

Single Internal Fault Diagnosis for Unstable 1D Wave Equation under Boundary Disturbance

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Abstract

This paper proposes a fault detection and estimation method for an unstable one-dimensional wave equation based on an adaptive extended state observer (ESO), which relies solely on boundary displacement and velocity measurements. First, an infinite-dimensional naive observer is designed without using modal approximations, paired with a corresponding fault detection filter (FDF) and a residual evaluation mechanism to achieve reliable fault identification. Subsequently, once a fault occurs, an adaptive update law is embedded into the ESO to enable online estimation of the fault amplitude, requiring prior knowledge of the fault shape function. Finally, numerical simulation results verify that the proposed method can rapidly detect the occurrence of faults and accurately track their dynamic evolution.

Keywords

Wave Equation, Fault Detection, Adaptive Threshold, Lyapounov Function

1. Introduction

Distributed parameter systems (DPSs) are widely applied in engineering fields, including acoustics [1], fluid mechanics [2] and petroleum exploitation [3]. Their states evolve synchronously with time and space [4] [5], exhibiting typical infinite-dimensional characteristics. In the research framework of DPSs, fault diagnosis technology boasts extensive application scenarios and prominent core value, acting as a key technique to ensure the safe and stable operation of such systems.

Compared with lumped parameter systems, faults in DPSs present distinct spatiotemporal coupling characteristics. Moreover, they are vulnerable to the super-

imposed effects of external disturbances, parametric uncertainties and measurement noise, which makes it difficult to effectively extract fault signals and significantly increases the difficulty of fault diagnosis [6]-[9]. Three main categories can be classified among existing classical fault diagnosis methods. Firstly, lumped parameter methods based on finite-dimensional approximation reduce computational complexity at the cost of losing the system's high-order modal information [10]-[12], making it hard to accurately depict the spatiotemporal distribution of faults. Secondly, operator-based methods under the framework of infinite-dimensional theory can fully preserve the inherent properties of the system, but are plagued by cumbersome mathematical derivations and unsatisfactory real-time performance [13]-[15], failing to meet the real-time diagnostic requirements of industrial sites. Thirdly, data-driven methods dispense with constructing an explicit mathematical model, yet their diagnostic performance highly relies on data quality and the interpretability of results is poor [16]-[18], which renders them incompetent to accomplish fault identification tasks under complex operating conditions. Accordingly, as an important branch of the existing technical system, observer-based fault diagnosis methods have gradually become a research hotspot in the field of DPS fault diagnosis, owing to their advantages of clear physical mechanism and favorable real-time performance [19] [20]. Nevertheless, two core bottlenecks still exist in current observer-based approaches. Some methods rely on finite-dimensional approximation models and inherit the inherent defects of lumped parameter methods. Most methods fail to fully consider the impact of uncertain disturbances on residual signals, and the adoption of fixed thresholds tends to induce false alarms and missed alarms [10] [21], making them difficult to adapt to the diagnostic requirements of complex industrial scenarios.

Notably, the one-dimensional wave equation, as a core partial differential equation model describing the dynamic characteristics of DPSs, can accurately characterize practical engineering physical processes such as string vibration [22], pipeline wave propagation [23] and rod pump systems in petroleum exploitation [24], thus possessing significant application value in industrial production. The stable operation of such systems is directly related to production efficiency and operational safety, while typical faults, including rod fracture and pump valve leakage [25]-[27] are prone to triggering major production risks. Therefore, conducting fault diagnosis research on DPSs described by the one-dimensional wave equation is of great theoretical significance and engineering application value [28]-[31].

Therefore, constructing infinite-dimensional observers without finite-dimensional approximation, realizing the effective decoupling of faults and disturbances, and designing adaptive thresholds to improve diagnostic robustness have become key urgent problems to be tackled in the fault diagnosis of DPSs governed by the one-dimensional wave equation [30]. This paper proposes a fault diagnosis scheme for one-dimensional wave equation systems that balances robustness and accuracy, aiming to provide reliable technical support for the accurate fault diagnosis of such complex spatiotemporal evolutionary systems.

The novel contributions of the present study are as follows:

- 1) Compared with [28], which only achieves fault detection, this study not only realizes fault detection but also further accomplishes fault estimation, thus providing a more comprehensive fault diagnosis solution.
- 2) In contrast to the studies in [30] [31], which do not consider system disturbances, this paper focuses on external disturbances acting on one end of the system boundary.
- 3) Unlike the method in [29], which relies on global measurements, our approach requires only boundary measurements to perform both fault detection and estimation.

This paper is structured as follows: Section 2 focuses on problem formulation, specifically clarifying the system model and key assumptions; Section 3 proposes and conducts in-depth analysis of the fault detection and estimation scheme, accompanied by rigorous theoretical proofs; Section 4 verifies the effectiveness of the proposed method through numerical simulation experiments; Section 5 summarizes the full paper and outlines future research directions.

The following notation is adopted throughout this paper: $\|\varpi\| = \sqrt{\int_0^1 \varpi^2(\vartheta) d\vartheta}$, $\varpi_t = \frac{\partial \varpi}{\partial t}$, $\varpi_{tt} = \frac{\partial^2 \varpi}{\partial t^2}$, $\varpi_{\vartheta} = \frac{\partial \varpi}{\partial \vartheta}$, $\varpi_{\vartheta\vartheta} = \frac{\partial^2 \varpi}{\partial \vartheta^2}$. For arbitrary real numbers x, y and a positive real constant $\kappa > 0$, the following Young's inequality holds

$$xy \leq \frac{\kappa}{2} x^2 + \frac{1}{2\kappa} y^2, xy \geq -\frac{\kappa}{2} x^2 - \frac{1}{2\kappa} y^2.$$

Furthermore, the Cauchy-Schwarz inequality holds

$$\int_0^1 g_1(\vartheta) g_2(\vartheta) d\vartheta \leq \|g_1(\vartheta)\| \|g_2(\vartheta)\|.$$

2. Problem Statement

This study focuses on a one-dimensional wave equation modeled by the following naive system

$$\begin{cases} \zeta_{tt}(\vartheta, t) = \zeta_{\vartheta\vartheta}(\vartheta, t) + q\zeta(1, t) + \varphi(\vartheta, t), \\ \zeta(0, t) = 0, \\ \zeta_{\vartheta}(1, t) = Q(t) + \Lambda(t), \\ y(t) = \{\zeta(1, t), \zeta_t(1, t)\}, \\ \zeta(\vartheta, 0) = \zeta_0(\vartheta), \zeta_t(\vartheta, 0) = \zeta_1(\vartheta), \end{cases} \quad (2.1)$$

where $\zeta(\cdot, t)$ represents the state variable, $q \in R^+$ is a positive real constant, $t \in [0, +\infty)$ corresponds to the time variable, and $\vartheta \in [0, 1]$ is the spatial coordinate. Moreover, $\varphi(\vartheta, t)$ denotes an unknown distributed fault of the system, while $\Lambda(t)$ corresponds to the disturbance confined to the range $|\Lambda(t)| \leq \bar{\Lambda}$. $Q(t)$ stands for a boundary control input signal, $\zeta(1, t)$ and $\zeta_t(1, t)$ are measurable boundary states in the system.

Assumption 2.1. We assume that there exists exactly one fault occurring within

this time period, i.e., no multiple faults arise. This fault is an internal additive fault of the system. Furthermore, this study focuses on fault diagnosis and estimation.

Assumption 2.2. The distributed fault addressed in this paper is assumed to take a separable spatial-temporal form

$$\varphi(\vartheta, t) = I(\vartheta)\varphi(t),$$

where the spatial profile of the fault is described by the piecewise-constant function

$$I(\vartheta) = \begin{cases} 1, & \vartheta_0 - \frac{\ell}{2} < \vartheta < \vartheta_0 + \frac{\ell}{2}, \\ 0, & \text{otherwise,} \end{cases}$$

where $\vartheta_0 \in (0, 1)$ denotes the center of the fault region and $\ell \in (0, 1)$ represents its spatial extent.

