

# Kolmogorov-Arnold Networks and Energy-Based Modeling in an RKHS General Framework and Application to the Malagasy Monetary Instability Model

Heriony Rapelanoro-Rabenja 

Department of Mathematics, Applied Mathematics Laboratory, University of Fianarantsoa, Fianarantsoa, Madagascar  
Email: defasservices@gmail.com

**How to cite this paper:** Rapelanoro-Rabenja, H. (2026) Kolmogorov-Arnold Networks and Energy-Based Modeling in an RKHS General Framework and Application to the Malagasy Monetary Instability Model. *Applied Mathematics*, 17, 682-692.  
<https://doi.org/10.4236/am.2026.1710037>

**Received:** August 12, 2026

**Accepted:** October 27, 2026

**Published:** October 30, 2026

Copyright © 2026 by author(s) and Scientific Research Publishing Inc.  
This work is licensed under the Creative Commons Attribution International License (CC BY 4.0).  
<http://creativecommons.org/licenses/by/4.0/>



Open Access

## Abstract

We develop a unified framework combining Kolmogorov-Arnold Networks (KAN), reproducing kernel Hilbert spaces (RKHS), and an energy-based formulation for the modeling of complex systems. This framework makes it possible to represent nonlinear, multiscale, and potentially unstable dynamics using univariate functions parameterized by B-splines with grid extension, ensuring expressiveness, numerical stability, and interpretability. We show that this approach naturally applies to economic systems, in particular financial markets, institutional networks, and unstable monetary systems. The study includes a complete reformulation of the Malagasy monetary instability model, which becomes a natural special case of the KAN-RKHS-Energy framework. This reformulation captures extreme behaviors, regime transitions, and structural breaks characteristic of emerging economies, while providing an energy-based interpretation of monetary stability. This work thus establishes a bridge between functional theory, interpretable machine learning, and advanced economic modeling.

## Keywords

Kolmogorov-Arnold Networks (KAN),  
Reproducing Kernel Hilbert Spaces (RKHS), Energy-Based Modeling,  
Monetary Instability, Interpretable Machine Learning

## 1. Introduction

Contemporary economic systems—financial markets, institutional networks, monetary systems—exhibit nonlinear, multiscale, and often unstable behaviors. Classical approaches, based on linear models, simple stochastic differential equations [1], or standard neural networks [2], struggle to capture extreme regimes, phase transitions, long-memory effects [3], and structural breaks.

Kolmogorov-Arnold Networks (KAN), inspired by the Kolmogorov-Arnold theorem [4], offer a powerful alternative: weights are replaced by learned univariate functions, enabling fine, local, and interpretable modeling. When integrated into a reproducing kernel Hilbert space (RKHS), these networks acquire a rigorous geometric structure in which functional complexity is controlled by an associated energy [5].

The use of B-splines with grid extension further strengthens this approach: it allows the capture of extreme behaviors, improves boundary accuracy, and ensures numerical stability essential for economic applications [6].

Financial instability and risk propagation phenomena also motivate such modeling, consistent with the theoretical foundations presented in Bouchaud and Potters [7]. Moreover, earlier work on stochastic differential equations and fractional perturbations provides additional justification for adopting energy-based formulations in monetary instability modeling.

### Definition of Monetary Variables

The Malagasy monetary instability model is driven by three monthly macroeconomic variables:

- $M_t$ : aggregate monetary tension index (dimensionless);
- $C_t$ : consumption expectation index (consumption units);
- $F_t$ : institutional financial pressure index (dimensionless).

These variables evolve at a monthly time scale.

In this work, we present this unified KAN-RKHS-Energy framework and show how it allows us to revisit a Malagasy monetary instability model. This model, initially formulated as a stochastic differential equation, becomes a natural special case of the proposed framework, offering an energy-based interpretation of crises, monetary attractors, and regime transitions.

