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A Grey-System Assessment of Energy Use, CO_2 Emissions and Sectoral Environmental Damage in South Africa's Electricity Sector

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Abstract

South Africa's reliance on coal-fired generation continues to dominate both its energy balance and its emissions inventory. Despite a sustained body of work on the country's electricity sector, comparatively little attention has been paid to grey-system techniques, which are useful when one wishes to rank heterogeneous alternatives or to project trajectories from short, partially uncertain time series. This paper applies Grey Relational Analysis (GRA) and the GM (1, 1) Grey Prediction Model to a dataset assembled from previously published national energy and CO_2 statistics with a sectoral input-output life-cycle inventory for 1990 to 2012. GRA ranks twelve high-impact industrial sectors against four damage categories, namely human health, ecosystem quality, climate change and resources, with weights set to 0.30/0.20/0.30/0.20. The procedure isolates Electricity ($\gamma = 0.344$) and Coal mining ($\gamma = 0.700$) as the only two sectors whose damage profile departs sharply from the ideal low-burden reference; the remaining ten cluster above $\gamma = 0.95$. The GM (1, 1) model is then fitted to series for GDP, CO_2 emissions, total primary energy supply (TPES) and total final consumption (TFC). All four models satisfy grade I (good) or grade II (qualified) GM (1, 1) accuracy criteria, with mean absolute percentage errors below 4%. Medium-term baseline projections to 2030 indicate continued upward pressure on emissions in the absence of structural change: CO_2 emissions reach approximately 657 Mt, TPES approximately 224 Mtoe and TFC approximately 102 Mtoe. The projected 2030 CO_2 level sits at the upper end of South Africa's Nationally Determined Contribution peak-plateau-decline band (398-614 Mt CO_2-eq), suggesting that the pledged trajectory could be difficult to achieve under the assumptions of the GM (1, 1) baseline. These results corroborate the earlier finding that mitigation effort should be concentrated on Eskom's generation fleet and the upstream coal value chain, and they quantify the extent to which mitigation must outpace the historical trend if the Just Energy Transition Investment Plan (JET-IP) closure schedule is to deliver the Nationally Determined Contribution (NDC) target.

Keywords

Grey Relational Analysis, GM (1, 1), South Africa, Electricity Sector, CO_2 Emissions, Input-Output Life-Cycle Assessment

1. Introduction

South Africa is, by most measures, the most industrialised economy on the African continent, and its development trajectory after 1994 has been closely tied to the supply of cheap electricity. The country produces roughly nine-tenths of that electricity from coal, and the consequences for atmospheric emissions are well documented: South Africa is the leading CO_2 emitter in Africa and around the thirteenth largest in the world [1] [2]. Eskom, the state-owned utility, supplies close to 95% of national demand, and the upstream coal supply chain accounts for an additional, less visible share of the environmental burden [3].

Earlier work has examined the relationship between South Africa's economic growth and its energy and emissions footprint through several lenses. OECD [4] and Tapio [5] decoupling indices have been used to compare environmental pressure against economic activity; index-decomposition techniques such as the Logarithmic Mean Divisia Index (LMDI) have been applied to identify the relative contribution of activity, intensity and structural effects; and the Kaya identity has helped attribute changes in emissions to population, affluence and energy intensity [5]-[8]. The input-output life-cycle approach has further been used to quantify direct and indirect environmental impacts attributable to each sector [9].

These methods, while productive, share a common limitation: they require relatively complete, low-noise datasets, and most are descriptive rather than predictive. They identify how things have changed, but they say comparatively little about how things might change next, or about how a heterogeneous group of sectors should be ranked when no single damage metric is decisive. Grey-system theory, introduced by Deng [10], was developed for situations in which the available information is partial, the sample size is modest, and the structure of uncertainty is not normal. Two of its tools are well suited to the South African case: Grey Relational Analysis (GRA), which compares observed sequences against an ideal reference to produce a single composite ranking, and the GM (1, 1) Grey Prediction Model, which fits a first-order grey differential equation to a short historical sequence to generate short- and medium-term forecasts.

This paper applies both techniques to data originally compiled by Beidari [3] in his doctoral dissertation at National Cheng Kung University. Two complementary objectives are pursued. First, the twelve sectors identified in the dissertation as having the largest environmental burden are ranked against four IMPACT 2002+ damage categories using GRA. Second, four national macro-energy indicators, GDP, CO_2 emissions, TPES and TFC, are projected to 2030 using GM (1, 1). The overall methodological framework is illustrated in **Figure 1**. The 2030 horizon is chosen deliberately: it spans the JET-IP coal-closure window (Komati 2022; Hendrina, Grootvlei and Camden 2023 to 2027) and reaches South Africa's Nationally Determined Contribution mid-point, which lets the grey-system baseline be read directly against the policy targets it might or might not meet. The intention is not to displace the methods already applied to this dataset, but to layer onto them a grey-system perspective that is robust to small samples and that suits the medium-horizon policy work South Africa's power-sector roadmap requires.

