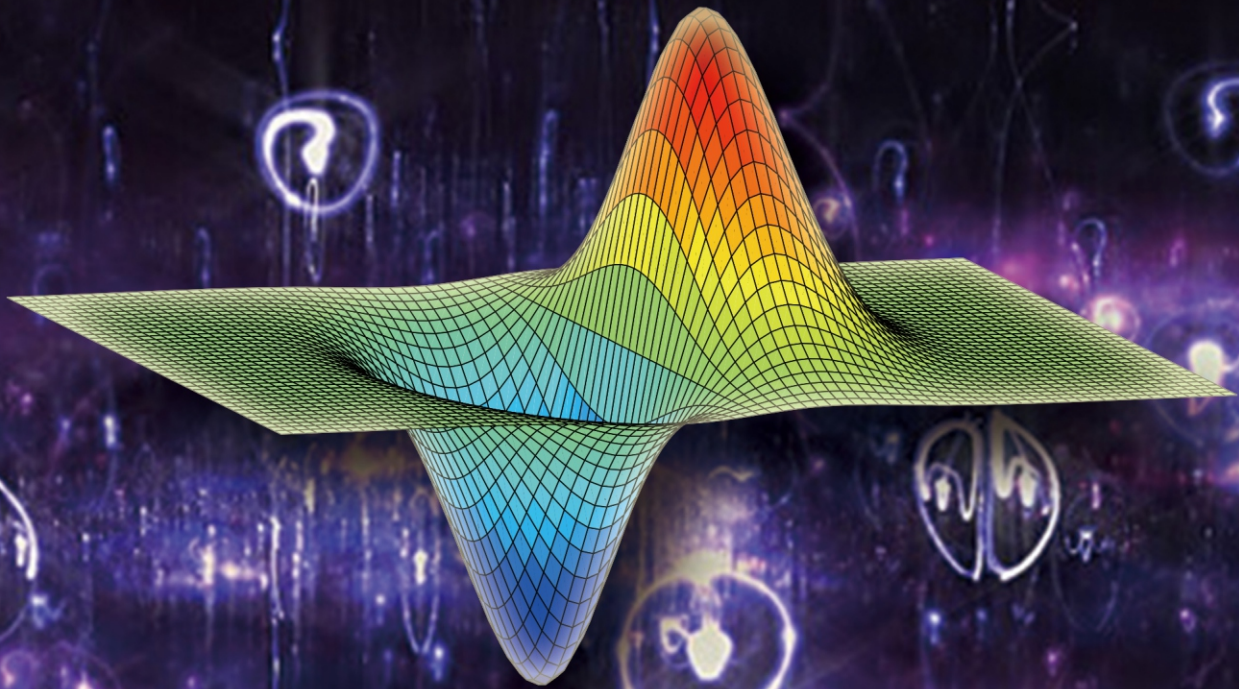


ISSN: 2160-0368 Volume 14, Number 6, June 2024



# Advances in Pure Mathematics



ISSN: 2160-0368



<https://www.scirp.org/journal/apm>

# Journal Editorial Board

ISSN 2160-0368 (Print) ISSN 2160-0384 (Online)

