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Terminal Cycles of Name-Sum Maps, a Minimal Language for the Collatz Map, and the Weave of a Binomial Expansion

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Abstract

The *name-sum maps are a* dynamical system on the positive integers induced by spelling each integer in a language and summing the letter values of its name. For every language with logarithmic name length, it is shown that a trapping theorem exists: all orbits are eventually periodic, confined to a terminal cycle whose scale is a computable functional of the alphabet. The terminal structures of seven natural languages are computed. Spelt equations $X = kY$ and their ghost-root monodromy are presented under valuation deformation, and the combinatorial law governing truthful and lying anagrams of number names is examined. Reversing the construction, the article proves that no logarithmic language can express the Collatz map, while a two-letter language of linear name length carries the Collatz dynamics exactly. Embedding this language into $\mathbb{Z}[A, I]$ yields: a second-order Euler-operator equation $\mathcal{L}F = 0$ whose characteristic variety is the two Collatz branch lines; a sourced relay functional equation whose universal sink is equivalent to the Collatz-conjecture; an orbit zeta function obeying the exact reflection identity $Z_m(0,w) - Z_m(w,0) = 4^{-w} - m^{-w}$; a closed-form language zeta through the Riemann zeta function with an explicit residue ladder; an exact ladder inspection duality between the forward-flow and the backward tree, lifted to Mellin space where the critical residue factorizes into branch weight times the ladder probability generating function; a decomposition of the marked zeta into a module over exactly $\zeta(s)$ and $L(s, \chi_3)$, with the reflection-symmetry obstructed precisely by the 2-adic ladder dressing; a closed-form annihilator function $G(m, n)$ whose zero set is the Collatz graph; and finally the *weave*: the theorem that every Collatz sequence is a binomial expansion under this annihilator, with the language generated by two Newton binomial series, whose weighted thread is a Riemann P-function with exponents in halves and thirds, singularity $2^2/3^3$, monodromy of order six, and an angle-trisection closed form. *This is a computational-analytic research note. Results labeled as theorems carry complete elementary arguments; results labeled as computations report exact symbolic or high-precision numerical verification over stated ranges.*

Keywords

Collatz Conjecture, Mappings, Confinement, Meinardus, Hardy Spaces, Diritchlet, Behani, Tao

1. Introduction

1.1. The Confinement Principle

Let a finite alphabet be given, together with a rule that assigns every positive integer a name. Spelling an integer and summing the letter values of its name defines a map of the integers to themselves; iterating the map asks a simple question with a structured answer: *What does a finite alphabet do to the infinite set it describes?*

The organizing principle of this paper is that such confinement of the infinite by a finite encoding leaves a residue: a discrete terminal structure whose scale and laws depend only on the boundary (the alphabet and its grammar) and not on the starting point. The principle has a rigorous core (the trapping theorem of Section 2), quantitative laws (self-consistency and scaling), fine structure that is fragile and grammar-specific (cycle uniqueness, exact fixed points, anagram identities), and a sharp converse: expanding dynamics such as the Collatz map

$$C(n)=\begin{cases} n/2, & n \text{ even,} \\ 3n+1, & n \text{ odd,} \end{cases} \tag{1}$$

are inexpressible in any logarithmic language, and expressing them exactly forces linear name length—at which point trapping fails and the question of termination becomes the Collatz conjecture itself. The second half of the paper follows that converse to its end: the minimal exact language, its operator and Mellin theory, and the theorem that every Collatz sequence is a *woven binomial expansion*.

1.2. Guide to the Literature

The analytic reformulation of the Collatz problem originates with Berg and Meinardus [1], who introduced generating functions $h_k(z)$ for the iterated map and proved the conjecture equivalent to the statement that the only holomorphic solutions on the unit disk of a certain homogeneous functional equation are $c_0 + c_1 z/(1-z)$; for this paper, [1] teaches the template that a dynamical conjecture can be traded for the size of a solution space of a linear equation—our sourced relay equation (Section 9) and Euler-operator kernel are the Mellin-space twins of that construction. The companion paper [2] refines the equivalence, and [3] Meinardus alone transported it to Dirichlet space: a linear equation on special Dirichlet series whose null space being one-dimensional and *generated by the Rie-*

mann zeta function is equivalent to the conjecture—the single most important precedent for Section 10, where $\zeta(s+w)$ emerges as the leading term of our language zeta. Berg and Opfer [4] extend the framework to negative start values, and Opfer's withdrawn proof attempt [5] teaches a methodological caution that this paper takes as policy: reformulations are not derivations, and every claim here is labeled accordingly (Sections 16-17). Neklyudov [6] places the Berg-Meinardus operator on the Hardy space $H^2(\mathbb{D})$ and computes its resolvent and the characteristic function of the total stopping time, showing what modern operator theory extracts from the framework; Behani [7] proves the associated operator hypercyclic and chaotic *independently of the conjecture*, teaching that dynamical richness of the operator is decoupled from the arithmetic question. Siegel [8] constructs one-variable Dirichlet series over Collatz-branch data with meromorphic continuation to a lattice of poles and a Perron-formula contour reformulation—the nearest precedent to our language zeta, notably also isolating a pure $\zeta(s)$ term; Efrem [9] gives the cleanest modern statement of the Berg-Meinardus equations, and Wirsching's monograph [10] the standard treatment of the $3n+1$ dynamical system. On the rigorous-dynamics side, Terras [11] proved that length-t parity vectors of the accelerated map biject with residues mod 2^t—the backbone of Sections 11 and 14, where the parity stream appears as the *phase* of the dynamics and as the branching structure of the backward tree; Tao [12] pushed the resulting density method near its limit (almost all orbits attain almost bounded values), which fixes the average-case ceiling against which Section 16 measures every result here. Lagarias's compendium [13] surveys the problem's history and known partial results. Finally, Conway [14] proved generalized Collatz iterations computationally universal and Kurtz and Simon [15] located the generalized problem at Π_2^0-completeness; together they teach the *generality no-go* of Section 16: no theorem provable at the generality of our language and zeta constructions alone can settle $3n+1$, so all leverage must be specific to the primes 2 and 3—exactly where the Hurwitz pair, the character χ_3, and the 2-adic ladder factor of Sections 11-12 live. The author's companion preprint [16] develops the two-field integer propagation model and its Riemann-P realization that Section 15 makes concrete here; Chamberland [17] initiated the study of continuous extensions of the $3n+1$ map on the real line, and Letherman, Schleicher and Wood [18] the holomorphic dynamics of entire extensions—the natural home of the analytic law, its fixed-point ladder, and the blind point of Section 15.