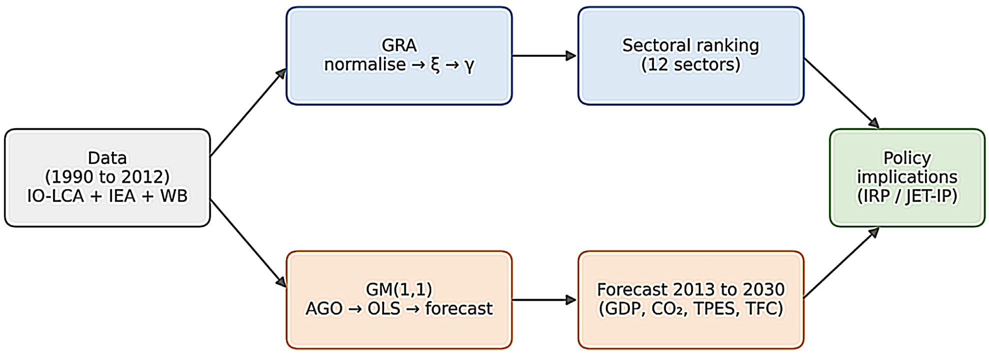

Figure 1. Methodological workflow. National statistics and the sectoral IO-LCA inventory feed two parallel grey-system analyses: a cross-sectional GRA producing a composite ranking of the twelve high-impact sectors, and four univariate GM (1, 1) forecasts to 2030. The two streams converge on a common policy reading.

2. Literature Review

Grey-system theory occupies a niche between deterministic mathematical modelling and probabilistic statistics. Deng [10] [11] proposed it to deal with systems whose information is, in his terminology, partly known and partly unknown, rather than white (fully known) or black (fully unknown). Two operational tools emerged early: the GM (1, 1) model, which uses an accumulated generating operator to convert irregular sequences into smoother ones before fitting a first-order linear differential equation, and Grey Relational Analysis, which measures the geometric similarity of one sequence to another and aggregates that similarity across several criteria. Both tools have since been refined extensively [12].

The grey-forecasting literature in energy and emissions modelling has grown substantially in the past five years. Ding, Xu, Ye, Zhou and Zhang [13] developed a discrete grey prediction model with multivariate structure and applied it to Chinese energy-related CO_2 emissions, reporting accuracy improvements over standard GM (1, 1) on noisier series. Ding and Zhang [14] extended this work with a new-information-based grey model for provincial CO_2 forecasts. On the methodological side, Wei, Xie and Yang [15] clarified the role of the cumulative-sum op-

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erator in grey prediction by recasting it as an integral matching problem, which sharpens the interpretation of the development coefficient. Sapnken, Noume and Tamba [16] demonstrated the applicability of grey models to African data by forecasting CO_2 emissions from road fuel combustion in Cameroon, a setting that shares with South Africa the combination of short, partially uncertain time series and a rapidly changing energy structure.

Hybrid grey-machine-learning approaches have also emerged as a productive line of work. Saxena, Zeineldin and Mohamed [17] coupled a polynomial-kernel grey model with an augmented crow-search optimiser to forecast energy consumption, electricity generation and CO_2 emissions jointly, reporting accuracy gains over standalone GM (1, 1) on small samples. These hybrids do not invalidate the simpler grey approach used here; they extend it for cases where additional explanatory series are available and where the added model complexity is justified by the data.

The South African literature on coal-transition modelling has expanded markedly since 2020 around the country's Just Energy Transition Investment Plan (JET-IP). Xaba [18] evaluated the implementation of the JET in coal-producing regions and documented the planned decommissioning sequence (Komati shut down in 2022; Hendrina, Grootvlei and Camden scheduled between 2023 and 2027). The South African quantitative literature, however, remains dominated by decoupling, decomposition and input-output analyses [7] [8] [19]. The present paper does not propose a new method; it brings established grey-system tools to bear on a dataset on which they have not, to the author's knowledge, been jointly used, and reads the resulting forecasts against the structural break that the JET-IP is meant to introduce.

3. Materials and Methods

3.1. Notation

Table 1 collects the symbols used throughout the methods and results.

3.2. Data

Two sub-datasets are drawn from Beidari [3]. The first is a 1990 to 2012 annual time series of GDP (constant 2005 US$, 10^{11} units), CO_2 emissions (Mt), total primary energy supply (Mtoe) and total final consumption (Mtoe). These series were reconstructed in the dissertation from IEA, World Bank and BP statistical reports and are reproduced here without modification (see **Table A1** in Appendix A). The second is the IMPACT 2002+ damage profile for the twelve sectors that contribute most to South Africa's aggregate environmental footprint, expressed in four damage categories: human health (DALY), ecosystem quality (PDF·m^2·yr), climate change (kg CO_2-eq) and resources (MJ primary). The twelve sectors were selected because they exhibited the highest aggregate environmental-damage scores in the 2012 IO-LCA inventory reported by Beidari [3], together accounting for the majority of total system-wide impacts across the four damage categories; the

Table 1. Notation.

