

Early Symbolic Notations for Mechanism Design in the 19th Century

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Abstract

Labels and symbolic notations are the basis for modelling and analysis of mechanical designs, particularly of mechanisms with features that have been proposed since the beginning of the nineteenth century, even for design purposes. They were proposed in several forms and solutions, starting with pioneer applications in catalogs and classifications of machines and mechanisms. Early symbolic notations of mechanisms, mainly due to Babbage, Reuleaux, and Masi, are presented and discussed in this paper with the aim of giving merit to those pioneer works and of tracing a historical evolution of the matter that is still fundamental for modern developments both in teaching and research. Modern symbolic notations have evolved from these early attempts with new mathematical features for design and analysis algorithms, with an increased interest in broader classifications and more efficient analysis and synthesis procedures.

Keywords

History of Mechanical Engineering, History of Mechanisms, Mechanism Design, Mechanism Modelling, Symbolic Notations, History of Mechanism Analysis

1. Introduction

Modeling is an important and fundamental phase in analysis and design activities in any discipline, especially in engineering, and more particularly in the field of mechanical systems. Furthermore, it has been and is being used effectively in teaching and training activities to introduce concepts and procedures for examining systems, especially mechanical ones. Modeling has been developed through concepts and methodologies that were initially purely descriptive and logical, culmi-

nating in graphical representations, even including analytical formulations, to concisely represent aspects and components of system structures and functions as well as their operation.

However, to the authors' knowledge, there is no specific interest or attention to the historical evolution of modeling concepts and methods that were used to support the introduction of new ideas, and especially new procedures. However, to the authors' knowledge, there is no specific interest or attention to the historical evolution of modeling concepts and methods that were used to support the introduction of new ideas, and especially new procedures. More specifically, the topics of notations and labeling for mechanism designs have not been addressed so far, and this paper is an attempt indeed to raise interest in such a subject also for better understanding of design and classification of mechanism design in the past and even in modern time. Particularly in mechanical engineering, modeling has evolved from often descriptive logics and methodologies to graphical and even analytical formulations. However, these formulations have been used for other purposes, so their evolution has gone unrecognized, while their effectiveness has been enhanced by the results of the procedures that used them.

More specifically, the topic of notation of mechanisms has not attracted interest in the studies of the historical evolution of technologies related to mechanisms and machines, most likely being considered an implicit tool in the treatment of the subjects of design and research on mechanism design. In fact, in the literature there are no specific works that address the topic, but attention is rather directed to the graphic aspects of the representation of machines and mechanisms, which is considered the fundamental technical language of the technology of machines and mechanisms. For example, in the works for classification and catalogue of mechanisms, the main attention was and is still paid to the kinematic modeling of the mechanism with the indication of link components, as shown in [Ceccarelli \(2004\)](#).

The history of notations is closely connected to the history of modeling in mechanical engineering since both show a shift from physical intuition to mathematical rigor as a transition to something ordered. The early models served to demonstrate ideas, test concepts, and communicate designs, but over time, it became clear how useful they were for making designs, experiments, and testing. As emblematic examples, we may mention that Heron of Alexandria described a model worked by steam as early as 120 A.D. ([Papadopoulos, 2007](#)), and that James Watt's development of the steam engine was significantly aided by his work on a model engine in the 18th century ([Dickinson, 2010](#)). In the 19th century the models became a well-recognized engineering tool as also extensively used in textbooks for teaching and particularly for formation of engineers. The solution of a scale problem, the establishment of similarity rules, and the application of infinitesimal calculus to actual engineering problems marked the transition from purely visual models to contemporary ones based on mathematics, up to computational models ([Ceccarelli, 2004](#); [Gibbings, 2014](#); [Chondros, 2021](#)).

Specifically, the analytical study and design of mechanisms were transformed when mathematics was incorporated into modeling. The development of a tool for inventing and synthesizing novel mechanisms was also necessary. Such a tool was found in the form of topological analysis of the structure of mechanisms, as indicated in Moon (2003). Its key innovation was scale-invariant reasoning: it could predict a mechanism's possible behaviors directly from qualitative inputs and a symbolic description, bypassing quantitative dimensional data. This advance, in turn, spurred rapid progress in the symbolic notation systems used for mechanism design.