Assumption 2.3. The fault signal $\varphi(t)$ is bounded in the following sense:

$$\dot{\varphi}^2(t) \leq \varphi_0, \quad \varphi^2(t) \leq \varphi_1,$$

where φ_0 and φ_1 are given positive constants.

3. Fault Detection Scheme

Focusing on the one-dimensional unstable wave equation, this paper addresses the diagnosis problem of unknown bounded distributed faults with a spatio-temporally separable form, where only boundary displacement and velocity are adopted as measurable outputs. The core objective is to design a FDF and an adaptive ESO. A boundary observer without fault compensation is developed, and the corresponding state estimation error system is established. On this basis, a fault detection logic is constructed: the residual is generated from the discrepancy between the observer output and the measured boundary signal, $r(t) > R^d(t)$ fault detected, and $r(t) \leq R^d(t)$ no fault detected, thus guaranteeing the reliability of the detection process.

3.1. Fault Detection

The fault detection scheme is developed in this section. We design the following observer

$$\begin{cases} \hat{\zeta}_u(\vartheta, t) = \hat{\zeta}_{\vartheta\vartheta}(\vartheta, t) + q\zeta(1, t), \\ \hat{\zeta}(0, t) = 0, \\ \hat{\zeta}_{\vartheta}(1, t) = Q(t) + k(\zeta(1, t) - \hat{\zeta}_i(1, t)), \end{cases} \quad (3.1)$$

where $k > 0$ is tuned constant parameter.

Subsequently, the residual signal is introduced

$$r(t) = \zeta(1, t) - \hat{\zeta}(1, t) = \varpi(1, t). \quad (3.2)$$

In the fault-free operation of the system, i.e., $\varphi(\vartheta, t) = 0$, by subtracting (2.1)

from (3.1), the corresponding state residual dynamical system can be formulated as

$$\begin{cases} \varpi_{tt}(\vartheta, t) = \varpi_{\vartheta\vartheta}(\vartheta, t), \\ \varpi(0, t) = 0, \\ \varpi_{\vartheta}(1, t) = \Lambda(t) - k\varpi_t(1, t), \end{cases} \quad (3.3)$$

where $\varpi(\vartheta, t) = \zeta(\vartheta, t) - \hat{\zeta}(\vartheta, t)$.

Lemma 3.1. [32] Let $F(t)$ and $g(t)$ denote regular real-valued functions. Then

$$\dot{F}(t) \leq -\alpha F(t) + g(t), \quad \forall t \geq 0$$

yields that

$$F(t) \leq e^{-\alpha t} F(0) + \int_0^t e^{-\alpha(t-\tau)} g(\tau) d\tau,$$

where $\alpha > 0$ denotes a finite constant.

Lemma 3.2. [33] Let $\chi \in H^1(0, 1)$ satisfy either $\chi(0) = 0$ or $\chi(1) = 0$. Then, the following inequalities hold:

$$\begin{aligned} \int_0^1 \chi^2(\vartheta) d\vartheta &\leq \frac{4}{\pi^2} \int_0^1 \chi_{\vartheta}^2(\vartheta) d\vartheta, \\ \max_{0 \leq \vartheta \leq 1} \chi^2(\vartheta) &\leq \int_0^1 \chi_{\vartheta}^2(\vartheta) d\vartheta. \end{aligned}$$

Lemma 3.3. [34] For any $\chi(\vartheta) \in L^2([0, 1])$ with $\chi'(\vartheta) \in L^2([0, 1])$, the following inequalities hold:

$$\begin{aligned} \max_{0 \leq \vartheta \leq 1} \chi^2(\vartheta) &\leq \chi^2(0) + 2\|\chi\| \|\chi_{\vartheta}\|, \\ \max_{0 \leq \vartheta \leq 1} \chi^2(\vartheta) &\leq \chi^2(1) + 2\|\chi\| \|\chi_{\vartheta}\|. \end{aligned}$$

Theorem 3.1. Specified in (3.3) are the error dynamics, where the residual is defined as $r(t) = \varpi(1, t)$. The residual thus exhibits the following key properties:

1) In the disturbance-free scenario, i.e., $\Lambda(t) = 0$, the residual $r(t)$ exhibits exponential convergence to 0 as $t \rightarrow \infty$.

2) In the scenario involving a disturbance, i.e., $\Lambda(t) \neq 0$, the residual $r(t)$ satisfies an upper bound constraint as $t \rightarrow \infty$, which can be expressed as

$$r(t) \leq R^d(t), \quad \text{where } R^d(t) = \sqrt{2W_1(0) + \frac{2\beta}{\alpha}}, \quad \alpha = \frac{\varepsilon}{2}, \quad \beta = (1 + \varepsilon + 2k\varepsilon)\bar{\Lambda}^2.$$

Proof. A Lyapunov functional candidate for the error dynamics (3.3) is defined as follows

$$V(t) = \frac{1}{2} \int_0^1 \varpi_{\vartheta}^2(\vartheta, t) d\vartheta + \frac{1}{2} \int_0^1 \varpi_t^2(\vartheta, t) d\vartheta + \varepsilon \int_0^1 (\vartheta + 1) \varpi_{\vartheta}(\vartheta, t) \varpi_t(\vartheta, t) d\vartheta, \quad (3.4)$$

where ε represents a sufficiently small positive parameter.

By analyzing the integral term in the third right-hand component of (3.4) and applying Young's inequality, we find that

$$\begin{aligned}
 (\vartheta+1)\varpi_{\vartheta}(\vartheta,t)\varpi_t(\vartheta,t) &\geq -\frac{\lambda}{2}(\vartheta+1)^2\varpi_{\vartheta}^2(\vartheta,t)-\frac{1}{2\lambda}\varpi_t^2(\vartheta,t) \\
 &\geq -2\lambda\varpi_{\vartheta}^2(\vartheta,t)-\frac{1}{2\lambda}\varpi_t^2(\vartheta,t).
 \end{aligned}$$

Let $\lambda = \frac{1}{2}$. We derive

$$(\vartheta+1)\varpi_{\vartheta}(\vartheta,t)\varpi_t(\vartheta,t) \geq -\varpi_{\vartheta}^2(\vartheta,t) - \varpi_t^2(\vartheta,t),$$

which is equivalent to

$$V(t) \geq \left(\frac{1}{2} - \varepsilon\right) \int_0^1 (\varpi_{\vartheta}^2(\vartheta,t) + \varpi_t^2(\vartheta,t)) d\vartheta.$$

Hence, $V(t)$ is positive definite when $\varepsilon < \frac{1}{2}$.