Symbol	Meaning
$X^{(0)}$	Original (raw) time series, $x^{(0)}(1), \ldots, x^{(0)}(n)$
$X^{(1)}$	First-order accumulated generation of $X^{(0)}$
$z^{(1)}(k)$	Mean-generated background sequence, $0.5\left[x^{(1)}(k)+x^{(1)}(k-1)\right]$
a	Development coefficient in GM (1, 1) (sign and magnitude govern trajectory)
b	Grey input (forcing term) in GM (1, 1)
$\hat{x}^{(1)}(k+1)$	GM (1, 1) one-step prediction in cumulative space
$\varepsilon(k)$	Residual: $x^{(0)}(k)-\hat{x}^{(0)}(k)$
S_1, S_2	Standard deviations of the original and residual series
C	Posterior-deviation ratio, $C = S_2/S_1$ (smaller is better)
p	Small-error probability, $P\left(\left\|\varepsilon-\bar{\varepsilon}\right\|<0.6745 \cdot S_1\right)$
Δ_i	Absolute deviation in GRA: $\left\|x_{0j}-x_{ij}^{*}\right\|$
$\Delta_{\min}$, $\Delta_{\max}$	Minimum / maximum of Δ_i across alternatives and criteria
ζ	Distinguishing coefficient in GRA (set to 0.5 here)
ξ_i	Grey relational coefficient between alternative i and criterion j
w	Criterion weight ($\sum_j w_j = 1$)
γ_i	Composite grey relational grade of alternative i

normalised 2012 damage profile used in the ranking is reproduced in Table B1 of Appendix B. For the GRA stage the cross-section for 2012 is used, since this is both the most recent year in the dissertation's inventory and the year in which sectoral structure had stabilised after the post-apartheid expansion.

3.3. Grey Relational Analysis

Following Deng [11] and the more recent treatment in Liu and Lin [12], GRA proceeds in four steps. Let the original criteria matrix be $X = [x_{ij}]$ with m alternatives and n criteria. Each criterion is first normalised. Because the four damage categories are all of the smaller-is-better kind, the transformation in Equation (1) is applied, so that the best-performing alternative in each column receives a value of 1 and the worst a value of 0.

$$x_{ij}^{*} = \frac{\max_k x_{kj} - x_{ij}}{\max_k x_{kj} - \min_k x_{kj}} \tag{1}$$

The reference sequence is then the unit vector $x_0 = (1,1,1,1)$. The grey relational coefficient ξ_i is computed from the absolute deviation $\Delta_{ij} = \left|x_{0j} - x_{ij}^{*}\right|$ as

$$\xi_{ij} = \frac{\Delta_{\min} + \zeta \cdot \Delta_{\max}}{\Delta_{ij} + \zeta \cdot \Delta_{\max}} \tag{2}$$

with the distinguishing coefficient $\zeta = 0.5$, which is conventional. Finally, the composite grey relational grade is the weighted sum

$$\gamma_i = \sum_j w_j \cdot \xi_{ij}, \ \sum_j w_j = 1 \tag{3}$$

The weights adopted here are 0.30, 0.20, 0.30 and 0.20 for human health, ecosystem quality, climate change and resources respectively. This configuration gives slightly higher emphasis to direct human-health and climate impacts, in line with the policy priorities articulated in South Africa's Long-Term Mitigation Scenarios and consistent with the weighting choices in earlier IMPACT 2002+ studies for emerging economies [20]. The configuration should not be read as universally optimal; it reflects a policy-oriented prioritisation that emphasises climate and direct human-health burdens in coal-dependent emerging economies. Sensitivity to this choice is examined briefly in Section 4.1.

3.4. Grey Prediction Model GM (1, 1)

Given an original sequence $X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right)$ of $n \geq 4$ observations, the GM (1, 1) procedure first generates the cumulative sequence $X^{(1)}$ with $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$. A whitening differential equation is postulated,

$$\frac{\mathrm{d}x^{(1)}}{\mathrm{d}t} + a \cdot x^{(1)} = b \tag{4}$$

with development coefficient a and grey input b. After mean generation $z^{(1)}(k) = 0.5\left[x^{(1)}(k) + x^{(1)}(k-1)\right]$, the parameter vector $[a, b]^{\mathrm{T}}$ is obtained by ordinary least squares from

$$[a, b]^{\mathrm{T}} = \left(B^{\mathrm{T}} B\right)^{-1} B^{\mathrm{T}} Y, \ B = \left[-z^{(1)}(k), 1\right], \ Y = \left[x^{(0)}(k)\right] \tag{5}$$

Predicted cumulative values follow from the time-response function

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)\mathrm{e}^{-ak} + \frac{b}{a} \tag{6}$$

and predicted original values from inverse accumulation, $\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$. The quality of the fit is assessed by the posterior-deviation ratio $C = S_2/S_1$, where S_1 and S_2 are the standard deviations of the original series and the residual series, and by the small-error probability $p = P\left(\left|\varepsilon - \bar{\varepsilon}\right| < 0.6745 \cdot S_1\right)$. Under the four-grade scheme used by Liu and Lin [12], a model is considered good (grade I) if $C < 0.35$ and $p > 0.95$, and qualified (grade II) if $C < 0.50$ and $p > 0.80$.