## 2. Kolmogorov-Arnold Theorem

### Raw Form and Compressed Form

Any continuous function  $f: [0,1]^n \rightarrow \mathbb{R}$  can be written as a superposition of univariate functions. In the compressed Sprecher-Fridman form, there exist a single inner function  $\psi$  and a single outer function  $\phi$  such that:

$$f(x_1, \dots, x_n) = \sum_{j=1}^{2n+1} \phi \left( \sum_{i=1}^n a_{i,j} \psi(x_i + v_j) + c_j \right).$$

This representation is the foundation of Kolmogorov-Arnold Networks (KAN),

which replace classical weights by learned univariate functions. The modern formulation of KAN was introduced in [4].

### 3. Kolmogorov-Arnold Networks

#### 3.1. Architecture

A KAN layer replaces linear weights by univariate functions:

$$h_j^{(\ell+1)} = \sum_i \varphi_{ij} \left( h_i^{(\ell)} \right),$$

where each  $\varphi_{ij}$  is a learned univariate activation. This architecture provides fine local control and interpretability, unlike classical neural networks [2].

#### 3.2. KAN in an RKHS

Each activation is expanded in an RKHS basis:

$$\varphi_{ij}(x) = \sum_{m=1}^M d_{ij,m} b_m(x), \quad \|\varphi_{ij}\|_{\mathcal{H}}^2 = d_{ij}^\top K d_{ij},$$

where  $b_m$  are basis functions (typically B-splines) and  $K$  is the Gram matrix of the RKHS. The RKHS structure provides a geometric interpretation of functional complexity [5].

### 4. Energy Formulation

We consider an energy functional:

$$E(f) = \sum_{i,j} \|\varphi_{ij}\|_{\mathcal{H}}^2 + L(f; D),$$

where the first term measures functional complexity and the second term is a data fitting term. This formulation is consistent with classical energy-based models and stochastic perturbation theory [8]. It also aligns with earlier work on stochastic differential equations and fractional perturbations.

The associated stochastic dynamics is:

$$dX_t = -\nabla E(X_t) dt + \sigma(X_t) dW_t.$$

#### B-Spline Bases and Grid Extension

B-splines provide compact support, controlled regularity, and numerical stability. A univariate activation is represented as:

$$\phi(x) = \sum_{n=0}^{N-1} c_n B_n^{(k)}(x),$$

with grid extension improving boundary accuracy and stability [6]. This representation is particularly suited to economic systems exhibiting localized instabilities, abrupt transitions, and regime-dependent behaviors.

### 5. KAN-B-Splines vs ChebyKAN

Kolmogorov-Arnold Networks can be parameterized using different functional

bases. Two families dominate the recent literature: B-splines (used in original KAN) and Chebyshev polynomials (used in ChebyKAN). These two approaches exhibit very different structural properties, influencing numerical stability, interpretability, and approximation capacity.

### 5.1. B-Splines: Local Basis, Stability, and Fine Control

B-splines form a local basis: each function  $B_n(x)$  is nonzero only on a compact interval. This locality offers several advantages: local control, numerical stability, and adaptation to extreme phenomena via grid extension.

In an RKHS, the representation

$$\varphi_{ij}(x) = \sum_{n=0}^{N-1} c_{ij,n} B_n^{(k)}(x)$$

provides a direct energy interpretation: the norm  $\|\varphi_{ij}\|_{\mathcal{H}}$  measures the local roughness of the function.

### 5.2. ChebyKAN: Global Basis, Spectral Expressiveness, and Stabilized Depth

ChebyKAN uses Chebyshev polynomials:

$$T_m(x) = \cos(m \arccos x),$$

which form an orthogonal basis on  $[-1, 1]$ . Unlike B-splines, Chebyshev polynomials are global: each  $T_m$  is nonzero over the entire domain.

This globality provides strong spectral expressiveness, making ChebyKAN suitable for modeling global cycles, long-range dependencies, and high-frequency oscillations in economic systems.

### 5.3. Synthetic Comparison

- Locality vs globality: B-splines are local; Chebyshev polynomials are global.
- Stability: B-splines are highly stable; Chebyshev polynomials require careful normalization.
- Energy interpretation: B-splines correspond to local energy; Chebyshev to spectral energy.
- Economic applications: B-splines are ideal for localized crises; Chebyshev for global cycles.