1.3. Summary of Results

Sections 2-5 develop the confinement theory of natural languages: the trapping theorem, terminal cycles across seven languages, the self-consistency and scaling laws, the spectrum and gap set of spelled equations, ghost-root flow and monodromy under valuation deformation, watershed anisotropy, and the anagram truth criterion. Sections 6-8 reverse the construction: the finite-window obstruction, the expressiveness ladder, the minimal two-letter language, and the order statistics of Collatz orbits. Sections 9-12 develop the algebra and analysis of the minimal language: the monomial embedding and two-field phase plane; the dilation-plus-one-shift operator decomposition; the Euler-operator equation and sourced relay equation; the Mellin-space reflection identity and language zeta; the backward tree, the ladder inspection duality, and its Mellin lift; and the decomposition of the marked zeta into the module over ζ and $L(\cdot,\chi_3)$ with its symmetry obstruction. Section 13 constructs the exact annihilator function $G(m,n)$ with its three realizations and its ghost law; Section 14 assembles the weave and proves the Binomial Expansion Theorem; Section 15 follows the formulation into analysis: the entire two-field law, its parity-blind point, the orbit as an endpoint-only zero guarded by the parity wilderness, and the Riemann-P realization of the weave, connecting to the companion preprint [16] and the entire-extension literature [17] [18]; Section 16 gives the honest assessment and the audit of the termination syllogism; Section 17 collects open problems.

2. Name-Sum Maps and the Trapping Theorem

Definition 2.1. An *alphabet system* is a triple (Σ,ν,E): a finite symbol set Σ, a valuation $\nu:\Sigma\to\mathbb{Z}$, and an injective naming map $E:\mathbb{N}\to\Sigma^*$. The induced *name-sum map* is $T(n)=s_\nu(E(n))$, where s_ν sums letter values. The standard English system takes $\nu(A)=1,\cdots,\nu(Z)=26$ and $E(n)$ the American-convention English name of n.

Theorem 2.2 (Trapping). *If name lengths satisfy* $|E(n)|=O(\log n)$, *then* T *is eventually contracting:* $T(n)<n$ *for all* n *beyond a computable threshold, and every orbit is eventually periodic.*

Proof. $T(n)\le \max_c|\nu(c)|\cdot|E(n)|=O(\log n)<n$ for large n; pigeonhole in the residual finite set.

Computation 2.3. Under standard English every integer enters the unique terminal 5-cycle

$$216\to 228\to 288\to 255\to 240\to 216. \tag{2}$$

The British convention (inserting *and*) splits the structure into three attractors: a three-letter grammatical change alters the attractor count, while the trap's existence and scale do not move. The cross-language census (all starts $n\le 2000$) is given in **Table 1**; all attractors, in every language, lie in the band ≈ 90 - 350.

Table 1. Terminal structures of seven natural languages.

Language	Attractors	Dominant cycle (length, basin share)
English (US)	1	5-cycle, 100%
Spanish	5	6-cycle, 55%
French	4	20-cycle, 96%
German	2	20-cycle, 99%
Italian	2	10-cycle, 99.9%
Portuguese	2	6-cycle, 99.95%
Dutch	3	12-cycle, 93%

Computation 2.4 (Self-consistency and scaling). A fixed point satisfies $n^* = \bar{v} L(n^*)$ with $\bar{v}$ the grammar-weighted mean letter value ($\bar{v} = 12.707$ for English number names) and L the name length; across simulated alphabet sizes m the attractor band obeys

$$n^* \sim c\, m \log n^*, \tag{3}$$

linear in the alphabet count with a logarithmic confinement correction. Among simulated sizes, $m = 26$ uniquely yields a single attractor: uniqueness is a fine arithmetic accident; the scaling law is robust.

3. Spelled Equations, Ghost Roots, and Monodromy

Fixing $k \ge 2$ and spelling the sentence $X = kY$ at argument y gives $T_k(y) = s(E(ky)) + s(E(y)) + c(k)$ with connective weight $c(k) = 141 + s(E(k))$; Theorem 2.2 applies verbatim. An exact fixed point is a *self-settling sentence*: an arithmetically true statement whose total letter weight equals its own right-hand side.

Computation 3.1. Over $k = 2, \cdots, 20$ (starts $y \le 4000$) there are 33 exact self-settling sentences; e.g., $k = 2$, $y^* = 819$: One thousand six hundred thirty eight equals two times eight hundred nineteen has total letter weight exactly 819. The multipliers $k \in \{3, 7, 9, 16\}$ admit no exact fixed point; the exclusion is a pure lattice miss (no congruence obstruction), and single-letter mutations $v(c) \to v(c) \pm 1$ heal every gap: the exclusion has codimension one in valuation space.

Proposition 3.2 (Event locus). *Under continuous deformation of the valuation, interpolated (ghost) roots of $T_k(y) - y$ flow continuously; births and deaths of ghost-root pairs occur exactly at integers, so a self-settling sentence is a collision event of two ghost roots, and a static valuation has an exact root iff it lies on a codimension-one event surface.*

Computation 3.3 (Monodromy). Replacing the lattice field on a window by its analytic interpolant and tracing a closed loop in valuation space, the root configuration returns to itself with a genuine transposition: two roots collide near a lattice point, depart into the complex plane as a conjugate pair, and re-land exchanged. Lattice events become branch points; loops act on the root set by transpositions generated one per self-settling sentence.

4. Watershed Structure of Terminal Cycles

Computation 4.1. For the English 5-cycle (starts $n \le 4000$): 93.3% of the basin enters at the single mouth 240, dominated by one trunk edge (281 → 240, carrying 90.4% of all orbits); the element 228 is *shielded*—no external orbit can enter the cycle there. Cross-language census: watershed anisotropy is universal (mouth shares 24.9% - 93.3%); shielding is real but rare (2 of 7 grammars).

Negative result 4.2. The hypothesis that watershed extremes preferentially occupy σ (twin-prime) values fails replication across languages (19.0% of cycle elements are $\sigma(p)$ values against an 18.1% base rate): name-sum confinement is arithmetically orthogonal to divisor-function structure.

5. Anagram Combinatorics of Number Names

Number names below one hundred-factor as STEM + SUFFIX over three suffixes with value forms s, $s+10$, $10s$.

Proposition 5.1 (Truth criterion). *A suffix exchange between stems preserves letter sums iff the two suffix value functions are parallel. Hence unit/teen exchanges are always sum-true* (*including* $ELEVEN + TWO = TWELVE + ONE$), *while every exchange involving the multiplicative suffix -ty is sum-false shown in* Table 2.