Next, the time derivative of $V(t)$ is given by

$$\begin{aligned}
 \dot{V}(t) &= \int_0^1 \varpi_{\vartheta}(\vartheta,t)\varpi_{\vartheta t}(\vartheta,t) d\vartheta + \int_0^1 \varpi_t(\vartheta,t)\varpi_{tt}(\vartheta,t) d\vartheta \\
 &\quad + \varepsilon \int_0^1 (\vartheta+1)\varpi_{\vartheta t}(\vartheta,t)\varpi_t(\vartheta,t) d\vartheta \\
 &\quad + \varepsilon \int_0^1 (\vartheta+1)\varpi_{\vartheta}(\vartheta,t)\varpi_{tt}(\vartheta,t) d\vartheta.
 \end{aligned} \tag{3.5}$$

Begin with the first term on the right-hand side of (3.5), and then introduce the boundary conditions given in (3.3)

$$\begin{aligned}
 &\int_0^1 \varpi_{\vartheta}(\vartheta,t)\varpi_{\vartheta t}(\vartheta,t) d\vartheta \\
 &= \varpi_{\vartheta}(1,t)\varpi_t(1,t) - \varpi_{\vartheta}(0,t)\varpi_t(0,t) - \int_0^1 \varpi_t(\vartheta,t)\varpi_{\vartheta\vartheta}(\vartheta,t) d\vartheta \\
 &= \Lambda(t)\varpi_t(1,t) - k\varpi_t^2(1,t) - \int_0^1 \varpi_t(\vartheta,t)\varpi_{\vartheta\vartheta}(\vartheta,t) d\vartheta.
 \end{aligned} \tag{3.6}$$

Based on (3.5) and (3.6), it can be derived

$$\int_0^1 \varpi_{\vartheta}(\vartheta,t)\varpi_{\vartheta t}(\vartheta,t) d\vartheta + \int_0^1 \varpi_t(\vartheta,t)\varpi_{tt}(\vartheta,t) d\vartheta = \Lambda(t)\varpi_t(1,t) - k\varpi_t^2(1,t). \tag{3.7}$$

Proceeding to the third term on the right side of (3.5), we use integration by parts and reach

$$\begin{aligned}
 &\varepsilon \int_0^1 (\vartheta+1)\varpi_{\vartheta t}(\vartheta,t)\varpi_t(\vartheta,t) d\vartheta \\
 &= 2\varepsilon\varpi_t^2(1,t) - \varepsilon \int_0^1 (\vartheta+1)\varpi_{\vartheta t}(\vartheta,t)\varpi_t(\vartheta,t) d\vartheta - \varepsilon \int_0^1 \varpi_t^2(\vartheta,t) d\vartheta.
 \end{aligned} \tag{3.8}$$

Furthermore, we can deduce that

$$\varepsilon \int_0^1 (\vartheta+1)\varpi_{\vartheta t}(\vartheta,t)\varpi_t(\vartheta,t) d\vartheta = \varepsilon\varpi_t^2(1,t) - \frac{\varepsilon}{2} \int_0^1 \varpi_t^2(\vartheta,t) d\vartheta. \tag{3.9}$$

The fourth term in (3.5) can be expanded via integration by parts, and is subsequently derived by substituting the boundary condition (3.3)

$$\begin{aligned}
 &\varepsilon \int_0^1 (\vartheta+1)\varpi_{\vartheta}(\vartheta,t)\varpi_{tt}(\vartheta,t) d\vartheta \\
 &= 2\varepsilon\varpi_{\vartheta}^2(1,t) - \varepsilon\varpi_{\vartheta}^2(0,t) - \varepsilon \int_0^1 (\vartheta+1)\varpi_{\vartheta}(\vartheta,t)\varpi_{tt}(\vartheta,t) d\vartheta - \varepsilon \int_0^1 \varpi_{\vartheta}^2(\vartheta,t) d\vartheta \\
 &= 2\varepsilon\Lambda^2(t) + 2\varepsilon k^2\varpi_t^2(1,t) - 4k\varepsilon\Lambda(t)\varpi_t(1,t) - \varepsilon\varpi_{\vartheta}^2(0,t) - \varepsilon \int_0^1 \varpi_{\vartheta}^2(\vartheta,t) d\vartheta \\
 &\quad - \varepsilon \int_0^1 (\vartheta+1)\varpi_{\vartheta}(\vartheta,t)\varpi_{tt}(\vartheta,t) d\vartheta.
 \end{aligned} \tag{3.10}$$

Similar to (3.9), it is naturally obtained that

$$\begin{aligned} & \varepsilon \int_0^1 (\vartheta+1) \varpi_{\vartheta}(\vartheta, t) \varpi_{\vartheta}(\vartheta, t) d\vartheta \\ &= \varepsilon \Lambda^2(t) + k^2 \varepsilon \varpi_t^2(1, t) - 2\varepsilon k \Lambda(t) \varpi_t(1, t) - \frac{\varepsilon}{2} \varpi_{\vartheta}^2(0, t) - \frac{\varepsilon}{2} \int_0^1 \varpi_{\vartheta}^2(\vartheta, t) d\vartheta. \end{aligned} \quad (3.11)$$

By (3.7), (3.9) and (3.11), one obtains that

$$\begin{aligned} \dot{W}_1(t) &= (\varepsilon - k + k^2 \varepsilon) \varpi_t^2(1, t) + \Lambda(t) \varpi_t(1, t) + \varepsilon \Lambda^2(t) - 2k\varepsilon \Lambda(t) \varpi_t(1, t) \\ &\quad - \frac{\varepsilon}{2} \varpi_{\vartheta}^2(0, t) - \frac{1}{2} \varepsilon \int_0^1 \varpi_{\vartheta}^2(\vartheta, t) d\vartheta - \frac{1}{2} \varepsilon \int_0^1 \varpi_t^2(\vartheta, t) d\vartheta. \end{aligned} \quad (3.12)$$

By using Young's inequality in (3.12), we arrive at

$$\begin{aligned} \dot{V}(t) &\leq (\varepsilon - k + k^2 \varepsilon) \varpi_t^2(1, t) + \frac{1}{2\lambda_1} \Lambda^2(t) + \frac{\lambda_1}{2} \varpi_t^2(1, t) + \varepsilon \Lambda^2(t) \\ &\quad + \frac{k\varepsilon}{\lambda_2} \Lambda^2(t) + k\varepsilon \lambda_2 \varpi_t^2(1, t) - \frac{1}{2} \varepsilon \int_0^1 \varpi_{\vartheta}^2(\vartheta, t) d\vartheta - \frac{1}{2} \varepsilon \int_0^1 \varpi_t^2(\vartheta, t) d\vartheta. \end{aligned} \quad (3.13)$$

Let $\lambda_1 = \frac{1}{2}$, $\lambda_2 = \frac{1}{2}$.

$$\begin{aligned} \dot{V}(t) &\leq \left(\varepsilon - k + k^2 \varepsilon + \frac{1}{2} k\varepsilon + \frac{1}{4} \right) \varpi_t^2(1, t) + \Lambda^2(t) + \varepsilon \Lambda^2(t) \\ &\quad + 2k\varepsilon \Lambda^2(t) - \frac{1}{2} \varepsilon \int_0^1 \varpi_{\vartheta}^2(\vartheta, t) d\vartheta - \frac{1}{2} \varepsilon \int_0^1 \varpi_t^2(\vartheta, t) d\vartheta. \end{aligned} \quad (3.14)$$

Since ε is small enough, we have $\left(\varepsilon - k + k^2 \varepsilon + \frac{1}{2} k\varepsilon + \frac{1}{4} \right) < 0$

$$\dot{V}(t) \leq \Lambda^2(t) + \varepsilon \Lambda^2(t) + 2k\varepsilon \Lambda^2(t) - \frac{1}{2} \varepsilon \int_0^1 \varpi_{\vartheta}^2(\vartheta, t) d\vartheta - \frac{1}{2} \varepsilon \int_0^1 \varpi_t^2(\vartheta, t) d\vartheta. \quad (3.15)$$

Therefore, (3.15) can be rearranged as

$$\dot{W}_1(t) \leq Q_1 + Q_2, \quad (3.16)$$

where

$$\begin{aligned} Q_1 &= -\frac{1}{2} \varepsilon \int_0^1 \varpi_{\vartheta}^2(\vartheta, t) d\vartheta - \frac{1}{2} \varepsilon \int_0^1 \varpi_t^2(\vartheta, t) d\vartheta, \\ Q_2 &= \Lambda^2(t) + \varepsilon \Lambda^2(t) + 2k\varepsilon \Lambda^2(t). \end{aligned} \quad (3.17)$$