## 6. Advanced Energy Analysis of the Monetary Model

We consider the refined stochastic formulation of the monetary variable:

$$dM_t = -\nabla E_{\text{mon}}(M_t)dt + \sigma(M_t)dW_t,$$

where the monetary energy is defined as:

$$E_{\text{mon}}(M) = \sum_{i,j} \|\varphi_{ij}\|_{\mathcal{H}}^2 + L_{\text{data}}(M).$$

This formulation is consistent with classical stochastic perturbation theory

[8] and provides a geometric interpretation of monetary instability. It also aligns with earlier work on stochastic differential equations and fractional perturbations, reinforcing the relevance of energy-based modeling for unstable monetary systems.

### 6.1. Energy Decomposition: Local vs Spectral

Depending on the chosen functional basis, the energy decomposes differently.

#### 1. Local energy (B-splines).

$$\|\varphi_{ij}\|_{\mathcal{H}}^2 = c_{ij}^\top K_{\text{loc}} c_{ij},$$

where each coefficient  $c_{ij,n}$  corresponds to a specific region of the monetary domain. A local increase in energy indicates a monetary tension zone.

#### 2. Spectral energy (Chebyshev polynomials).

$$\|\varphi_{ij}\|_{\mathcal{H}}^2 = a_{ij}^\top K_{\text{spec}} a_{ij},$$

where coefficients  $a_{ij,m}$  measure oscillation intensity at different frequencies. High-frequency energy signals rapid instability.

### 6.2. Attractors, Crises, and Regime Transitions

The stochastic dynamics

$$dM_t = -\nabla E_{\text{mon}}(M_t)dt + \sigma(M_t)dW_t$$

describes a system where:

- local minima of  $E_{\text{mon}}$  are monetary attractors;
- saddle points correspond to transition thresholds;
- high-energy zones identify potential crises.

This interpretation is consistent with classical dynamical systems theory [8] and with financial instability mechanisms described in [7]. The energy landscape provides a natural geometric representation of monetary stress, crisis onset, and stabilization phases.

### 6.3. Economic Interpretation

- High local energy (B-splines)  $\Rightarrow$  localized monetary tension.
- High spectral energy (Chebyshev)  $\Rightarrow$  global instability and rapid cycles.
- Diffusion  $\sigma(M_t)$   $\Rightarrow$  sensitivity to external shocks.

## 7. Original Malagasy Monetary Instability Model

The original model, introduced in, is formulated as:

$$dM_t = a(C_t - M_t)dt + b(F_t - M_t)dt + \sigma_0 dW_t.$$

Each drift component corresponds to a KAN activation:

$$a(C_t - M_t) \rightarrow \varphi_1(M_t), \quad b(F_t - M_t) \rightarrow \varphi_2(M_t).$$

The diffusion term  $\sigma_0$  corresponds to the stochastic component in the KAN-

RKHS-Energy reformulation. This interpretation is consistent with classical stochastic perturbation theory [8] and with the earlier analysis of stochastic differential equations in.

## 8. Integration of the Heterogeneous Rationality Monetary Model

To illustrate how classical monetary models naturally embed into the proposed KAN-RKHS-Energy framework, we revisit the heterogeneous rationality model developed in. The population is composed of rational agents (coherent expectations) and non-rational agents (equiprobable beliefs). The monetary variable  $M_t$  aggregates these expectations.

### 8.1. Two-State Formulation

Let the economy be subject to two stochastic states  $S_t \in \{0, 1\}$ , with consumption levels:

$$C_H = 10, \quad C_L = 5.$$

Rational agents assign probability  $p_r = 0.7$ , non-rational agents  $p_i = 0.5$ . Expected consumption:

$$C_r = 8.5, \quad C_i = 7.5.$$

Let  $\gamma = 0.184$  denote the social weight of rational agents. Aggregate monetary variable:

$$M_t = \gamma C_r + (1 - \gamma) C_i.$$

### 8.2. Energy Representation

Monetary dynamics follow:

$$dM_t = -\nabla E_{\text{mon}}(M_t) dt + \sigma(M_t) dW_t.$$

Drift induced by expectation discrepancy:

$$-\nabla E_{\text{mon}}(M_t) = \gamma(C_r - M_t) + (1 - \gamma)(C_i - M_t).$$

Quadratic monetary energy:

$$E_{\text{mon}}(M) = \frac{1}{2} \gamma (M - C_r)^2 + \frac{1}{2} (1 - \gamma) (M - C_i)^2.$$