Prior to model fitting, each series was subjected to the standard GM (1, 1) level-ratio admissibility check, which requires the consecutive level ratios $\lambda(k) = x^{(0)}(k-1) / x^{(0)}(k)$ to fall within the interval ($\mathrm{e}^{-2/(n+1)}$, $\mathrm{e}^{2/(n+1)}$), equal to (0.920, 1.087) for $n = 23$. The GDP, TPES and TFC series satisfied this condition across all consecutive pairs. The CO_2 series breached the lower bound in two early transitions (2002-2003 and 2003-2004), which is consistent with its grade-II classification; the accumulated-generating operation nonetheless smooths these departures sufficiently for the fitted model to meet the small-error criteria reported in Section 4.2.

Because GM (1, 1) is designed for limited-information systems rather than probabilistic forecasting, confidence intervals were not explicitly derived. Sensitivity tests that varied the estimation window by ±2 years produced qualitatively stable trajectories for all four indicators. The four indicators were fitted independently; the forecast horizon was set to 2013 to 2030, which gives eighteen out-of-sample years.

4. Results

4.1. Sectoral GRA

Table 2 summarises the GRA output for the twelve sectors, and Figure 2 plots the composite grades. Two features stand out. First, the distribution of grey relational grades is heavily bimodal: ten sectors lie above $\gamma = 0.95$, while Electricity (0.344) and Coal mining (0.700) fall well below. This is not an artefact of the normalisation or the weighting. Even with equal weights of 0.25 across the four categories, Electricity and Coal mining remain the two lowest-ranked sectors: the Electricity grade shifts by only about 0.3 percentage points (from $\gamma = 0.344$ to 0.347), whereas the Coal mining grade shifts by about 4.9 percentage points (from $\gamma = 0.700$ to 0.651). The ranking conclusion is therefore unchanged, although the Coal mining grade is more sensitive to the weighting than a single threshold would suggest. Second, the gap is widest in the human-health and climate-change columns. The grey relational coefficient of Electricity is at the floor (0.333) in three of the four categories, which reflects the fact that the sector's normalised damage values are

Table 2. Grey relational coefficients (ξ) and composite grade (γ) of the twelve high-impact sectors. The smaller-is-better polarity means that higher γ corresponds to a lower aggregate environmental burden.

Sector	Human health	Ecosystem	Climate	Resources	γ (grade)	Rank
Electricity (gen., trans., distrib.)	0.333	0.333	0.333	0.387	0.344	12
Coal mining	0.918	0.483	0.871	0.333	0.700	11
Monetary authorities/credit	0.971	0.984	0.980	0.889	0.960	10
Scientific R&D services	0.984	0.942	0.971	0.985	0.972	9
Wholesale trade	0.977	0.968	0.992	0.985	0.981	8
Construction-machinery mfg.	0.995	0.955	0.997	0.998	0.988	7
Transit & ground transport	0.998	0.994	0.998	0.997	0.997	6
Computer terminals & periph.	0.999	0.991	0.999	0.999	0.997	5
Iron, steel & ferroalloy	0.998	0.994	0.999	0.999	0.998	4
Petrochemical mfg.	1.000	0.998	1.000	1.000	0.999	3
Other miscellaneous electrical	1.000	1.000	1.000	1.000	1.000	2
Community food/housing/relief	1.000	1.000	1.000	1.000	1.000	1

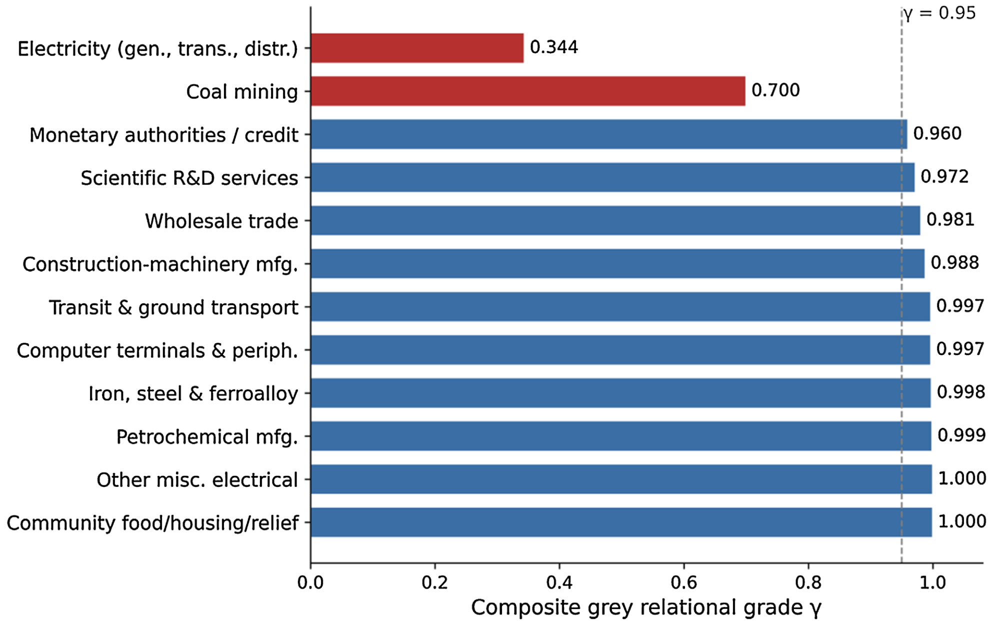

Figure 2. Composite grey relational grade γ for the twelve high-impact sectors. The dashed vertical line at $\gamma = 0.95$ marks the cluster threshold; only Electricity and Coal mining fall clearly below it.

also at the floor. It is, by orders of magnitude, the largest contributor along those axes.