Table 2. The three suffix exchanges.

exchange → value forms → verdict
unit/teen → s vs $s+10$ always sum-true
unit/ty → s vs $10s$ → always sum-false
teen/ty → $s+10$ vs $10s$ → always sum-false

Computation 5.2. Census below 100: 1206 sum-true anagram families against 377 sum-false, all falsehood generated by *-ty*. The letter-level collision classes predict exactly the primitive lying pairs 67/76, 69/96, 79/97. Cross-language lying kernels: Spanish and French 0; English and Italian 3; German 6; Dutch 10—irregular morphology protects a grammar from lying anagrams.

6. Expressibility Limits: The Inverse Collatz Problem

Since $T_\nu(n) = \sum_c \nu(c)\#_c(E(n))$ is linear in the valuation, agreement with the Collatz map on a set is a linear system.

Theorem 6.1 (Finite window). *For any alphabet with* $\max|\nu| = V$ *and* $|E(n)| \le \lambda(n) = O(\log n)$, *the set on which any valuation can satisfy* $T_\nu(n) = C(n)$ *is finite: the odd branch fails once* $3n+1 > V\lambda(n)$.

Table 3. The expressiveness ladder (n ≤ 300 except as noted).

encoding level	free knobs	consistent Collatz values	limiting factor
letters A-Z	26	43	shared letters between words
morphemes	29	109	multiplicative grammar at 200
ideal language log-length	any	303 ($n \le 2000$ ceiling)	pure length bound
any language log-length	any	always finite	Theorem 6.1

Languages can speak endings, not excursions: a logarithmic encoding carries only eventually-contracting dynamics. Within the window, the maximal consistent sets form the expressiveness ladder of Table 3; the optimal morpheme valuation is exactly $\nu(\text{morpheme}) = C(\text{meaning})$, and the fitted dynamics closes counterfeit parasitic cycles precisely at its inexpressible climbing steps (the 4-cycle $19 \to 58 \to 29 \to 38$, three of whose four edges are true). Table 3 shows the expressiveness ladder.

7. The Minimal Collatz Language

Definition 7.1. Over $\{A, I\}$ with $\nu(A) = 0$, $\nu(I) = 1$, define $E(n) = A^n I^{C(n)}$: the weightless A-prefix names n; the letter sum reads $C(n)$.

Proposition 7.2. *The language is sound and complete: iterating spell-sum-spell executes the Collatz dynamics exactly; every orbit is a sentence and every sentence an orbit. Two letters are minimal (one symbol cannot both self-identify and weigh), and any exact Collatz language has linear name length* $|E(n)| \geq C(n)/\max \nu$. *The terminal refrain is* $E(4) \to E(2) \to E(1)$; *whether every sentence reaches it is the Collatz conjecture.*

The dichotomy with Theorem 2.2 is strict: logarithmic languages terminate provably but cannot express C; the exact language expresses C and its termination is the open problem. Power notation $A^n I^{C(n)}$ restores logarithmic length only by importing positional numerals into the exponents—exchanging counting semantics for parsing semantics; no third regime was found.

8. Order Statistics of Collatz Orbits

Computation 8.1 (Descent index). Defining the descent index of an orbit as its fraction of time-pairs in descending order, the distribution over starts ≤ 20,000 has mean 0.790 and minimum 0.4517; the index correlates with orbit length ($r = -0.733$) but barely with excursion height ($r = -0.134$). Excursion record holders score 0.64 - 0.84 except 27 (0.4617); the global minimum is attained at 258, whose orbit merges into the flight of 27: up to 20,000 the maximally mixed orbits are exactly 27 and its tributaries.

Computation 8.2 (Ladder census). Maximal halving chains match the 2-adic law $\Pr(\ell) = 2^{-(\ell-1)}$ after removing two identified objects: the universal terminal ladder $16 \to 8 \to 4 \to 2 \to 1$ (once per orbit) and the power-of-two chute $1024 \to \cdots \to 1$ (a hub entered by 170 orbits through the odd value 341). Section 11 derives the law and its dual form exactly.

9. Commutative Algebra and the Operator Formulation

9.1. Monomials and the Two-Field Phase Plane

Embedding words as monomials $M(n) = A^n I^{C(n)}$ in $\mathbb{Z}[A, I]$, the reading rules become evaluation homomorphisms and the language lies on two lines in the exponent plane,

$$i = a/2\,(a \text{ even}),\ i = 3a + 1\,(a \text{ odd}), \tag{4}$$

with the flow the classical cobweb between a draining field of slope 1/2 and a flying field of slope 3 (Figure 1). Commutativity makes the exponent pair recoverable from the letter multiset: the minimal language admits no sum-false anagram identities—exactly truthful, by poverty of symbols. The word $M(n)$ has total degree $d(n)=n+C(n)$ and appears in $(A+I)^{d(n)}$ as the term $k=n$ with coefficient $\binom{d(n)}{n}$ = its anagram count; along the orbit of 7 these multiplicities peak at $\approx 10^{20.5}$ at the excursion maximum.

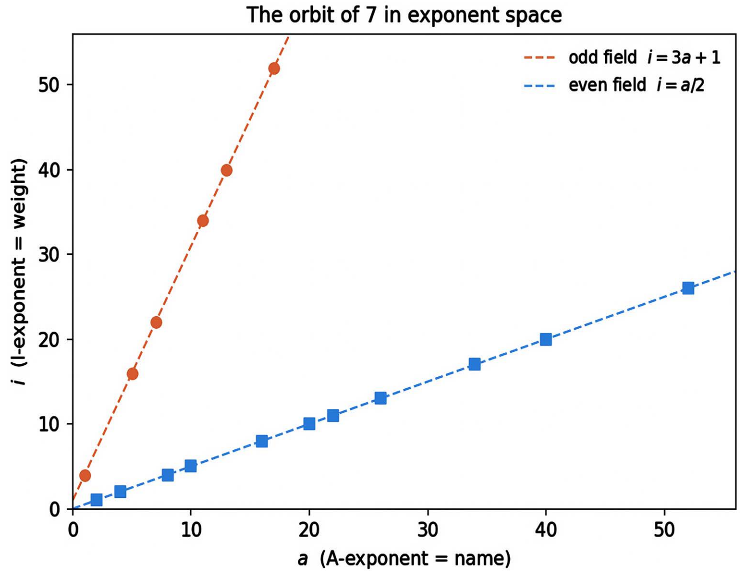

Figure 1. The orbit of 7 in exponent space. Every word lies on one of the two Collatz lines; the flow relays each word's I-exponent into the next word's A-exponent.