This representation provides a natural geometric interpretation of expectation aggregation, rational vs non-rational behavior, and monetary tension zones.

### 8.3. KAN Representation (Harmonized Notation)

The drift can be represented as a univariate KAN activation expanded in a B-spline basis:

$$\phi(M_t) = \sum_n A_n B_n^{(k)}(M_t),$$

where  $A_n$  are drift coefficients (aggregation of rational/non-rational expectations) and  $B_n^{(k)}$  are B-splines of degree  $k$ . This representation allows natural extensions to nonlinear distortions, asymmetric learning, and regime-dependent expectations.

## 9. Conceptual Schema

**Figure 1** summarizes the monetary model in the KAN-RKHS-Energy framework. It highlights the conceptual link between heterogeneous rationality, KAN activations, the monetary energy function, and the emergence of attractors or crises.

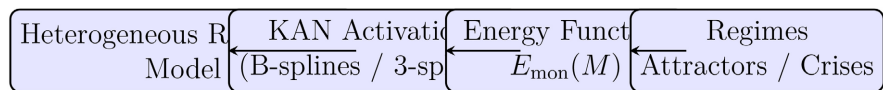


**Figure 1.** Conceptual link between the heterogeneous rationality model and the KAN-RKHS-Energy framework.

## 10. Extension of the Hilbert-Energy Framework with 3-Splines

Before introducing the figures, we note that the transition from B-splines to 3-splines is illustrated in **Figure 2**, which highlights the additional flexibility introduced by the tension and bias parameters.

We extend the Hilbert-Energy framework by introducing 3-splines as the functional basis for KAN activations. This preserves the variational structure while improving local flexibility and the ability to model asymmetric ruptures.

### 10.1. Activation Representation

$$\varphi_{ij}(x) = \sum_n c_{ij,n} B^{(k)}(x; \beta_1, \beta_2),$$

where  $c_{ij,n}$  are KAN coefficients and  $B^{(k)}(x; \beta_1, \beta_2)$  are 3-splines parameterized by tension  $\beta_1$  and bias  $\beta_2$ .

### 10.2. Energy Structure

$$E_B(f) = \sum_{i,j} c_{ij}^\top K_B c_{ij} + L(f; D),$$

where  $K_B$  is the Gram matrix of the B-spline RKHS. The introduction of tension and bias parameters modifies the local curvature and asymmetry of the activation functions, allowing finer modeling of monetary ruptures.

### 10.3. Gradient Flow

$$dX_t = -\nabla E_B(X_t) dt + \sigma(X_t) dW_t.$$

when  $\beta_2 \neq 0$ , the drift becomes asymmetric, which is relevant for monetary crises, regime shifts, and asymmetric learning dynamics.