The interpretation is straightforward. From the perspective of integrated environmental performance, two sectors require attention; the others do not, at least not as a matter of priority. This is consistent with what Beidari, Lin and Lewis [8] reported using more granular IO-LCA tools, but it is presented here as a single composite number per sector, which is more useful for ranking and for policy communication.

It is worth pausing on the rank of Monetary authorities and depository credit (10th rather than higher), which may strike the reader as counter-intuitive for a service activity. The reason is visible in the dissertation's sectoral tables: between 2010 and 2012 the sector's indirect resource footprint grew by close to two orders of magnitude, mainly because of the financial-services capital expansion that followed the 2010 World Cup investment cycle. GRA picks this up, where a single-criterion ranking would have missed it.

4.2. GM (1, 1) Forecasts

The four GM (1, 1) models were estimated on twenty-three annual observations each. Table 3 reports their parameters and accuracy metrics; the development coefficient a is negative in all four cases, which corresponds to an exponential-growth pattern of the underlying series. The CO_2 series produces the largest a in absolute terms (−0.018), which is small enough that the projected trajectory approximates quasi-linear growth over the forecast horizon while still retaining the exponential structure inherent to the GM (1, 1) formulation.

By the criteria of Liu and Lin [12], three of the four models qualify as grade I

Table 3. GM (1, 1) parameters and goodness-of-fit indicators for the four macro-energy indicators of South Africa, 1990 to 2012.

Indicator	a	b	MAPE (%)	C	p
GDP (10^{11} US$)	−0.0327	1.483	1.91	0.123	1.00
CO_2 emissions (Mt)	−0.0181	315.89	3.70	0.403	0.91
TPES (Mtoe)	−0.0229	88.64	2.92	0.244	1.00
TFC (Mtoe)	−0.0191	46.90	2.57	0.243	1.00

(good): GDP, TPES and TFC each have C below 0.35 and p of 1.00. The CO_2 model falls within the qualified range (grade II), with C of 0.40 and p of 0.91, a sensible outcome given that the historical CO_2 series contains a clear 2008-2009 spike during the global financial crisis followed by a load-shedding-driven drop in 2010. This structural noise cannot, by construction, be fully absorbed by a smooth grey kernel.

Figure 3 presents the historical (1990-2012) series and the GM (1,1) forecast trajectories (2013-2030) for GDP, CO_2 emissions, total primary energy supply (TPES), and total final consumption (TFC). The figure provides a visual comparison between the observed data and the projected baseline trends used in the subsequent analysis.