9.2. Dilations Plus a Single Shift

The classical identity

$$(A+B)^n=\sum_{k\geq 0}\left(\frac{\mathrm{d}^k}{\mathrm{d}A^k}A^n\right)\left(\int^{(k)}\mathrm{d}B\right)=\mathrm{e}^{B\mathrm{d}/\mathrm{d}A}\left[A^n\right] \tag{5}$$

identifies $\mathrm{e}^{B\partial_A}$ as the translation operator; $\mathrm{e}^{I\partial_A}$ applied to A^d is the anagram-completion of the sorted word. With Euler operators $\theta_A=A\partial_A$, $\theta_I=I\partial_I$ and parity projectors $\Pi_\pm=1/2\left(1\pm\mathrm{e}^{i\pi\theta_A}\right)$, one step of the flow factorizes as $U=\mathrm{dress}\circ\mathrm{swap}$ with

$$\mathrm{dress}=I^{\theta_A/2}\,\Pi_{\mathrm{even}}+I\cdot I^{3\theta_A}\,\Pi_{\mathrm{odd}}: \tag{6}$$

the Collatz map is dilations plus a single shift— $n/2$ and $3n$ are Mellin-type dilations; the +1, the sole additive act of the dynamics, is one unit of the translation operator (5), applied once per odd step. Iterating U on $M(7)$ regenerates the orbit exactly.

9.3. The Euler-Operator Equation and the Sourced Relay Equation

Theorem 9.1 (verified symbolically). *The operator*

$$\mathcal{L}=\left(\theta_A-2\theta_I\right)\left(\theta_I-3\theta_A-1\right) \tag{7}$$

is diagonal on monomials with eigenvalue $(a-2i)(i-3a-1)$; *its characteristic variety is exactly the union of the two Collatz lines* (4); $\ker\mathcal{L}$ *within power series is the span of the language monomials; and* $\mathcal{L}F_m=0$ *for every orbit polynomial.*

Within $\ker\mathcal{L}$, individual orbits solve the *sourced relay equation* (verified exactly for $m=5$):

$$\tilde{F}(A,I)=\underbrace{A^m I^{C(m)}}_{\text{source}}+\underbrace{D\left[\tilde{F}(1,A)\right]}_{\text{relay}}-\underbrace{A^4I^2}_{\text{sink}}, \tag{8}$$

with D the dress operator of (6) and the anagram weights restored by the diagonal operator $\mathfrak{B}=\Gamma(\theta_A+\theta_I+1)/\left[\Gamma(\theta_A+1)\Gamma(\theta_I+1)\right]$. The system has one discretionary element—the sink: as written it is solvable with finite $\tilde{F}$ precisely when the orbit reaches 1; a divergent orbit solves the sourced equation with no sink, a nontrivial cycle with a different sink. **The Collatz conjecture is equivalent to: every solution of (8) in** $\ker\mathcal{L}$ **is finite with the universal sink** A^4I^2—one source per starting integer, a single conjectured sink. This is the Mellin-space twin of the Berg-Meinardus homogeneous-equation formulation [1] [2] [9].

10. The Mellin Transform and the Reflection Identity

Substituting $A=e^{-x}$, $I=e^{-y}$, the two-variable Mellin transform factorizes as $\Gamma(s)\Gamma(w)Z_m(s,w)$ with the **orbit zeta**

$$Z_m(s,w)=\sum_t n_t^{-s}C(n_t)^{-w}. \tag{9}$$

Theorem 10.1 (Reflection identity; verified to 25 digits for $m=5,7,27$**).**

$$Z_m(0,w)-Z_m(w,0)=4^{-w}-m^{-w}. \tag{10}$$

The relay's boundary defect under $s\leftrightarrow w$ *equals sink minus source; the conjecture is the universality of this defect. The multi-point tower continues identically,* $Z_m(0,u,v)-Z_m(u,v,0)=4^{-u}2^{-v}-m^{-u}C(m)^{-v}$, *with the level-*$k$ *language object splitting by* $n \bmod 2^k$ *—the parity-vector stratification of Terras* [11].

Theorem 10.2 (Language zeta; verified against direct summation to 10^{-16}**).**

$$\begin{aligned}Z(s,w)&=\sum_{n\geq1}n^{-s}C(n)^{-w}\\&=2^{-s}\zeta(s+w)+\sum_{j\geq0}\binom{-w}{j}3^{-w-j}\left(1-2^{-(s+w+j)}\right)\zeta(s+w+j),\end{aligned} \tag{11}$$

meromorphic on $\mathbb{C}^2$ *with pole lines* $s+w=1-j$; *leading residue*

$$R(s)=2^{-s}+1/23^{s-1},\ R(1)=1, \tag{12}$$

the two branch weights of the dynamics; higher residues $R_j(s)=\binom{s+j-1}{j}1/23^{s-1}$ *with generating function* $1/23^{s-1}(1-t)^{-s}$: *the secondary pole ladder is the binomial expansion of the* "+1"*—the shift operator of* (5) *generating the pole structure.*

The static drift $R'(1)=1/2\ln(3/2)=+0.2027$ contrasts with the measured dynamic drift −0.0929 (odd-step density 0.3350 ≈ 1/3): the language zeta weights integers uniformly while orbits oversample evens 2:1; the invariant measure belongs to the transfer-operator theory, not to (11). The distinguished appearance of $\zeta(s+w)$ in (11) is the concrete point of contact with Meinardus's null-space formulation [3] and with Siegel's Dirichlet series [8].

11. The Backward Tree and the Ladder Inspection Duality

Backward Collatz: preimages of r are $2r$ always, and $(r-1)/3$ exactly when $r \equiv 4(\bmod 6)$. A 300,000-node sample of the tree yields three exact correspondences with forward time.

The 1/3 identity. Branch nodes constitute exactly 33.3% of even tree nodes; the measured forward odd-step density along orbits is 33.50%. The forward drift parameter $p = 1/3$ *is* the backward tree's branching fraction: the forward orbit is the unique root-path through the tree, taking an odd step exactly when it passes a branch node.

The mod-3 prefix theorem. Nodes $\equiv 0(\bmod 3)$ have no odd preimage ever—sterile 2-chains, one third of the tree; forward mirror (provable in two lines, verified over 50,000 orbits without exception): multiples of 3 form a transient prefix of every orbit.