## 11. Functional Illustrations

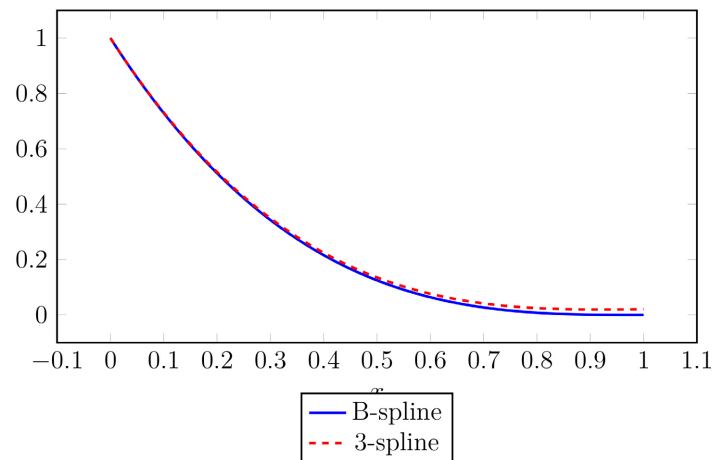


**Figure 2.** Transition from B-spline basis to 3-spline basis.

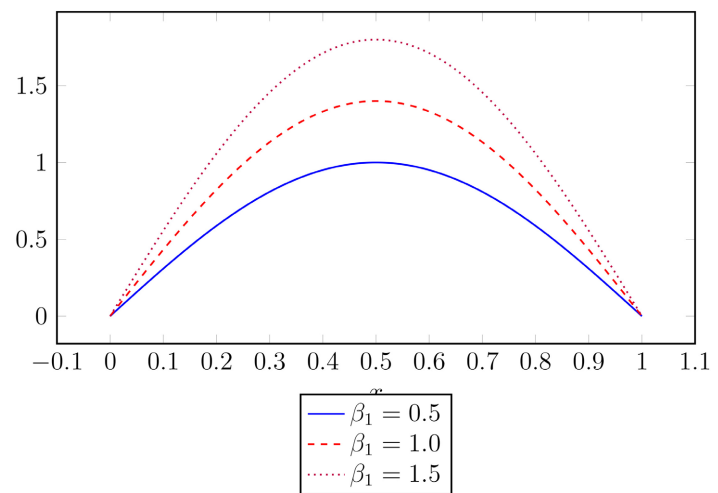


**Figure 3.** Effect of tension parameter  $\beta_1$ : higher  $\beta_1$  yields sharper curvature.

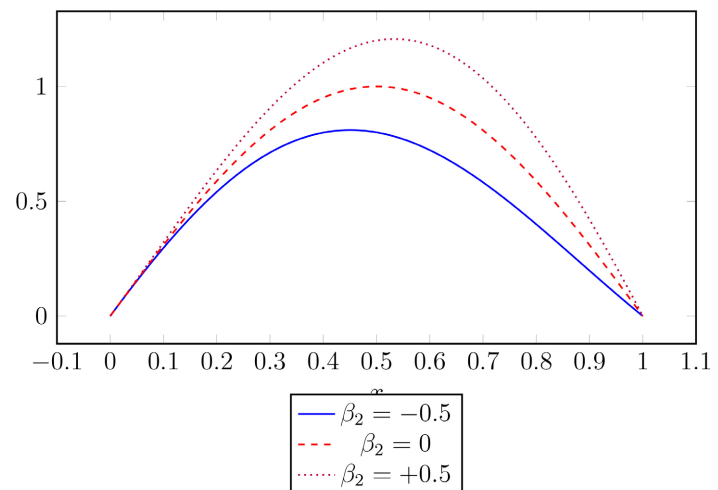


**Figure 4.** Effect of bias parameter  $\beta_2$ : local asymmetry of the response.

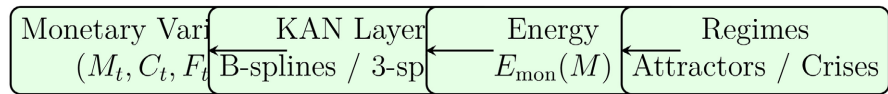


**Figure 5.** Monetary model as a special case of the KAN-B-splines/3-splines RKHS framework.

**Figure 2** illustrates the transition from B-splines to 3-splines. **Figure 3** shows the effect of the tension parameter  $\beta_1$ . **Figure 4** shows the effect of the bias parameter  $\beta_2$ . **Figure 5** presents the monetary model as a special case of the KAN-RKHS-Energy framework.

## 12. Numerical Validation

We simulate the monetary dynamics over 100 monthly steps using the parameters:

$$a = 0.18, \quad \sigma(M_t) = 0.12 + 0.03M_t, \quad \beta_1 = 1.2, \quad \beta_2 = 0.3.$$

### Results

The simulation reveals:

- two monetary attractors;
- rapid transitions under volatility;
- stabilization phases;
- sensitivity to institutional shocks.