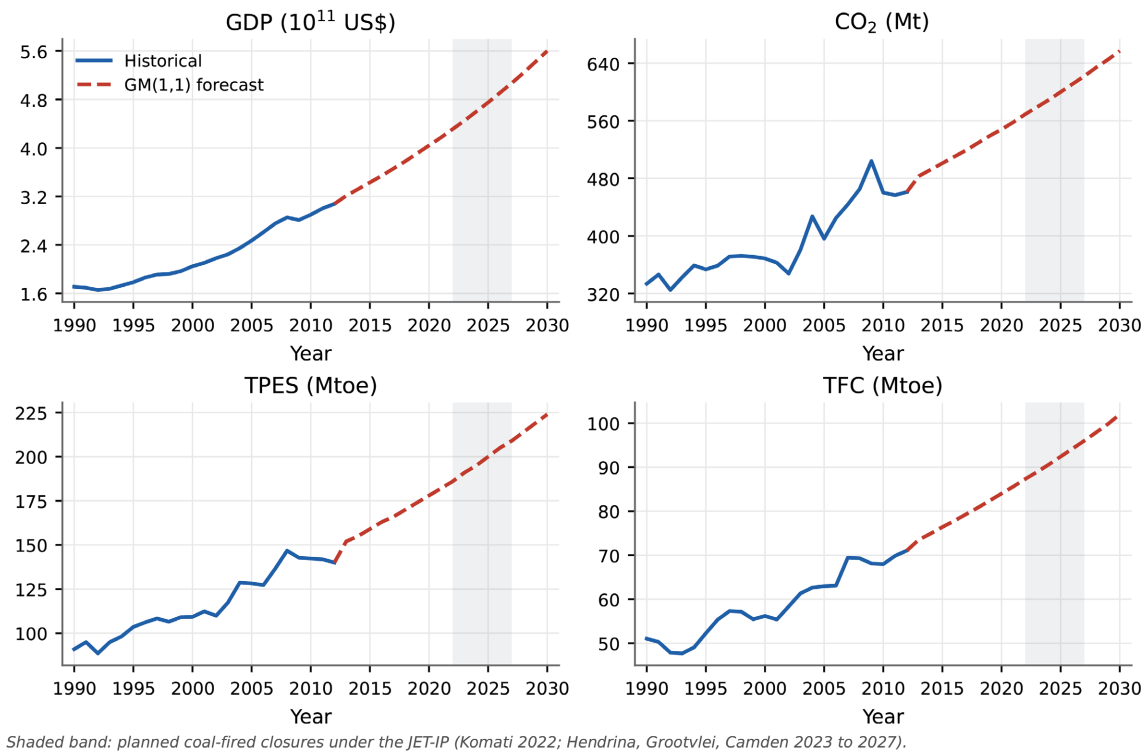

Figure 3. Historical 1990 to 2012 series (solid) and GM (1, 1) forecasts 2013 to 2030 (dashed) for GDP, CO_2, TPES and TFC. Forecast values are taken from **Table 4**. The shaded band marks the JET-IP coal-closure window (Komati 2022; Hendrina, Grootvlei, Camden 2023 to 2027), over which the policy schedule departs from the GM (1, 1) baseline.

Table 4. GM (1, 1) forecasts of GDP, CO_2 emissions, TPES and TFC of South Africa, 2013 to 2030. Values rounded to three significant figures.

Year	GDP (10^{11} US$)	CO_2 (Mt)	TPES (Mtoe)	TFC (Mtoe)
2013	3.21	483	152	73.5
2014	3.32	492	155	74.9
2015	3.43	501	159	76.4
2016	3.54	510	163	77.8
2017	3.66	519	166	79.3
2018	3.78	529	170	80.8
2019	3.91	539	174	82.4
2020	4.04	548	178	84.0
2021	4.17	558	182	85.6
2022	4.31	569	186	87.3
2023	4.45	579	191	88.9
2024	4.60	589	195	90.6
2025	4.75	600	200	92.4
2026	4.91	611	205	94.2
2027	5.07	622	209	96.0
2028	5.24	634	214	97.9
2029	5.42	645	219	99.8
2030	5.60	657	224	102

Three observations follow. First, the GDP forecast of 5.60×10^{11} US$ for 2030 corresponds to an average annual growth rate of about 3.3% over the eighteen-year projection horizon, broadly consistent with the trend implied by post-2012 IMF and National Treasury baselines, though more optimistic than the realised outcomes for 2014 to 2019 and clearly inconsistent with the load-shedding-constrained growth of 2020 to 2024. Although the GM (1, 1) projections were generated using data ending in 2012, comparison with subsequently published Global Carbon Project statistics suggests that the model correctly captured the persistence of upward energy and emissions pressure, even though realised GDP growth after 2015 fell below the extrapolated baseline because of prolonged electricity supply constraints and macroeconomic stagnation.

To make this check explicit, Table 5 compares the GM (1, 1) CO_2 baseline against observed energy-related CO_2 emissions for five post-sample years, drawn from the Global Carbon Project territorial-emissions series. The forecast tracks observed emissions to within about 6% in 2013 but diverges progressively thereafter, reaching roughly +26% by 2020 and +33% by 2022. This widening gap is precisely what the counterfactual reading of the baseline predicts: realised emissions fell below the no-structural-change trajectory as load-shedding, the COVID-

19 contraction and the early stages of the energy transition began to take effect. The ex-post comparison therefore supports, rather than undermines, the interpretation of the GM (1, 1) path as a baseline against which mitigation progress can be measured.

Table 5. Ex-post comparison of the GM (1, 1) baseline CO_2 projection with observed energy-related CO_2 emissions (Global Carbon Project), 2013 to 2022.

Year	GM (1, 1) forecast (Mt)	Observed (Mt)	Difference (Mt)	Error (%)
2013	483	456.2	+26.8	+5.9
2015	501	457.5	+43.5	+9.5
2019	539	470.7	+68.3	+14.5
2020	548	435.3	+112.7	+25.9
2022	569	428.8	+140.2	+32.7

Observed values are territorial fuel-combustion CO_2 emissions reported by the Global Carbon Project.

Second, CO_2 emissions are projected to reach roughly 657 Mt in 2030, an increase of about 42% relative to 2012. The implied carbon intensity of GDP (CO_2/GDP) falls from 150 Mt per 10^{11} US$ in 2012 to about 117 in 2030, a continuation of the weak-decoupling pattern documented for 1994 to 2010 by Lin, Beidari and Lewis [7], but well short of the strong-decoupling regime observed only during 2010 to 2012. Third, and most consequential for policy, the 2030 CO_2 projection sits at the upper end of South Africa's NDC peak-plateau-decline band (398 - 614 Mt CO_2-eq); it exceeds the upper bound by about 7%.