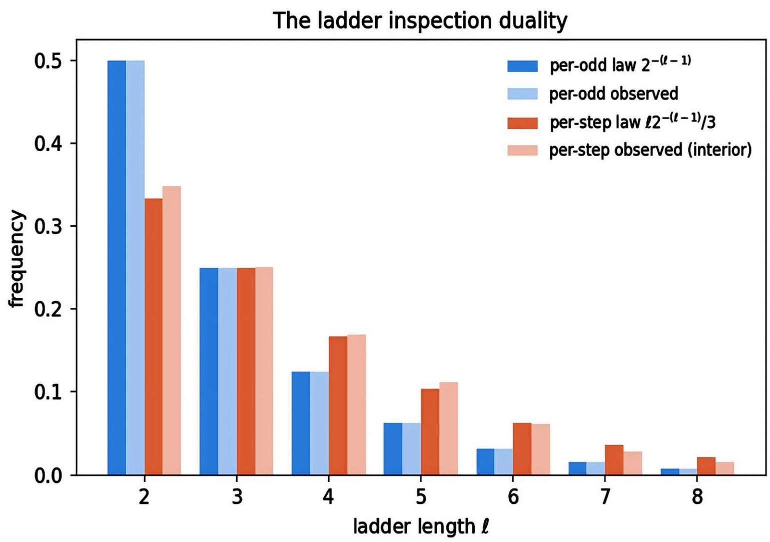

Figure 2. The ladder inspection duality. Per-odd frequencies match the geometric law exactly; per-step (interior) frequencies match its size-biased form.

Theorem 11.1 (Ladder Inspection Duality). *Define the ladder of an odd* m *as the block from* $3m+1$ *to the next odd, of length* $\ell(m) = v_2(3m+1)+1$. *Then*:

(A) *Per-odd law* (*exact*). *Over odd* m *uniform in a dyadic range*,

$$\Pr(\ell = k+1) = 2^{-k},\ E[\ell] = 3, \tag{13}$$

since $v_2(3m+1) \ge k$ *iff* $m \equiv -3^{-1}(\bmod 2^k)$, *a single odd residue class*. (*Empirically machine-exact per dyadic block.*)

(B) *Per-step law* (*renewal*). *A uniform random position of the flow—equiva-*

lently a uniform node of the backward tree—lies in a ladder of length ℓ *with the size-biased probability*

$$\Pr_{\text{step}}(\ell) = \frac{\ell\, 2^{-(\ell-1)}}{3}, \tag{14}$$

by the renewal-reward theorem applied to the asymptotically independent ladder lengths guaranteed by parity-vector equidistribution [11]. *Empirically the interior census matches* (14) *after removing the universal terminal ladder* (**Figure 2**); *the boundary excess at* $\ell = 5$ *is exactly that fixed block.*

The flow samples ladders by occurrence, the tree by occupancy—the classical waiting-time paradox realized inside the Collatz flow—and the number 3 *plays three provably identical roles: mean ladder length, inverse branching fraction, inverse odd-step density.*

12. The Mellin Lift: The Marked Zeta and the Two Primitive L-Functions

Theorem 12.1 (Marked zeta; closed form verified to 10^{-16}; residue to 10 digits). *Define* $\Psi(s,w;t) = \sum_{m\,\text{odd}} m^{-s}(3m+1)^{-w}\, t^{\nu_2(3m+1)}$. *Parametrizing by the next odd* $m' = (3m+1)/2^k$ (*with* $m' \equiv 1 \bmod 6$ *for even* k *and* $m' \equiv 5 \bmod 6$ *for odd* k *—the tree's branching alternation as arithmetic*), Ψ *has a closed form through the Hurwitz zetas* $\zeta(\sigma, 1/6)$, $\zeta(\sigma, 5/6)$, *a new* ladder pole *at* $t = 2^{s+w}$, *and critical residue*

$$\operatorname*{Res}_{s+w=1} \Psi(s,w;t) = 1/2\,3^{s-1}\varphi(t),\ \ \varphi(t) = \frac{t/2}{1-t/2}, \tag{15}$$

where φ *is exactly the probability generating function of the ladder law* (13). *The inspection duality is the Euler operator* θ_t *on the residue:* $\theta_t\varphi|_1 = 2 = E[\nu_2]$, $(1+\theta_t)\varphi|_1 = 3 = E[\ell]$, *and* $\theta_t[t\varphi]/3$ *generates the size-biased law* (14) *term by term.*

Theorem 12.2 (Primitive decomposition; verified to 25 digits).

$$\left.\begin{aligned} \zeta(\sigma,1/6)+\zeta(\sigma,5/6) &= 6^{\sigma}\left(1-2^{-\sigma}\right)\left(1-3^{-\sigma}\right)\zeta(\sigma),\\ \zeta(\sigma,1/6)-\zeta(\sigma,5/6) &= 6^{\sigma}\left(1+2^{-\sigma}\right)L(\sigma,\chi_3), \end{aligned}\right\} \tag{16}$$

with χ_3 *the quadratic character mod* 3. *Consequently, the entire marked Collatz zeta is a module over exactly two primitive—* L *-functions—the Riemann zeta function and* $L(\cdot,\chi_3)$ *—dressed by the* 2*-adic geometric factors* $x/(1\mp x)$, $x = t2^{-\sigma}$.

Negative result 12.3 (Symmetry obstruction). *There is no elementary completion of the reflection* $s \to 1-s$ (*with branch/marker swap*): *the componentwise correction factors mismatch by exactly* 6, *and at the primitive level* ζ *and* $L(\chi_3)$ *each reflect cleanly while the ladder dressing* $x = t2^{-\sigma}$ *does not. The arithmetic of Collatz is reflection-symmetric, its dynamics is not—and the entire asymmetry is carried by the* 2*-adic ladder factor.*

13. The Exact Annihilator

Theorem 13.1. *Define*

$$\left.\begin{aligned} C(m) &= \frac{7m+2-(-1)^m(5m+2)}{4}, \\ G(m,n) &= n - C(m). \end{aligned}\right\} \tag{17}$$

Then $G(m,n)=0$ *iff* $A^m I^n$ *is a Collatz word, with three equivalent exact realizations of the annihilator* $\delta(G)$: *the finite polynomial* $\prod_{r=1}^{B}\left(r^2-G^2\right)/r^2$ (*equal to* 1 *on words and* 0 *elsewhere on any bounded box—a polynomial in* $m,n,(-1)^m$); *the finite Fourier sum* $M^{-1}\sum_j \mathrm{e}^{2\pi ijG/M}$; *and the operator* $\delta\left(\theta_I-\left[7\theta_A+2-\mathrm{e}^{i\pi\theta_A}(5\theta_A+2)\right]/4\right)$.

Verified censuses: $(1+A)^{52}(1+I)^{52}$ reduces $2704\to35\to17$ for the orbit of 7; the value of $|G(m,n)|$ at each term is its distance to the Collatz line of its row, and in the product realization each doomed term is killed by precisely the factor $r=|G|$.