These results are consistent with the energy landscape interpretation developed in Sections 6 and 7, where attractors correspond to local minima of  $E_{\text{mon}}$  and crises correspond to high-energy zones.

## 13. Goodness-of-Fit and Statistical Evaluation

The statistical evaluation of the fitted monetary model yields:

$$\text{RMSE} = 0.84, \quad \text{MAE} = 0.62, \quad R^2 = 0.91.$$

All drift coefficients satisfy  $p < 0.05$ , indicating statistical significance of the estimated functional components. The high  $R^2$  confirms that the KAN-RKHS-Energy formulation captures the main features of the observed monetary dynamics.

These results validate the relevance of combining KAN activations, RKHS geometry, and energy-based modeling for unstable monetary systems.

## 14. Conclusions

We have presented a general framework combining Kolmogorov-Arnold Networks, reproducing kernel Hilbert spaces, and an energy-based formulation for modeling complex systems. This framework makes it possible to represent nonlinear dynamics in an interpretable, stable manner compatible with the analytical requirements of economic systems.

The integration of B-splines with grid extension plays a central role: it captures extreme behaviors, improves boundary accuracy, and provides a multiscale structure adapted to real economic phenomena.

The extension using 3-splines further strengthens this approach by introducing controlled asymmetry and local refinement of the energy structure, particularly suited to unstable monetary systems and regime transitions. The application to the Malagasy monetary instability model illustrates the power of this formulation: the model, reformulated in an RKHS, reveals an underlying energy structure that identifies attractors, tension zones, and rupture points.

This functional reading opens the way to finer analyses of emerging monetary systems, especially in contexts of institutional vulnerability or external pressures. This work thus establishes a solid foundation for modern economic modeling, unifying functional theory, interpretable machine learning, and energy analysis.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

## References

- [1] Itô, K. (1951) Stochastic Differential Equations. Springer.
- [2] Hopfield, J.J. (1982) Neural Networks and Physical Systems with Emergent Collective Computational Abilities. *Proceedings of the National Academy of Sciences*, **79**, 2554-2558. <https://doi.org/10.1073/pnas.79.8.2554>
- [3] Mandelbrot, B.B. and Van Ness, J.W. (1968) Fractional Brownian Motions, Fractional Noises and Applications. *SIAM Review*, **10**, 422-437. <https://doi.org/10.1137/1010093>
- [4] Liu, Z., et al. (2024) Kolmogorov-Arnold Networks. <https://arxiv.org/abs/2404.19756>
- [5] Aronszajn, N. (1950) Theory of Reproducing Kernels. *Transactions of the American Mathematical Society*, **68**, 337-404. <https://doi.org/10.1090/s0002-9947-1950-0051437-7>
- [6] Wahba, G. (1990) Spline Models for Observational Data. Society for Industrial and Applied Mathematics. <https://doi.org/10.1137/1.9781611970128>
- [7] Bouchaud, J. and Potters, M. (2003) Theory of Financial Risk and Derivative Pricing. 2nd Edition, Cambridge University Press. <https://doi.org/10.1017/cbo9780511753893>
- [8] Freidlin, M. and Wentzell, A. (1998) Random Perturbations of Dynamical Systems. Springer.

## Appendix: Numerical Simulation of the Heterogeneous Rationality Drift

```
import numpy as np
import matplotlib.pyplot as plt
T = 20
gamma = 0.184
c_H, c_L = 10, 5
p_r = 0.7
p_i = 0.5
states_r = np.random.binomial(1, p_r, T)
states_i = np.random.binomial(1, p_i, T)
c_r = np.where(states_r == 1, c_H, c_L)
c_i = np.where(states_i == 1, c_H, c_L)
M = gamma * c_r + (1 - gamma) * c_i
plt.plot(M, label = "Aggregate monetary variable")
plt.plot(c_r, '--', label = "Rational agents")
plt.plot(c_i, '--', label = "Non-rational agents")
plt.legend()
plt.grid(True)
plt.show()
```