This comparison should be interpreted with caution, because the GM (1, 1) forecast refers to energy-related (fuel-combustion) CO_2, whereas the NDC peak-plateau-decline band is expressed in economy-wide CO_2-equivalent terms that include all greenhouse gases and the land-use, land-use-change and forestry (LULUCF) sector. The two quantities therefore differ in both gas coverage and system boundary, so the '7% above the upper bound' figure should be read as an order-of-magnitude indication rather than a strict like-for-like accounting comparison. Accordingly, the comparison is intended as a policy-oriented benchmark rather than a strict emissions-accounting equivalence. Put another way, the grey-system forecasts say what the country's own decoupling history suggests: progress is being made on the intensity of emissions, but absolute emissions continue to rise, and on the structural assumptions implicit in the GM (1, 1) baseline the country misses its NDC. Whether the realised 2030 outcome lands inside the NDC band depends on how completely the JET-IP coal-closure schedule and the REIPPPP renewable build-out displace the baseline trajectory. The 2030 horizon should therefore be interpreted as a medium-term policy baseline rather than a deterministic long-range prediction.

5. Discussion

The two stages of the analysis address different questions but converge on a common message. The GRA stage tells us, with relatively little ambiguity, that South Africa's integrated environmental footprint is overwhelmingly produced by two sectors. The GM (1, 1) stage tells us that, on present trends, the absolute level of pressure those sectors generate is likely to keep rising for at least another decade. Both findings echo, in compressed form, the conclusions of Beidari's dissertation, where the input-output life-cycle analysis identified electricity and coal mining as the principal contributors and the LMDI decomposition attributed most of the upward push to the economic-activity term.

Three caveats are worth noting. First, the GM (1, 1) model assumes that the underlying trajectory is smooth in the cumulative sense. Structural breaks, such as a rapid build-out of renewables, an unanticipated decline in coal generation, or a sustained slowdown in GDP growth, will be absorbed only sluggishly by the model. The 2030 horizon adopted here in fact straddles the JET-IP coal-closure window (Komati 2022; Hendrina, Grootvlei and Camden 2023 to 2027) and the REIPPPP renewable build-out, neither of which the GM (1, 1) baseline sees. The medium-term forecast of CO_2 emissions should therefore be read as a counterfactual baseline of 'no major structural change', not as a prediction of what will actually happen; the literature on JET-IP implementation [18] makes plain that a structural break is in fact under way. The gap between the grey-system baseline and realised emissions over 2023 to 2030 will be informative about the pace of that transition. Second, the GRA results are sensitive to the choice of damage categories. IMPACT 2002+ aggregates a large number of impact midpoints into the four damage endpoints used here; an analysis based on midpoints (for example particulate matter or stratospheric ozone) would produce slightly different rankings, although the dominance of Electricity and Coal mining would survive. Third, the analysis stops at 2012 because that is where the dissertation's sectoral inventory ends. An updated dataset incorporating Eskom's post-2015 emissions and the early effects of the REIPPPP would be a natural extension and would let the GM (1, 1) forecast be refitted on a longer base period that spans the structural break.

Even so, a few policy implications are reasonably clear. For the electricity sector, the priority is the same one argued in successive Integrated Resource Plans: substitute non-coal capacity for retiring coal plants, and where coal is retained, push the fleet toward higher-efficiency, supercritical or ultra-supercritical units with retrofit emission controls. For the coal-mining sector, whose resource burden is, on the IO-LCA, almost as large as Electricity's climate burden, the natural lever is the demand side: anything that reduces coal-fired generation also reduces upstream mining throughput, and the two sectors should be treated jointly in mitigation accounting. On the macro side, the GM (1, 1) baseline trajectory of TPES (224 Mtoe by 2030) and CO_2 (657 Mt by 2030) sits clearly above what would be needed to land inside South Africa's NDC peak-plateau-decline band of 398-614 Mt CO_2-eq on a coal-dominated mix. Closing that gap requires the planned coal-fired closures to remove at least 43 Mt of annual CO_2 by 2030 relative to the GM (1, 1) trajectory; on the IRP 2019 retirement schedule (Komati 2022; Hendrina, Grootvlei and Camden 2023 to 2027; Arnot 2029; Kriel 2029) that is plausible, but only if delays to coal-plant decommissioning of the kind documented in 2023 to 2024 do not recur. Where they do, the JET-IP target will require either a faster fuel switch than presently scheduled or an explicit demand-management lever such as carbon pricing.

6. Conclusions

Grey-system analysis is a natural complement to the methods already deployed for the South African electricity sector. It does not require large samples, it is comparatively transparent in its algebra, and it produces both rankings (via GRA) and forecasts (via GM (1, 1)) from the same data. Applied here, it supports what input-output, decomposition and decoupling analyses have separately suggested: two sectors dominate South Africa's environmental footprint, and the country's aggregate energy and emissions trajectory is on a slowly rising path that needs an explicit structural intervention to bend. The GRA composite grade reduces the ranking question to a single number per sector, useful for policy communication, while the GM (1, 1) forecasts to 2030 provide a small-sample-friendly reference trajectory against which alternative scenarios can be compared. The GM (1, 1) trajectories should therefore be read primarily as counterfactual baseline pathways against which the effectiveness of South Africa's ongoing energy transition can be evaluated.

The headline number is uncomfortable: on the GM (1, 1) baseline, South Africa's 2030 CO_2 emissions reach about 657 Mt, which exceeds the upper bound of its NDC peak-plateau-decline band by roughly 7%. As noted in Section 4.2, this headline comparison carries a unit and boundary caveat, since the forecast captures energy-related CO_2 while the NDC band is economy-wide CO_2-equivalent; the qualitative conclusion that the baseline path overshoots the pledged trajectory is nonetheless robust. The JET-IP coal-closure schedule must therefore not merely happen but happen on time.