<https://www.scirp.org/journal/apm>

## Editor-in-Chief

Prof. Leo Depuydt

Brown University, USA

## Editorial Board

Dr. Taher S. H. Abdelmonem

Prof. Ravi P. Agarwal

Prof. Rafik Aramyan

Prof. Józef Banas

Prof. Serdar Beji

Prof. Constantin Buse

Prof. Clemente Cesarano

Prof. M. P. Chaudhary

Prof. Chuanmiao Chen

Prof. Xiaoxing Chen

Dr. Claudio Cuevas

Prof. Dragan S. Djordjevic

Dr. Ozgur EGE

Prof. Hala J. El-Khozondar

Prof. Rifa Jarallah El-Khozondar

Prof. Ahmed M. A. El-Sayed

Prof. Cristescu Gabriela

Prof. Hongya Gao

Prof. Aleksander Grytczuk

Prof. Yuxia Guo

Dr. Sevdzhan Hakkaev

Dr. Chengchun Hao

Prof. Tianxiao He

Prof. Wei He

Prof. Tingzhu Huang

Prof. Huzoor H. Khan

Prof. Mostafa M. A. Khater

Prof. Abdessatar Khelifi

Prof. Shaban Khidr

Prof. Toshitake Kohno

Prof. Vichian Laohakosol

Prof. V. Lokesha

Dr. Jingjing Ma

Prof. Elimhan N. Mahmudov

Dr. Lucia Marino

Prof. Ming Mei

Dr. Vishnu Narayan Mishra

Dr. Farkhod Mukhamadiev

Dr. M. T. Mustafa

Dr. Mahammad A. Nurmammadov

Prof. Jyotindra C. Prajapati

Dr. Marius Rădulescu

Dr. Madhu Ram

Prof. Jingli Ren

Dr. Gábor Sági

Prof. Bayram Sahin

Prof. Samir H. Saker

Dr. Bessem Samet

Dr. Navneet Lal Sharma

Dr. Yongli Song

Prof. Hari M. Srivastava

Prof. Yeong-Jeu Sun

Dr. Néstor Thome

Prof. Aydin Tiryaki

Prof. Gabor Toth

Prof. Amitabha Tripathi

Prof. Nodari Vakhania

Prof. Qingwen Wang

Dr. Yanjin Wang

Prof. Changjin Xu

Dr. Jasang Yoon

Prof. Mohamed F. Yousif

Dr. Shijun Zheng

Prof. Changjiang Zhu

Mansoura University, Egypt

Texas A&M University-Kingsville, USA

Russian-Armenian University, Armenia

Bronislaw Markiewicz State School of Higher Vocational Education in Jaroslaw, Poland

Istanbul Technical University, Türkiye

West University of Timisoara, Romania

Uninettuno University, Italy

International Scientific Research and Welfare Organization, India

Central South University, China

Fuzhou University, China

Federal University of Pernambuco, Brazil

University of Nis, Serbia

Celal Bayar University, Türkiye

Islamic University of Gaza, Palestine

Al-Aqsa University, Palestine

Alexandria University, Egypt

Aurel Vlaicu University of Arad, Romania

Hebei University, China

University of Zielona Góra and Jan Pawel II Western, Poland

Tsinghua University, China

University of Kansas, USA

Chinese Academy of Sciences, China

Illinois Wesleyan University, USA

Nanjing Normal University, China

University of Electronic Science & Technology of China, China

Aligarh Muslim University, India

Xuzhou Medical University, China

Carthage University, Tunisia

Beni-Suef University, Egypt

The University of Tokyo, Japan

Kasetsart University, Thailand

Vijayanagara Sri Krishnadevaraya University, India

University of Houston-Clear Lake, USA

Istanbul Technical University, Türkiye

Catania University, Italy

McGill University, Canada

Indira Gandhi National Tribal University, India

National University of Uzbekistan, Uzbekistan

Qatar University, Qatar

Shamakhy Astrophysical Observatory, Azerbaijan National Academy of Sciences, Azerbaijan

Charotar University of Science and Technology, India

Institute of Mathematical Statistics and Applied Mathematics “Gheorghe Mihoc-Caius Iacob” of the Romanian Academy, Romania

University of Jammu, India

Zhengzhou University, China

Alfréd Rényi Institute of Mathematics and BUTE, Hungary

Inonu University, Türkiye

Mansoura University, Egypt

Tunis College of Sciences and Techniques, Tunisia

National Rail and Transportation Institute (NRTI) Vadodara, India

Tongji University, China

University of Victoria, Canada

I-Shou University, Chinese Taipei

Universitat Politècnica de València, Spain

Izmir University, Türkiye

Rutgers University, USA

Indian Institute of Technology, India

Georgian Academy of Sciences, Mexico

Shanghai University, China

Institute of Applied Physics and Computational Mathematics, China

Guizhou Key Laboratory of Economics System Simulation, China

University of Texas Rio Grande Valley, USA

The Ohio State University, USA

Georgia Southern University, USA

South China University of Technology, China

# Table of Contents

**Volume 14    Number 6**

**June 2024**

## **New Asymptotic Results on Fermat-Wiles Theorem**

K. K. Prosper, K. K. Vincent, T. F. Emmanuel.....421

## **Stochastic Maximum Principle for Optimal Advertising Models with Delay and Non-Convex Control Spaces**

G. Guatteri, F. Masiero .....442

## **Euler Product Expressions of Absolute Tensor Products of Dirichlet $L$ -Functions**

H. Tanaka, S.-Y. Koyama .....451

## **On Perron's Formula and the Prime Numbers**

M. M. Anthony .....487

## **Evaluation of Generalized Error Function via Fast-Converging Power Series**

S. Beji .....495

# Advances in Pure Mathematics (APM)

## Journal Information

### SUBSCRIPTIONS

The *Advances in Pure Mathematics* (Online at Scientific Research Publishing, <https://www.scirp.org/>) is published monthly by Scientific Research Publishing, Inc., USA.

#### Subscription rates:

Print: \$79 per issue.

To subscribe, please contact Journals Subscriptions Department, E-mail: [sub@scirp.org](mailto:sub@scirp.org)

### SERVICES

#### Advertisements

Advertisement Sales Department, E-mail: [service@scirp.org](mailto:service@scirp.org)

#### Reprints (minimum quantity 100 copies)

Reprints Co-ordinator, Scientific Research Publishing, Inc., USA.

E-mail: [sub@scirp.org](mailto:sub@scirp.org)

### COPYRIGHT

#### Copyright and reuse rights for the front matter of the journal:

Copyright © 2024 by Scientific Research Publishing Inc.

This work is licensed under the Creative Commons Attribution International License (CC BY).