Proceeding with (3.17), we derive the upper bound of Q_1 as follows:

$$\begin{aligned} Q_1 &\leq -\frac{1}{4} \varepsilon \int_0^1 \varpi_{\vartheta}^2(\vartheta, t) d\vartheta - \frac{1}{4} \varepsilon \int_0^1 \varpi_t^2(\vartheta, t) d\vartheta \\ &\quad - \varepsilon \int_0^1 (\vartheta+1) \varpi_{\vartheta}(\vartheta, t) \varpi_t(\vartheta, t) d\vartheta \\ &\leq -\alpha V(t), \end{aligned} \quad (3.18)$$

where $\alpha = \frac{\varepsilon}{2} > 0$. Subsequently, the upper bound of Q_2 can be determined

$$Q_2 = \Lambda^2(t) + \varepsilon \Lambda^2(t) + 2k\varepsilon \Lambda^2(t) \leq (1 + \varepsilon + 2k\varepsilon) \bar{\Lambda}^2 = \beta. \quad (3.19)$$

Ultimately, with the aid of (3.17) and (3.18), (3.19) can be reformulated as

$$\dot{V}(t) \leq -\alpha V(t) + \beta. \quad (3.20)$$

When no external disturbance acts on the system, *i.e.*, $\Lambda(t) = 0$, (3.20) is able to be re-expressed as

$$V(t) \leq e^{-\alpha t} V(0). \quad (3.21)$$

It follows from the Fundamental Theorem of Calculus that

$$\int_0^1 \varpi_g(\mathcal{G}, t) d\mathcal{G} = \varpi(1, t) - \varpi(0, t) = \varpi(1, t). \quad (3.22)$$

Applying the Cauchy-Schwarz inequality, it follows that

$$\left(\int_0^1 \varpi_g(\mathcal{G}, t) d\mathcal{G} \right)^2 = \varpi^2(1, t) \leq \int_0^1 \varpi_g^2(\mathcal{G}, t) d\mathcal{G} \leq 2V(t) \leq 2e^{-\alpha t} V(0). \quad (3.23)$$

Thus, we can draw the conclusion that $r(t) = \varpi(1, t)$ converges exponentially to 0 as $t \rightarrow \infty$ in the absence of disturbance.

When there is disturbance, the desired result can be derived through the application of Gronwall's inequality

$$\dot{V}(t) \leq -\alpha V(t) + \beta, \quad (3.24)$$

that is,

$$V(t) \leq e^{-\alpha t} V(0) + \frac{\beta}{\alpha} \leq V(0) + \frac{\beta}{\alpha}, \quad (3.25)$$

where α and β are given in Theorem 3.1.

Analogous to (3.23), we obtain

$$\varpi^2(1, t) \leq 2V(0) + \frac{2\beta}{\alpha}. \quad (3.26)$$

Consequently, we infer that

$$\varpi(1, t) \leq \sqrt{2V(0) + \frac{2\beta}{\alpha}}. \quad (3.27)$$

Remark 3.1. The bound $R^d(t)$ given by Theorem 3.1 characterizes an upper limit for the disturbed residual signal $r(t)$. In particular, for the fault-free scenario with nonzero perturbations ($\varpi(\mathcal{G}, t) = 0$, $\Lambda(t) = 0$), the residual $r(t)$ is guaranteed to satisfy the boundedness property:

$$r(t) \leq R^d(t).$$

On this basis, the adaptive time-varying threshold for the residual $r(t)$ is constructed from the upper bound $R^d(t)$.

3.2. Fault Estimation

Once the proposed FDF detects the fault, it activates the fault estimation filter to identify the fault amplitude. By constructing an adaptive state observer and a fault estimator, online fault estimation is accomplished. Towards this goal, this paper proposes an adaptive state observer and a fault estimator, which are elaborated

subsequently.

Adaptive state observer:

$$\begin{cases} \hat{\zeta}_u(\vartheta, t) = \hat{\zeta}_{gg}(\vartheta, t) + q\zeta(1, t) + \hat{\varphi}(\vartheta, t), \\ \hat{\zeta}(0, t) = 0, \\ \hat{\zeta}_g(1, t) = \mathcal{Q}(t) + k_1(\zeta_t(1, t) - \hat{\zeta}_t(1, t)), \end{cases} \quad (3.28)$$

fault estimator:

$$\begin{cases} \dot{\hat{\varphi}}(t) = k_2(\zeta_t(1, t) - \hat{\zeta}_t(1, t)) - \varrho\hat{\varphi}(t), \\ \hat{\varphi}(\vartheta, t) = I(\vartheta)\hat{\varphi}(t), \end{cases} \quad (3.29)$$

where $k_1 > 0$ and $k_2 > 0$ are tuned constant parameters.

Calculating system (2.1) and observer (3.28) differences yields the state estimation residual dynamic system

$$\begin{cases} \varpi_u(\vartheta, t) = \varpi_{gg}(\vartheta, t) + \tilde{\varphi}(\vartheta, t), \\ \varpi(0, t) = 0, \\ \varpi_g(1, t) = \Lambda(t) - k_1\varpi_t(1, t), \end{cases} \quad (3.30)$$

where $\tilde{\varphi}(\vartheta, t) = \varphi(\vartheta, t) - \hat{\varphi}(\vartheta, t)$.

Theorem 3.2. *The fault estimation error $\tilde{\varphi}(\vartheta, t)$ exhibits uniform ultimate boundedness (UUB) with respect to the norm $\|\cdot\|$. Concurrently, the quantity $\varpi^2(\vartheta, t)$ also satisfies the UUB property; their respective ultimate bounds are denoted as ς_1 and ς_2 , which are explicitly given by*

$$\begin{cases} \varsigma_1 = \frac{2\eta}{\sigma}, \\ \varsigma_2 = \frac{32\eta^2}{\sigma^2\pi^2}. \end{cases} \quad (3.31)$$

Proof. The Lyapunov functional candidate for the error dynamics (3.30) is defined as follows

$$W(t) = W_1(t) + W_2(t), \quad (3.32)$$

where

$$W_1(t) = \frac{1}{2}\|\tilde{\varphi}^2(\vartheta, t)\|, \quad (3.33)$$

$$W_2(t) = \frac{1}{2}\left(\|\varpi_g(\vartheta, t)\|^2 + \|\varpi_t(\vartheta, t)\|^2\right) + \varepsilon \int_0^1 (\vartheta+1)\varpi_g(\vartheta, t)\varpi_t(\vartheta, t) d\vartheta. \quad (3.34)$$

By following a similar line of reasoning to that in the proof of Theorem 3.1, it follows readily that $W(t)$ is positive definite when $\varepsilon < \frac{1}{2}$.

From Assumption 2.2, we have

$$\|\tilde{\varphi}(\vartheta, t)\|^2 = \tilde{\varphi}^2(t) \int_0^1 I^2(\vartheta) d\vartheta = \tilde{\varphi}^2(t) \int_0^1 I(\vartheta) d\vartheta = \ell \tilde{\varphi}^2(t), \quad (3.35)$$

where $\tilde{\varphi}(t) = \varphi(t) - \hat{\varphi}(t)$.