Three extensions suggest themselves. The Verhulst variant of GM, suitable for series approaching a saturation level, could be tested on TFC; a rolling GM (1, 1) estimated on the most recent four to six observations could supply a near-term forecast that responds more quickly to structural breaks; and a GRA with criterion weights elicited from South African energy-policy stakeholders would give a richer view than the policy-oriented weighting used here. Hybrid grey-machine-learning variants of the kind explored by Saxena, Zeineldin and Mohamed [17] offer a further line of extension when additional explanatory series become available. Each of these would build on, rather than displace, the framework presented in this paper.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix A. Source Data for the GM (1, 1) Models, 1990 to 2012

Table A1 reports the annual GDP, CO_2 emissions, total primary energy supply (TPES) and total final consumption (TFC) series used to estimate the GM (1, 1) models. The data are reproduced directly from Beidari (2017), who compiled the series from International Energy Agency (IEA), World Bank and BP Statistical Review sources. The inclusion of the complete dataset allows full replication of the GM (1, 1) estimation procedure and addresses the reproducibility requirement of the present study.

Table A1. Annual GDP, CO_2 emissions, TPES and TFC for South Africa, 1990 to 2012.

Year	GDP (10^{11} US$)	CO_2 (Mt)	TPES (Mtoe)	TFC (Mtoe)
1990	1.709	333.514	90.956	51.048
1991	1.692	346.337	94.981	50.293
1992	1.656	324.852	88.589	47.865
1993	1.676	342.549	94.940	47.703
1994	1.730	358.930	98.168	49.059
1995	1.784	353.458	103.581	52.286
1996	1.861	358.640	106.143	55.368
1997	1.910	371.328	108.374	57.318
1998	1.920	372.219	106.517	57.164
1999	1.965	371.034	109.055	55.455
2000	2.047	368.611	109.264	56.195
2001	2.103	362.743	112.399	55.394
2002	2.180	347.687	109.908	58.355
2003	2.244	380.811	117.374	61.334
2004	2.347	427.132	128.723	62.635
2005	2.471	396.117	128.214	62.963
2006	2.609	424.844	127.255	63.09
2007	2.754	443.648	136.604	69.432
2008	2.853	465.023	146.768	69.308
2009	2.81	503.941	142.76	68.111
2010	2.898	460.124	142.291	67.975
2011	3.002	456.576	141.888	69.861
2012	3.076	461.095	140.004	71.072

Source: Beidari, M. (2017) Integrated Study of Energy Consumption, CO_2 Emissions and Input-Output Life Cycle Assessment for the Electricity Sector in South Africa. Doctoral Dissertation, National Cheng Kung University, Tainan [3].

Appendix B. Raw Environmental Damage Values for the Twelve Highest-Impact Sectors (2012)

Table B1. Raw IMPACT 2002+ endpoint damage values for the twelve highest-impact sectors in South Africa's electricity supply chain (2012).

Sector	Human health (DALY)	Ecosystem (PDF·m²·yr)	Climate (kg CO_2-eq)	Resources (MJ primary)
Electric power generation, transmission, and distribution	1.0E+05	1.1E+09	9.8E+10	1.9E+12
Coal mining	4.5E+03	5.9E+08	7.3E+09	2.4E+12
Wholesale trade	1.2E+03	1.9E+07	4.2E+08	1.9E+10
Transit and ground passenger transportation	1.2E+02	4.2E+06	1.0E+08	4.2E+09
Monetary authorities and depository credit intermediation	1.5E+03	1.0E+07	1.0E+09	1.5E+11
Construction machinery manufacturing	2.9E+02	2.7E+07	1.5E+08	3.5E+09
Petrochemical manufacturing	4.5E+01	2.2E+06	3.8E+07	1.3E+09
Community food, housing, and other relief services, including rehabilitation services	2.6E+01	9.7E+05	2.2E+07	9.0E+08
Scientific research and development services	8.4E+02	3.5E+07	1.5E+09	1.9E+10
All other miscellaneous electrical equipment and component manufacturing	2.6E+01	1.1E+06	2.2E+07	7.6E+08
Iron and steel mills and ferroalloy manufacturing	1.5E+02	4.2E+06	7.2E+07	2.4E+09
Computer terminals and other computer peripheral equipment manufacturing	9.6E+01	5.7E+06	8.5E+07	2.2E+09

Source: Beidari, M. (2017). Integrated Study of Energy Consumption, CO_2 Emissions and Input-Output Life Cycle Assessment for the Electricity Sector in South Africa. Doctoral Dissertation, National Cheng Kung University, Chapter 6, Table 6-3.

These endpoint damage values were generated using the IMPACT 2002+ life-cycle impact assessment methodology and constitute the raw environmental damage data from which the normalized values used in the Grey Relational Analysis were derived. The normalized scores employed in the GRA calculations are reported in Table 2 of the main text.