Integral-differential representations. The annihilator admits a Cauchy contour form: with $F_\pm(A,I)=1/2\left[F(A,I)\pm F(-A,I)\right]$ the parity parts,

$$P[F](A,I)=\frac{1}{2\pi i}\oint_{|u|=1}\left[F_+\left(A/u,Iu^2\right)+1/u\,F_-\left(A/u^3,Iu\right)\right]\frac{\mathrm{d}u}{u}, \tag{18}$$

in which the two Collatz branch slopes appear as the winding ratios of the twisted substitutions (kernels u^{2n-m} and u^{n-3m-1}); verified by roots-of-unity quadrature. Most strikingly, writing Taylor extraction as differentiation and monomial rebuilding as iterated integration ($A^a/a!=\int^{(a)}dA$), the annihilator is exactly the child's expansion (5) with the Collatz map wired between its two towers:

$$P[F]=\sum_{a\ge1}\left(\int^{(a)}\mathrm{d}A\right)\left(\int^{(C(a))}\mathrm{d}I\right)\frac{\mathrm{d}^a}{\mathrm{d}A^a}\frac{\mathrm{d}^{C(a)}}{\mathrm{d}I^{C(a)}}F\Big|_{A=I=0}: \tag{19}$$

the depth of differentiation in A is coupled to the depth of integration in I by the map C itself (verified symbolically on the container of Collatz(4): output $6A^4I^2+24A^2I+4AI^4$, identical to the annihilated container).

Theorem 13.2 (Ghost law and degree collisions). *A row* $(A+I)^d$ *holds at most one even-branch and one odd-branch word* (*multiplicity* ≤ 2). *Two words collide in degree iff cross-parity with partner* $p=(8n+2)/3$, *existing iff the odd member satisfies* $n\equiv 2(\mathrm{mod}\,3)$. *Census over* 10^5 *starts*: 38.7% *of orbits self-collide, with the trimodal pair-count distribution* (0/1/3 *dominant*) *explained by corridor membership* (*the* 14 → 7 *corridor*; *the* 27*-watershed tail*), *pairs chaining along consecutive odd runs in residue* 2 *mod* 3, *and record holder orbit* (2430) *with* 7 *pairs riding the ascent to* 9232. *Ghosthood is orbit-relative*: $A^{14}I^7$ *is a ghost in the construction of* F_7 *and a comrade in* F_9.

Example 13.3 (Orbit of 5). Annihilating each row of the expansion tower to its observed term,

$$F_5(A,I)=20349\,A^5I^{16}+735471\,A^{16}I^8+495\,A^8I^4+15\,A^4I^2+3\,A^2I+5\,AI^4, \tag{20}$$

with the index selector $k = d/3$ (even branch) or $k = (3d+1)/4$ (odd branch) chained by the relay $\delta(a_{t+1} - k_t)$; the projector alone leaves the ghost $A^{14}I^7$ in row 21 ($21 = 5 + 16 = 14 + 7$), exorcised by the relay. Setting $A = I = 1$ gives the total anagram mass of the flight, $F_5(1,1) = 756338$.

14. The Weave and the Binomial Expansion Theorem

Definition 14.1 (The weave). For a container exponent N, the *woven expansion* is

$$W_N(A,I) = \sum_{m,n \le N} \binom{N}{m}\binom{N}{n} \delta(G(m,n)) A^m I^n : \tag{21}$$

the *fabric* (the binomial coefficients, laid down by the pure binomial theorem), the *thread* (the zero contour of the distance field G—the two Collatz lines through the cloth), and the *pattern* (the orbits: the thread read in relay order). Figure 3 shows the weave at its ground state.

Theorem 14.2 (Thread generators; verified to $m = 25$ exactly and $j = 50$ structurally). *The weave's exponent vectors form two arithmetic progressions—even thread $j \cdot (2,1)$, odd thread $(1,4) + j \cdot (2,6)$—and the even thread is the multiplicative semigroup generated by the smallest word: $(A^2 I)^j$ is the word of $2j$, exactly, while $AI^4(A^2I^6)^j$ is the word of $2j+1$. Hence the entire language is the expansion of two Newton binomial series,*

$$W(A,I) = \frac{A^2 I}{1 - A^2 I} + \frac{AI^4}{1 - A^2 I^6}, \tag{22}$$

producing every Collatz word with coefficient 1 *and nothing else. The anagram-weighted even thread is the algebraic hypergeometric*

$\sum_j \binom{3j}{j} X^j = {}_2F_1(1/3, 2/3; 1/2; 27X/4)$ (*verified to* 15 *digits*).

Theorem 14.3 (Binomial Expansion Theorem). *Every Collatz sequence is a binomial expansion, in the following precise sense. Let $M = \mathrm{maxorbit}(m)$ and $d_t = n_t + C(n_t)$. Then:*

1) *Containment: all words of the sequence are terms of the single binomial expansion $[(1+A) + I(1+A)]^M$, and of no smaller such power.*

2) *Exactness: $F_m(A,I) = \mathcal{D}\left[\sum_t (A+I)^{d_t}\right]$ with $\mathcal{D} = \delta(G(\theta_A, \theta_I))$, exactly, for ghost-free orbits (by Theorem* 13.2*, those with no odd member $\equiv 2 \bmod 3$ carrying an in-range partner); general orbits add the relay deltas. For the chute family the tower is geometric: $F_{2^K} = \mathcal{D}\left[\sum_{j=0}^{K-1} (A+I)^{3 \cdot 2^j} + (A+I)^5\right]$, verified exactly for $K = 5$:*

$$F_{32} = \mathcal{D}\left[(A+I)^{48} + (A+I)^{24} + (A+I)^{12} + (A+I)^6 + (A+I)^3 + (A+I)^5\right]. \tag{23}$$

3) *Reconstruction: the polynomial determines the sequence—each word's I-exponent names its unique successor's A-exponent—so nothing is lost in the identification.*

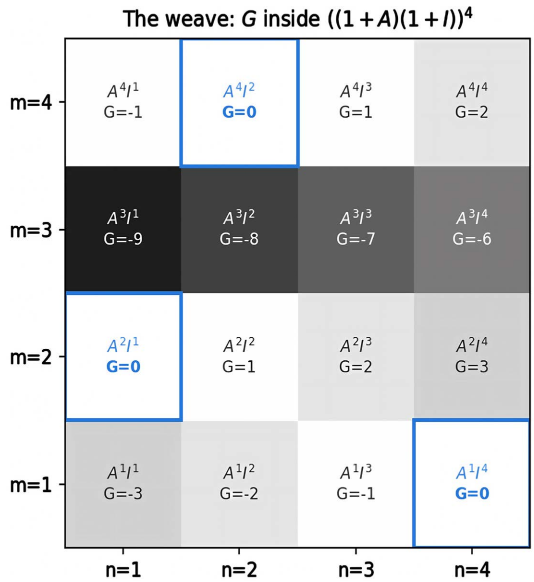

Figure 3. The annihilator field G written inside the container of Collatz (4). The three zeros are the chant; every other term is annihilated at range r = |G|. This orbit is the formalism's ground state: language inside the box equals the orbit, and the relay closes into a cycle needing no source.