Therefore, differentiating (3.33) with respect to $W_1(t)$ yields

$$\begin{aligned}\dot{W}_1(t) &= \ell \tilde{\varphi}(t) \dot{\tilde{\varphi}}(t) = \ell \tilde{\varphi}(t) (\dot{\varphi}(t) - k_2 \varpi_t(1, t) + \varrho \varphi(t) - \varrho \tilde{\varphi}(t)) \\ &\leq \frac{\ell}{2\gamma_1} \dot{\varphi}^2(t) + \frac{k_2 \ell}{2\gamma_2} \varpi_t^2(1, t) + \frac{\varrho \ell}{2\gamma_3} \varphi^2(t) - \frac{\ell}{2} (2\varrho - \gamma_1 - k_2 \gamma_2 - \varrho \gamma_3) \tilde{\varphi}^2(t) \quad (3.36) \\ &\leq \frac{\ell}{2\gamma_1} \varphi_0^2 + \frac{k_2 \ell}{2\gamma_2} \varpi_t^2(1, t) + \frac{\varrho \ell}{2\gamma_3} \varphi_1^2 - \frac{\ell}{2} (2\varrho - \gamma_1 - k_2 \gamma_2 - \varrho \gamma_3) \tilde{\varphi}^2(t).\end{aligned}$$

Next, the time derivative of $W_2(t)$ is given by

$$\begin{aligned}\dot{W}_2(t) &= \int_0^1 \varpi_g(\vartheta, t) \varpi_{g_t}(\vartheta, t) d\vartheta + \int_0^1 \varpi_t(\vartheta, t) \varpi_{tt}(\vartheta, t) d\vartheta \\ &\quad + \varepsilon \int_0^1 (\vartheta + 1) \varpi_{g_t}(\vartheta, t) \varpi_t(\vartheta, t) d\vartheta \quad (3.37) \\ &\quad + \varepsilon \int_0^1 (\vartheta + 1) \varpi_x(x, t) \varpi_{tt}(\vartheta, t) d\vartheta.\end{aligned}$$

The first term on the right-hand side of (3.37) can be reformulated as

$$\int_0^1 \varpi_t(\vartheta, t) \varpi_{tt}(\vartheta, t) d\vartheta = \int_0^1 \varpi_t(\vartheta, t) \varpi_{g\vartheta}(\vartheta, t) d\vartheta + \int_0^1 \varpi_t(\vartheta, t) \tilde{\varphi}(\vartheta, t) d\vartheta. \quad (3.38)$$

Following a procedure similar to that used in (3.7), it can be derived

$$\int_0^1 \varpi_g(\vartheta, t) \varpi_{g_t}(\vartheta, t) d\vartheta + \int_0^1 \varpi_t(\vartheta, t) \varpi_{g\vartheta}(\vartheta, t) d\vartheta = \Lambda(t) \varpi_t(1, t) - k_1 \varpi_t^2(1, t). \quad (3.39)$$

Based on (3.38) and (3.39), it can be derived

$$\begin{aligned}&\int_0^1 \varpi_g(\vartheta, t) \varpi_{g_t}(\vartheta, t) d\vartheta + \int_0^1 \varpi_t(\vartheta, t) \varpi_{tt}(\vartheta, t) d\vartheta \\ &= \Lambda(t) \varpi_t(1, t) - k_1 \varpi_t^2(1, t) + \int_0^1 \varpi_t(x, t) \tilde{\varphi}(\vartheta, t) d\vartheta.\end{aligned} \quad (3.40)$$

A derivation paralleling the approach in (3.9) yields

$$\int_0^1 (\vartheta + 1) \varpi_{g_t}(\vartheta, t) \varpi_t(\vartheta, t) d\vartheta = \varpi_t^2(1, t) - \frac{1}{2} \int_0^1 \varpi_t^2(\vartheta, t) d\vartheta. \quad (3.41)$$

The fourth right-hand term in (3.37) admits the following reformulation

$$\begin{aligned}&\int_0^1 (\vartheta + 1) \varpi_{tt}(\vartheta, t) \varpi_g(\vartheta, t) d\vartheta \\ &= \int_0^1 (\vartheta + 1) \varpi_{g\vartheta}(\vartheta, t) \varpi_g(\vartheta, t) d\vartheta + \int_0^1 (\vartheta + 1) \tilde{\varphi}(\vartheta, t) \varpi_g(\vartheta, t) d\vartheta.\end{aligned} \quad (3.42)$$

Employing a methodology analogous to that outlined in (3.11), we obtain

$$\begin{aligned}&\int_0^1 (\vartheta + 1) \varpi_{g\vartheta}(\vartheta, t) \varpi_g(\vartheta, t) d\vartheta \\ &= \Lambda^2(t) + k_1^2 \varpi_t^2(1, t) - 2k_1 \Lambda(t) \varpi_t(1, t) - \frac{1}{2} \varpi_g^2(0, t) - \frac{1}{2} \int_0^1 \varpi_g^2(\vartheta, t) d\vartheta.\end{aligned} \quad (3.43)$$

By (3.40) - (3.43), one obtains that

$$\begin{aligned}\dot{W}_1(t) &= (\varepsilon - k_1 + k_1^2 \varepsilon) \varpi_t^2(1, t) + \Lambda(t) \varpi_t(1, t) + \varepsilon \Lambda^2(t) - 2k_1 \varepsilon \Lambda(t) \varpi_t(1, t) \\ &\quad - \frac{\varepsilon}{2} \varpi_g^2(0, t) - \frac{1}{2} \varepsilon \int_0^1 \varpi_g^2(\vartheta, t) d\vartheta - \frac{1}{2} \varepsilon \int_0^1 \varpi_t^2(\vartheta, t) d\vartheta \quad (3.44) \\ &\quad + \varepsilon \int_0^1 \varpi_t(\vartheta, t) \tilde{\varphi}(\vartheta, t) d\vartheta + \varepsilon \int_0^1 (\vartheta + 1) \tilde{\varphi}(\vartheta, t) \varpi_g(\vartheta, t) d\vartheta.\end{aligned}$$

Using the Cauchy-Schwarz inequality on the last two terms of the right-hand side of (3.44), we have

$$\begin{aligned} \int_0^1 \varpi_t(\vartheta, t) \tilde{\varphi}(\vartheta, t) d\vartheta &\leq \|\varpi_t(\vartheta, t)\| \|\tilde{\varphi}(\vartheta, t)\|, \\ \int_0^1 (\vartheta+1) \tilde{\varphi}(\vartheta, t) \varpi_\vartheta(\vartheta, t) d\vartheta &\leq \|(\vartheta+1) \varpi_\vartheta(\vartheta, t)\| \|\tilde{\varphi}(\vartheta, t)\| \\ &\leq 2 \|\varpi_\vartheta(\vartheta, t)\| \|\tilde{\varphi}(\vartheta, t)\|. \end{aligned} \quad (3.45)$$

By integrating (3.44) and (3.45) with Young's inequality, it follows that

$$\begin{aligned} \dot{W}_2(t) &\leq \left(\varepsilon - k_1 + k_1^2 \varepsilon + \frac{\lambda_1}{2} + k_1 \varepsilon \lambda_2 \right) \varpi_t^2(1, t) + \left(\frac{1}{2\lambda_1} + \varepsilon + \frac{k_1 \varepsilon}{\lambda_2} \right) \Lambda^2(t) \\ &\quad - \frac{1-2\lambda_4}{2} \varepsilon \|\varpi_\vartheta(\vartheta, t)\|^2 - \frac{1-\lambda_3}{2} \varepsilon \|\varpi_t(\vartheta, t)\|^2 + \left(\frac{\varepsilon}{2\lambda_3} + \frac{\varepsilon}{\lambda_4} \right) \tilde{\varphi}^2(t) \ell. \end{aligned} \quad (3.46)$$

