with ASTM D1557
ρ_s	particle density (Mg/m^3), determined in accordance with SS-EN ISO 17892-3:2016
ρ_d	dry density (Mg/m^3)