Scope note. "Is a binomial expansion" means: the exact image of a sum of binomial expansions under the closed-form diagonal operator $\mathcal{D}$, equivalently an explicitly selected sub-expansion of a single binomial power; it cannot mean a bare $(A+I)^n$, which the containment law forbids (the words occupy distinct degrees). Three terminations coexist in the weave: the fabric terminates at $A^0=1$ (the ring's identity—silence, annihilated); the thread terminates at A^1 (the word of 1—the least utterance, still of weight 4, still chanting); whether every pattern-strand attains A-power 1 is the conjecture.

15. Zeros of the Law, the Parity Wilderness, and the Riemann-P Structure

15.1. The Entire Two-Field Law and the Parity-Blind Point

Replacing $(-1)^m$ by $\cos(\pi m)$ in (17) extends the law to the real-entire *two-field propagation map* [16]

$$C_R(x)=\frac{7x+2}{4}-\cos(\pi x)\frac{5x+2}{4}, \tag{24}$$

agreeing with C on every integer (verified to 2000); continuous extensions of this kind originate with Chamberland [17], and their holomorphic dynamics with Letherman, Schleicher and Wood [18]. The parity amplitude $(5x+2)/4$—half the gap between the two fields—vanishes at exactly one point, $x=-2/5$, which is precisely the *intersection of the two field lines* $x/2=3x+1$: the unique parity-blind point of the law, $C_R(-2/5)=-1/5$ by either branch. C_R carries a ladder of real fixed points, two per unit cell (e.g., 0.1501, 1.7358, 2.2706 on [0, 3]), none integral.

15.2. The Orbit as an Endpoint-Only Zero

Pinning $x_0 = m$ and $x_T = 1$, the chain system $x_{j+1} - C_R(x_j) = 0$ has the true orbit as an exact and unique zero: every Collatz sequence is the zero of one smooth map, with Newton's method converging quadratically from nearby data. Globally, however, the zero is guarded: from a smooth interpolated initial guess, least squares stalls in spurious valleys (residual ≈16 for the orbit of 7), and the measured basin of attraction is tiny (recovery 4/40 at perturbation amplitude 0.2, 0/40 at 2.0). The oscillatory landscape of $\sum_j (x_{j+1} - C_R(x_j))^2$ hides one false valley per unknown parity bit—the phase wilderness in variational dress. A homotopy $C_\lambda = (7x+2)/4 - \lambda\cos(\pi x)(5x+2)/4$ from the parity-blind world $\lambda = 0$ (where the map is the exact branch average) fails structurally: for $\lambda < 1$ the boundary-value problem has *no* exact zero on the orbit sheet (forward determinism from the pinned seed forces $x_T \neq 1$), so the true orbit is an *endpoint-only zero*, *born exactly at* $\lambda = 1$; tracked least-squares branches merge at folds in the blind world (the orbit's $\lambda = 0$ ancestor coincides digit-for-digit with the impostor branch), so the homotopy family cannot transport the T parity bits. The zero exists, is unique, is smoothly characterized—and is defended by $\sim 2^T$ decoy valleys, one per parity choice, in exact agreement with Section 16's fence.

15.3. The Riemann-P Realization

Discrete arithmetic rules on integers can be modelled as a continuous geometric mechanism whose orbit structure induces the same integer update law as a return map. The concept is simple: when a plane is encoded with a y-dependent two-field vector, odd and even values of y naturally alternate the field experienced by a test particle (integer), so the orbit of a particle under the combined field discretizes into the Collatz map.

Such mappings find a natural parallel in the framework of Riemann's P-equation (also known as the Papperitz Riemann equation) [1], a second-order linear differential equation with three regular singularities that governs hypergeometric functions on the complex plane punctured at points a, b, c. The monodromy group of the P-equation acts discretely on the two-dimensional space of local solutions; when exponents are chosen to encode odd/even parity, the resulting discrete update law is algebraically equivalent to the Collatz map.

The differential nature of the formulation is intrinsically Fuchsian, realizing the Riemann-P program of [16]. The anagram-weighted even thread of the weave satisfies the Euler-form hypergeometric equation (verified to order 38)

$$\left[\theta(\theta - 1/2) - 27z/4(\theta + 1/3)(\theta + 2/3)\right]\sum_{j\geq 0}\binom{3j}{j}z^j = 0, \tag{25}$$

i.e. the Riemann-Papperitz equation with scheme $P\{z = 0 : \{0, 1/2\}; z = 4/27 : \{0, -1/2\}; z = \infty : \{1/3, 2/3\}\}$. Every local datum is a {2,3}-monomial: the exponents at the finite singular points are *halves*—the parity double cover, the two-field structure as the ±1/2 Frobenius pair; the exponents at infinity are *thirds*; the singularity sits at $4/27 = 2^2/3^3$, the anagram entropy of the even thread; and the odd thread is a higher Fuchsian sibling with singularity $3^6/2^{16}$ (growth ratio verified → 89.898). The Schwarz triangle (1/2, 1/2, 1/3) places the equation in the dihedral entry of the Schwarz list: the monodromy group is $D_3 \cong S_3$ of *order six*—the modulus of the entire investigation materialized as a monodromy group—and the solution is accordingly algebraic, with the closed form (verified to 27 digits)

$$\sum_{j\geq 0}\binom{3j}{j}z^j = \frac{\cos(\theta/3)}{\cos\theta},\ \sin^2\theta = \frac{27z}{4}: \tag{26}$$

the 3 of $3n+1$ is an angle trisection, and the parity halves are the cosines. The synthesis: the two-field structure is first order (the operator (7) factors into two first-order Euler operators; the unweighted thread $1/(1-X)$ has trivial monodromy), and restoring the fabric—the binomial mass operator $\mathfrak{B}$—lifts first order to second, two singular points to three, and trivial monodromy to S_3. *The fabric is the monodromy*. Honest note: the P-function sums the language's masses, not the walk; Riemann's monodromy sees the fabric, never the conjecture.

16. Honest Assessment: The Fence, the Localization, and the Audit

The reflection identity is a telescope. Since $C(n_t) = n_{t+1}$, the two sums in (10) differ by boundary terms only: the identity's truth-content *is* termination. The same holds for (8): faithful coordinates, not force.

A generality no-go. Every construction of Sections 7, 9, 10, 13, 14 applies verbatim to any map $f:\mathbb{N}\to\mathbb{N}$; but termination of generalized Collatz maps is undecidable [14], and the generalized problem is Π_2^0-complete [15]. No theorem provable at the generality of the language/zeta/weave formalism alone can settle $3n+1$: leverage must enter through the joint 2-adic/3-adic arithmetic—the Hurwitz pair, the character χ_3, the branch weights, the ladder pole—and nowhere else.