This paper is an attempt to fill the absence of consideration of the history of notation in the historical examination of the evolution of mechanical engineering, and especially of mechanism design. It focuses specifically on the historical evolution of the schematization of mechanism design parameters, which has led to formal logical expressions with notation characteristics useful in mathematical formulations. In particular, this work presents a report of the historical evolution of notation modes for the structure of mechanisms with the intent of stimulating further historical work and research to better determine the contributions and impacts that graphic and analytical aspects of mechanism notations had on the development of knowledge and theories in the fields of the theory, design, and practice of machines and mechanisms.

2. Modelling of Mechanisms

Label notation can be considered a fundamental tool in the final phase of a modeling process that starts from examining the real system in its multidisciplinary components up to creating specific models for the fields of interest. Figure 1 summarizes the process of schematization and modeling of modern mechatronic systems that may give specific models that are used for mechanical analysis and design in terms of kinematics and dynamics of components of a machine. In general, a modeling process is planned with a segmentation of the aspects of interest in sequential order towards the model for the problem target after the main field is focused among the multidisciplinary sectors for a mechatronic analysis.

In Figure 1, a general flow of this sequential segmentation is represented, referring to mechanical aspects of modeling when the aim is to reach graphic representation with mechanical drawings, with the identification of mechanical design in its main components, and finally with the kinematic design modeling. This last modelling activity focuses on kinematic parameters of the characteristics and motion capabilities of the mechanisms that compose a machine to identify the parameters with specific labels. Label modelling can refer to kinematic evaluation procedures within which the symbolic notation of mechanisms plays an important role. In general, a label is a symbol for parameters that can be either variables or invariable measures of an entity. The reverse process in Figure 1 indicates design activities that may run from abstract modeling of aspects of dynamics and kinematics up to the definition of the machine design through identification

of components for a practical mechatronic construction, including similar reverse processes also for the other multidisciplinary aspects.

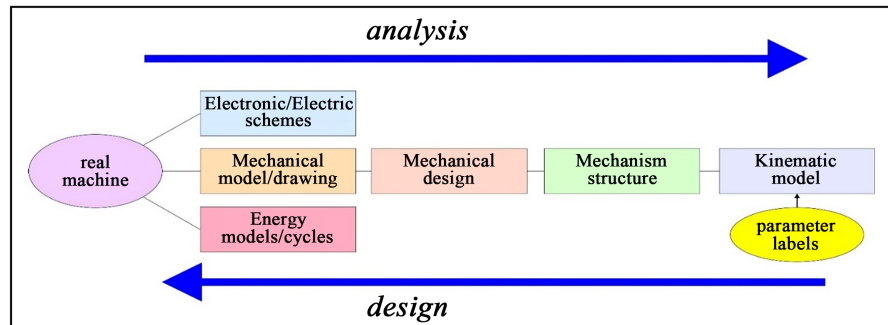


Figure 1. Modelling process of a mechatronic machine towards a kinematic model with parameter labels.

Figure 2 shows the specific modeling procedure that can be used in research, design, and training of mechanism designs, with a step-by-step activity that, in analysis, gives a kinematic model using adequate link and joint schemes with design and operation parameters that are formulated with kinematic representation with appropriate notations. Thus, labels are used to indicate specific design structure or operation variables, and notation can be used with its mathematical features to indicate other aspects and structure characteristics. Similar to **Figure 1**, **Figure 2** also indicates how an inverted process can be used for the design and synthesis of mechanisms. In both activities, labeling with proper notation can be considered an important step not only for identification of main parameters, but it can also be instrumental for efficient calculation procedures towards a numerical characterization of results both in analysis and synthesis of mechanisms.