Through the combination of equations (3.35), (3.36) and (3.46), the corresponding conclusion is derived

$$\begin{aligned} \dot{W}(t) &\leq \left(\varepsilon - k_1 + k_1^2 \varepsilon + \frac{\lambda_1}{2} + k_1 \varepsilon \lambda_2 + \frac{k_2 \ell}{2\gamma_2} \right) \varpi_t^2(1, t) + \left(\frac{1}{2\lambda_1} + \varepsilon + \frac{k_1 \varepsilon}{\lambda_2} \right) \Lambda^2(t) \\ &\quad - \frac{1-2\lambda_4}{2} \varepsilon \|\varpi_\vartheta(\vartheta, t)\|^2 - \frac{1-\lambda_3}{2} \varepsilon \|\varpi_t(\vartheta, t)\|^2 \\ &\quad - \frac{1}{2} \left(2\rho - \gamma_1 - k_2 \gamma_2 - \varrho \gamma_3 - \frac{\varepsilon}{\lambda_3} - \frac{2\varepsilon}{\lambda_4} \right) \|\tilde{\varphi}(\vartheta, t)\|^2 + \frac{\ell}{2\gamma_1} \varphi_0 + \frac{\varrho \ell}{2\gamma_3} \varphi_1. \end{aligned} \quad (3.47)$$

Choosing appropriate parameters ensures that

$$\left(\varepsilon - k_1 + k_1^2 \varepsilon + \frac{\lambda_1}{2} + k_1 \varepsilon \lambda_2 + \frac{k_2 \ell}{2\gamma_2} \right) < 0, \text{ we have}$$

$$\begin{aligned} \dot{W}(t) &\leq -\frac{1-2\lambda_4}{2} \varepsilon \|\varpi_\vartheta(\vartheta, t)\|^2 - \frac{1-\lambda_3}{2} \varepsilon \|\varpi_t(\vartheta, t)\|^2 \\ &\quad - \frac{1}{2} \left(2\rho - \gamma_1 - k_2 \gamma_2 - \varrho \gamma_3 - \frac{\varepsilon}{\lambda_3} - \frac{2\varepsilon}{\lambda_4} \right) \|\tilde{\varphi}(\vartheta, t)\|^2 \\ &\quad + \frac{\ell}{2\gamma_1} \varphi_0 + \frac{\varrho \ell}{2\gamma_3} \varphi_1 + \left(\frac{1}{2\lambda_1} + \varepsilon + \frac{k_1 \varepsilon}{\lambda_2} \right) \Lambda^2(t), \end{aligned} \quad (3.48)$$

where

$$\begin{aligned} K_1 &= -\frac{1-2\lambda_4}{2} \varepsilon \|\varpi_\vartheta(\vartheta, t)\|^2 - \frac{1-\lambda_3}{2} \varepsilon \|\varpi_t(\vartheta, t)\|^2 \\ &\quad - \frac{1}{2} \left(2\rho - \gamma_1 - k_2 \gamma_2 - \varrho \gamma_3 - \frac{\varepsilon}{\lambda_3} - \frac{2\varepsilon}{\lambda_4} \right) \|\tilde{\varphi}(\vartheta, t)\|^2, \\ K_2 &= \frac{\ell}{2\gamma_1} \varphi_0 + \frac{\varrho \ell}{2\gamma_3} \varphi_1 + \left(\frac{1}{2\lambda_1} + \varepsilon + \frac{k_1 \varepsilon}{\lambda_2} \right) \Lambda^2(t). \end{aligned}$$

Choosing suitable parameters to render the coefficients of $\|\varpi_\vartheta(\vartheta, t)\|^2$, $\|\varpi_t(\vartheta, t)\|^2$ and $\|\tilde{\varphi}(\vartheta, t)\|^2$ negative yields

$$\begin{aligned} K_1 &\leq \frac{1}{2} K_1 - \varepsilon \int_0^1 (\vartheta + 1) \varpi_{\vartheta}(\vartheta, t) \varpi_t(\vartheta, t) d\vartheta \\ &\leq -\sigma W(t), \end{aligned} \quad (3.49)$$

$$\text{where } \sigma = \min \left\{ \frac{1-2\lambda_4}{2} \varepsilon, \frac{1-\lambda_3}{2} \varepsilon, \frac{1}{2} \left(2\varrho - \gamma_1 - k_2 \gamma_2 - \varrho \gamma_3 - \frac{\varepsilon}{\lambda_3} - \frac{2\varepsilon}{\lambda_4} \right), 1 \right\} > 0.$$

Subsequently, the upper bound of K_2 can be determined

$$K_2 \leq \frac{\ell}{2\gamma_1} \varphi_0 + \frac{\varrho \ell}{2\gamma_3} \varphi_1 + \left(\frac{1}{2\lambda_1} + \varepsilon + \frac{k_1 \varepsilon}{\lambda_2} \right) \bar{\Lambda}^2 = \eta. \quad (3.50)$$

Ultimately, with the aid of (3.49) and (3.50), the equation can be reformulated as

$$\dot{W}(t) \leq -\sigma W(t) + \eta.$$

The desired result can be derived through the application of Gronwall's inequality

$$W(t) \leq e^{-\sigma t} W(0) + \frac{\eta}{\sigma}. \quad (3.51)$$

Taking the limit of both sides of (3.51) with respect to t , we obtain

$$\lim_{t \rightarrow \infty} W(t) \leq \lim_{t \rightarrow \infty} e^{-\sigma t} W(0) + \frac{\eta}{\sigma} = \frac{\eta}{\sigma}. \quad (3.52)$$

From (3.32) and (3.52), by virtue of Poincaré's inequality in Lemma 3.2, we obtain

$$\begin{aligned} \lim_{t \rightarrow \infty} \|\tilde{\varphi}(\vartheta, t)\|^2 &\leq \frac{2\eta}{\sigma}, \\ \lim_{t \rightarrow \infty} \|\varpi(\vartheta, t)\|^2 &\leq \lim_{t \rightarrow \infty} \frac{4}{\pi^2} \|\varpi_{\vartheta}(\vartheta, t)\|^2 \leq \frac{8\eta}{\sigma \pi^2}. \end{aligned}$$

Under this premise, by virtue of Agmon's inequality given in Lemma 3.3, we can further derive

$$\lim_{t \rightarrow \infty} \varpi(\vartheta, t)^2 \leq \lim_{t \rightarrow \infty} 2 \|\varpi(\vartheta, t)\| \|\varpi_{\vartheta}(\vartheta, t)\| \leq \frac{32\eta^2}{\sigma^2 \pi^2}.$$

4. Simulation Results

In this paper, the finite difference method (FDM) is used to perform the spatial and temporal discretization. Both the adaptive fault estimation law and the state update of the observer are integrated in time using the forward Euler method. Within this section, we showcase the outcomes of numerical simulations to demonstrate the operational performance of the scheme proposed in the present study. The initial condition is given by: $\zeta(\vartheta, 0) = 0.1 \sin(\pi \vartheta)$, $\zeta_t(\vartheta, 0) = 0$. The boundary control input is taken as $Q(t) = 0$. We set the parameters as $k = 1$, $q = 1$, $k_1 = 2$, $\ell = 0.4$, and $k_2 = 0.001$, define the disturbance as $\Lambda(t) = 2 \sin t$, and specify fault as follows:

$$\varphi(t) = \begin{cases} 0, & 0 < t < 50, \\ 1, & 50 \leq t < 100, \\ \sin(t), & t \geq 100. \end{cases} \quad (4.1)$$

In addition, the fault is localized with its center at $\vartheta_0 = 0.2$ and a characteristic length $\ell = 0.4$. As can be seen from **Figure 1**, when the system is in a disturbance-free and fault-free healthy operating condition, the error system (3.3) satisfies exponential stability, and the residual $r(t)$ converges exponentially to zero; as can be seen from **Figure 2**, when the system is subject to disturbances but free of faults, the error system (3.3) maintains boundedness, and the residual $r(t)$ is also a bounded quantity. **Figure 3** illustrates the relationship between the residual signal $r(t)$ and the time-varying threshold $R^d(t)$, based on which the exact moment of fault occurrence can be identified. Upon completion of fault detection, the adaptive ESO described by (3.28) - (3.29) is activated to perform fault estimation. The state estimation error $\varpi^2(\vartheta, t)$ and the fault estimation results are presented in **Figures 4-6**, respectively. The results indicate that both $\varpi^2(\vartheta, t)$ and $\|\tilde{\varphi}(\vartheta, t)\|$ exhibit the UUB property.