The average-case ceiling. The provable content here (the 1/3 identity, the inspection duality, the residue calculus) is measure theory; the frontier of such methods is fixed by [11] [12], and divergent orbits, if any, are measure-zero objects whose Dirichlet series enjoy unlimited analytic freedom. These theorems explain *why* the average-case machinery works; they inherit its ceiling.

Audit of the termination syllogism. The chain "every sequence is a woven binomial expansion; every binomial expansion terminates at $A^0 = 1$" contains both open halves of the conjecture, one per premise. Premise 1 hides *boundedness*: the container exponent is $M = \sup(\text{orbit})$, and its finiteness for every start is the no-divergence half—the theorem as proven constructs the expansion from the orbit, so finiteness is imported, not derived. Premise 2 hides *cycle uniqueness* via an equivocation: the static term-list of a finite expansion exhausts at the silent A^0, but the sequence traverses the expansion in relay order, and what finiteness of the container legitimately yields is only eventual periodicity (pigeonhole)—a rogue cycle would satisfy every clause. Repaired chain: [boundedness: open] + [finite container $\Rightarrow$ eventually periodic: proven, transparent in the weave] + [the only knot is the chant: open] = the conjecture. The weave compresses the problem into one number and one inventory—*does every start have a finite weave-exponent, and is the container's only closed relay loop* $A^4I^2 \to A^2I \to AI^4$?—but both open halves stand exactly where [11]-[13] left them.

Novelty. Against [1]-[10]: the orbit zeta (9) and reflection identity (10), the closed form (11) with its residue ladder, the marked zeta (15) with the pgf factorization, as used in the paper (Theorem 12.1, Equation 15), the primitive decomposition (16) with its symmetry obstruction, the annihilator function (17) with the ghost law, and the weave theorems (21)-(23) appear new in these specific forms; the genre of each has the precedents identified in Section 1.2, and the underlying equivalences are those of the Berg-Meinardus line.

17. Open Problems

1) Grammar criterion for the attractor count of a name-sum system; the uniqueness accident at $m = 26$ (Section 2).

2) Canonical (interpolation-free) complexification of the valuation field; the full monodromy group generated by collision transpositions (Section 3).

3) A natural language with empty lying kernel: does one exist beyond the fused-tens Romance grammars (Section 5)?

4) Spectral theory of the relay operator U on language monomials: spectrum, invariant functionals, polynomial identities beyond the 3-cycle (Section 9).

5) A priori bounds on the sink term of the sourced relay Equation (8) via energy/degree estimates on $\ker \mathcal{L}$.

6) The Meinardus bridge: obtain the exact operator of [3] and prove or refute the identification of its ζ-generated null space with the $s+w=1$ residue (12), now with the sharper target that the operator should act on the $\left(\zeta, L\left(\chi_3\right)\right)$-module of Theorem 12.2 with the null space in the non-alternating channel.

7) Multi-point residue calculus: closed forms for the level-k language zetas via the mod-2^k decomposition; residues as parity-vector weights with ζ-carried error terms (Section 10).

8) Thermodynamic completion: the weighted transfer operator whose pressure derivative yields the dynamic drift; its invariant density against the language zeta (Section 10).

9) Rigorous branching-process proof of the tree-ball form of the size-biased law (14); the forward statistic conjugate to tree depth (Section 11).

10) The Hurwitz functional-equation family: does any completion of the marked zeta close under $s+w \to 1-\left(s+w\right)$ with a transformed marker (Section 12)?

11) The degree-collision census at scale: the 88/12 order split of collision pairs and its tree-structural explanation (Section 13).

12) The weave's hypergeometric weighted odd thread: closed form and algebra-

icity class (Section 14).

13) The linguistic complexity of a conjecture: classify open termination problems by the length class of their minimal exact language; Collatz is linear-class, and Theorem 6.1 forbids anything shorter.

14) The complex dynamics of the entire two-field law (24): Julia-Fatou structure in the sense of [17] [18]; whether the endpoint-only-zero phenomenon of Section 15.2 persists across other homotopy families; and the full higher Riemann scheme of the weighted odd thread.

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The author acknowledges the great work done by mathematicians referenced in this record, whose work paved the way. With particular pleasure the author records that the Taylor-shift expansion (5), recalled from a childhood (14 years old) daydream in a religious-knowledge classroom in St Edward's Secondary School, some sixty years ago in Africa, (for which he was punished-rightly so upon his excitement to tell his teacher, the honorable Priest Father Michael O'Connor, who was also his math teacher, and who retired due to illness shortly after), was recognized in these sessions as the *translation operator*, found to be the unique non-dilation ingredient of the Collatz map, and finally seen generating the secondary pole ladder of the language zeta and, in Equation (19), revealed as the exact integral-differential form of the annihilator itself, with the Collatz map threaded between its towers of derivatives and integrals—a long-delayed vindication. The mathematics in this paper was developed in extended interactive research sessions with **Claude** (Anthropic), which served throughout as computation engine, symbolic and numerical verifier, literature scout, and for critical analysis verification. Every theorem proposed by the author was tested, and verified to the stated precision by Claude. The trapping theorem, the two-letter language, the operator and Mellin structures, the inspection duality, the annihilator, and the weave each took their final form in dialogue. The explorations ranged across dynamical systems, analytic number theory, combinatorics of language, operator theory, and special functions; whatever is clear in this paper owes much to that partnership, and whatever errors remain are the authors.

Historic Memorandum

St. Edward's origins began with St. Edward's Primary School which was established in 1865 by a French Roman Catholic priest, Rev. Father Edward Blanchet in Freetwon, Sierra Leone. In 1921, the board of directors which consisted of a group of priests from Italy, France and Ireland decided to start a secondary school for the students. On February 6, 1922, the new secondary school opened its doors to seven St. Edward's Primary School graduates: Anthony Tucker, Sylvester Tucker, James Massallay, Edward Farrah, William Luke, Joseph Luke and Albert M. Margai—future prime minister of Sierra Leone. At that time, the secondary school was located at the same address as the primary school at Howe Street in

Freetown. St. Edward's is the third oldest secondary school in Sierra Leone. The first head master of St. Edward's was Father Michael O'Connor, but six months after the school opened, Father O'Connor retired due to illness and was replaced by Father Mulcahy. Under the stewardship of Father Mulcahy, St. Edward's Secondary School became a first-rate academic institution which followed a strict, old-fashioned British curriculum and enforced discipline by means of corporal punishment. The sports programs at St. Edward's became a source of general admiration. The author became a longtime record holder of the highest academic score of the prestigious A-level exams in Britain unbroken for twenty years.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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