<http://creativecommons.org/licenses/by/4.0/>

#### Copyright for individual papers of the journal:

Copyright © 2024 by author(s) and Scientific Research Publishing Inc.

#### Reuse rights for individual papers:

Note: At SCIRP authors can choose between CC BY and CC BY-NC. Please consult each paper for its reuse rights.

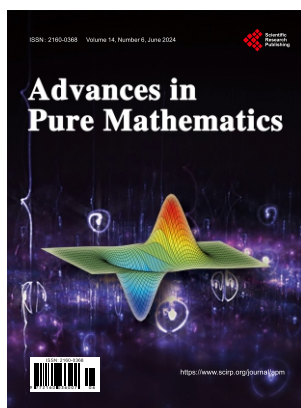
#### Disclaimer of liability

Statements and opinions expressed in the articles and communications are those of the individual contributors and not the statements and opinion of Scientific Research Publishing, Inc. We assume no responsibility or liability for any damage or injury to persons or property arising out of the use of any materials, instructions, methods or ideas contained herein. We expressly disclaim any implied warranties of merchantability or fitness for a particular purpose. If expert assistance is required, the services of a competent professional person should be sought.

### PRODUCTION INFORMATION

For manuscripts that have been accepted for publication, please contact:

E-mail: [apm@scirp.org](mailto:apm@scirp.org)



**Call\_for\_Papers**

# Advances in Pure Mathematics (APM)

ISSN 2160-0368 (Print) ISSN 2160-0384 (Online)  
<https://www.scirp.org/journal/apm>

**Advances in Pure Mathematics (APM)** is an international journal dedicated to the latest advancements in ordered algebraic structures. The goal of this journal is to provide a platform for scientists and academicians all over the world to promote, share, and discuss various new issues and developments in different areas of ordered algebraic structures.

## Subject Coverage

This journal invites original research and review papers that address the following issues. Topics of interest include, but are not limited to:

- |                             |                                                       |                                                     |
|-----------------------------|-------------------------------------------------------|-----------------------------------------------------|
| • Applied Harmonic Analysis | • Functional Differential Equations                   | • Partial Differential Equations                    |
| • Approximation Theory      | • Fuzzy Mathematics                                   | • Probability                                       |
| • Asymptotic Methods        | • Geometry                                            | • Reaction-Diffusion Problems, Discrete Mathematics |
| • Bifurcation Theory        | • Geometry of Banach Spaces                           | • Scattering Theory                                 |
| • Coding Theory             | • Integral Equations                                  | • Semigroups of Operators                           |
| • Combinatorics             | • Mathematical Biology, including Population Dynamics | • Signal Theory                                     |
| • Complex Analysis          | • Mathematical Physics                                | • Special Functions                                 |
| • Convex Analysis           | • Matrix Theory                                       | • Statistics                                        |
| • Difference Equations      | • Moment Theory                                       | • Stochastic Process                                |
| • Differential Equations    | • Number Theory                                       | • Topology and Applications                         |
| • Dynamical Systems         | • Numerical Analysis                                  | • Transport Theory                                  |
| • Fixed-Point Theory        | • Operator Theory                                     | • Ultrametric Analysis                              |
| • Fourier Analysis          | • Optimal Control                                     | • Variational Methods                               |
| • Fractals                  | • Optimization Wavelets                               |                                                     |
| • Fractional Analysis       |                                                       |                                                     |

We are also interested in short papers (letters) that clearly address a specific problem, and short survey or position papers that sketch the results or problems on a specific topic. Authors of selected short papers would be invited to write a regular paper on the same topic for future issues of the APM.

## Notes for Intending Authors

Submitted papers should not have been previously published nor be currently under consideration for publication elsewhere. Paper submission will be handled electronically through the website. All papers are refereed through a peer review process. For more details about the submissions, please access the website.

## Website and E-Mail

<https://www.scirp.org/journal/apm> E-mail: [apm@scirp.org](mailto:apm@scirp.org)

## What is SCIRP?

Scientific Research Publishing (SCIRP) is one of the largest Open Access journal publishers. It is currently publishing more than 200 open access, online, peer-reviewed journals covering a wide range of academic disciplines. SCIRP serves the worldwide academic communities and contributes to the progress and application of science with its publication.

## *What is Open Access?*

All original research papers published by SCIRP are made freely and permanently accessible online immediately upon publication. To be able to provide open access journals, SCIRP defrays operation costs from authors and subscription charges only for its printed version. Open access publishing allows an immediate, worldwide, barrier-free, open access to the full text of research papers, which is in the best interests of the scientific community.

- High visibility for maximum global exposure with open access publishing model
- Rigorous peer review of research papers
- Prompt faster publication with less cost
- Guaranteed targeted, multidisciplinary audience



**Website: <https://www.scirp.org>**  
**Subscription: [sub@scirp.org](mailto:sub@scirp.org)**  
**Advertisement: [service@scirp.org](mailto:service@scirp.org)**