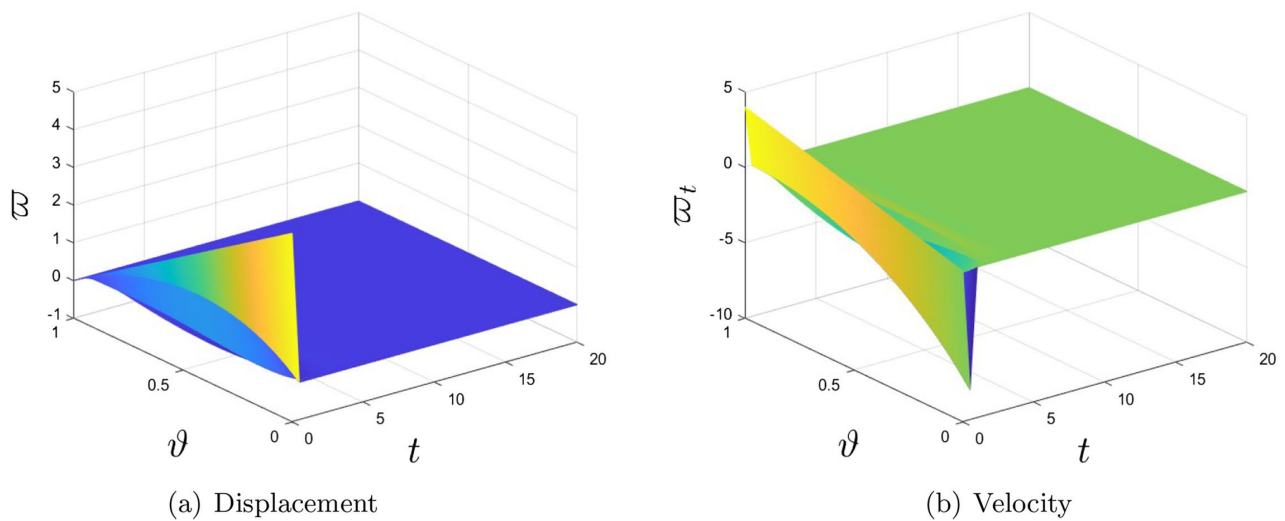


Figure 1. Trajectory evolution for Case 1.

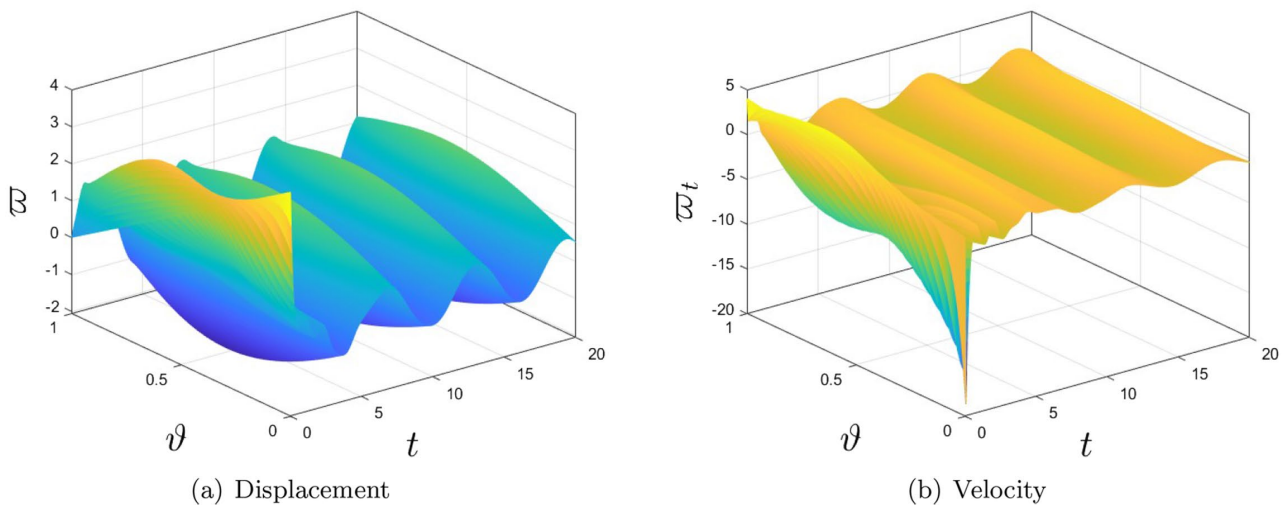


Figure 2. Trajectory evolution for Case 2.

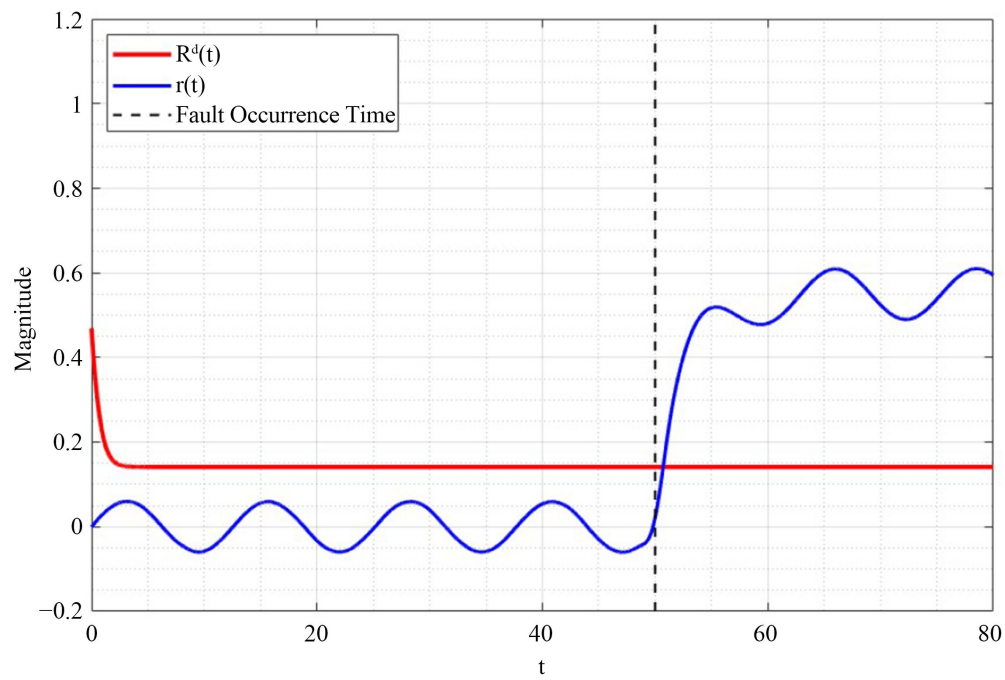


Figure 3. Detection residual $r(t)$ and threshold $R^d(t)$.

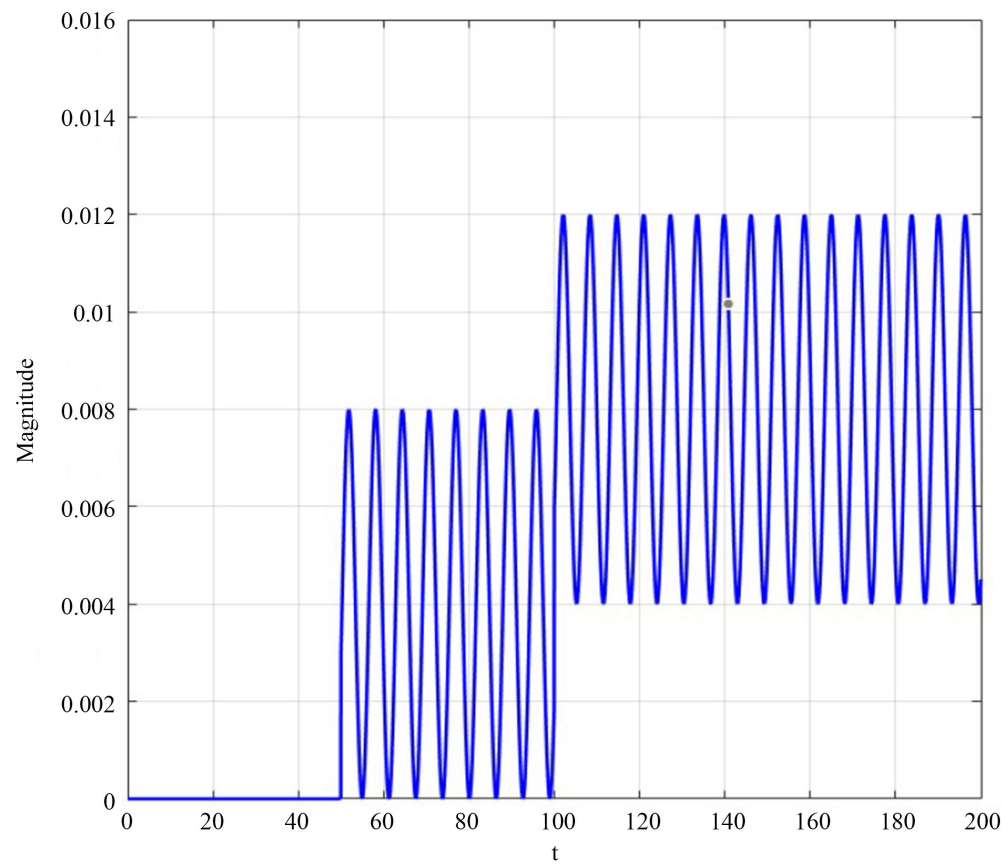


Figure 4. Trajectory of state estimation error $\varpi^2(\mathcal{G}, t)$.

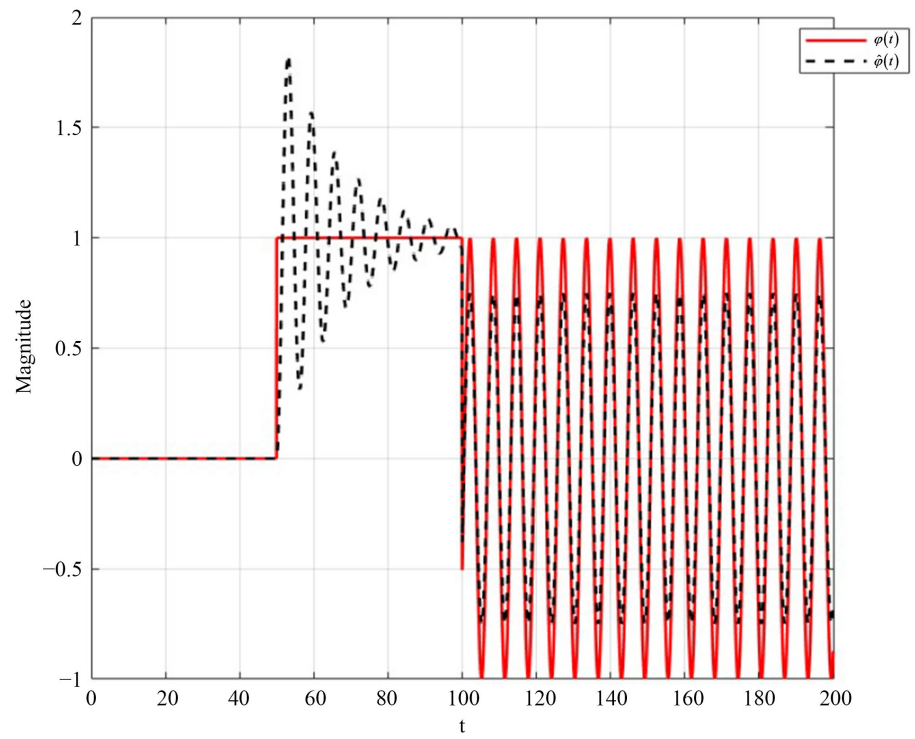


Figure 5. Trajectory of the fault $\varphi(t)$ and its estimation $\hat{\varphi}(t)$.

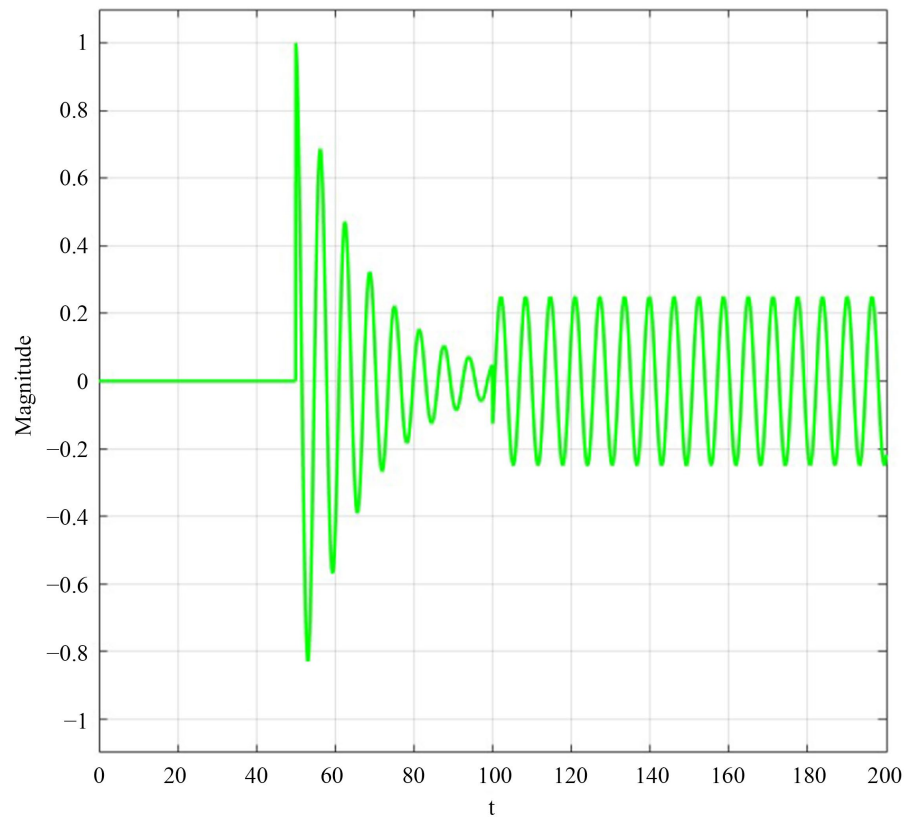


Figure 6. Trajectory of the fault estimation error $\tilde{\varphi}(t)$.

5. Conclusion

This paper proposes a fault diagnosis scheme based on an adaptive ESO for a DPS described by an unstable one-dimensional wave equation with boundary disturbances. The scheme adopts a spatial-temporal decoupling design and requires only boundary measurements to achieve fault detection and estimation. By introducing an adaptive threshold strategy, it effectively reduces the risks of false alarms and missed detections. Both theoretical analysis and simulation results demonstrate that the proposed method can rapidly detect faults, accurately estimate fault dynamics, and ensure that the estimation error satisfies UUB. The theoretical convergence and stability guarantee derived in this study apply specifically to a single separable internal additive fault, subject to the uniform boundedness of its time derivative. Future work may extend this approach to more complex elastic vibration systems, such as fault diagnosis in beam equations.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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