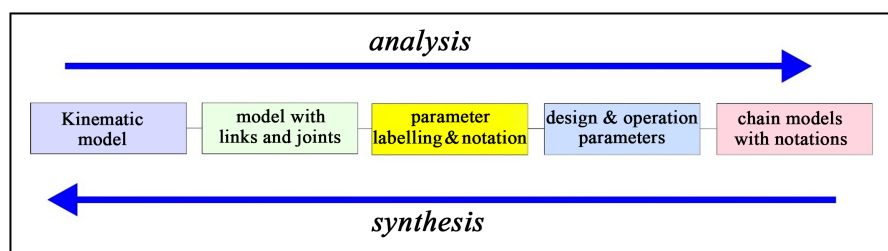


Figure 2. Modelling of mechanisms as schemes with parameter identification by labels and notations.

Construction principles in terms of design relationships can be identified by conceptual rules in the assembly of structures and mechanisms. These rules, which are often expressed in the past in descriptive or logical forms such as structural constraints, can be formulated in mathematical form with features that can be expressed and elaborated with appropriate computational algebras. Specific characteristics of structures and functions can be represented with mathematical to-

pologies so that a symbolic notation formulation can be characterized by variables (parameters), mathematical structure (topology), and computational structure (calculus algebra).

The main requirements for a symbolic notation are summarized in the diagram in **Figure 3** with characteristics that can be expected for a formulation useful in analysis and design algorithms. The need to indicate with a symbol corresponds to the need to identify significant parameters in the structure of a mechanism either in analysis or design problems. Labels can be considered a first step of a notation whose better elaboration can lead to significant formulations with great potential in computational aims.

In the field of mechanisms, a symbolic evaluation approach has evolved from descriptive solutions for classification purposes up to graphical symbolisms for analytical approaches with attempts in algebraic formulations useful in design procedures, arriving at the more sophisticated systems used in modern expert systems for computer-aided design. The structure and characteristics summarized in **Figure 3** can be recognized both in the earliest attempts and in the most modern systems of symbolic formulation of mechanisms.

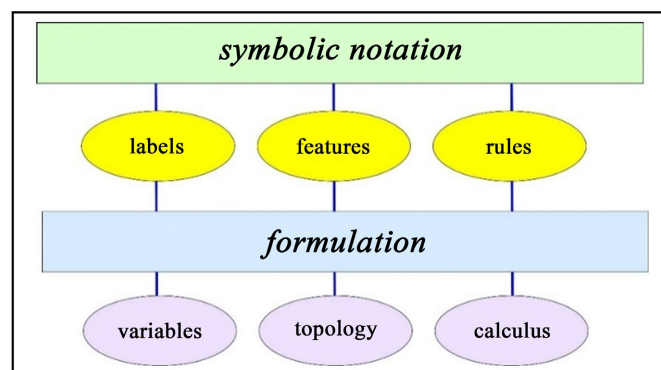


Figure 3. The main requirements for a symbolic notation.

3. Early Notations

Mechanism classification in the 18th and 19th centuries was motivated by the variety of designed machines and practical applications. It was based on tables and abstract notations, similar to those in biology and chemistry. Monge, Laboulaye, Babbage, Hachette, Reuleaux, Grashof, Willis, Masi, Silvester, and many others developed notations for typical components and their functions, based on investigations of machinery structure, part motions, and intended applications. The first modeling using labels can be recognized in the classification studies and mechanisms of the early 19th century, where such symbolic notations were used primarily to indicate the characteristics of the classification and/or the classes or categories that were determined by it. Only in a few cases of specific analysis of specific mechanisms was the indication of structural or functional parameters used to specify the structural and functional characteristics of the examined mechanism as an example of the typologies of the classes or categories

of classification.

An emblematic example and also the first treatise on machine classification accepted as a reference for the entire nineteenth century and beyond is the work (Lanz & Betancourt, 1808), developed according to the principles of Gaspard Monge, in which the classification is parameterized according to the type of input-output movement (Figure 4). Figure 4(a) shows the summary table, and in the detail of Figure 4(b), the labels of the classification of the categories are used as letters of the alphabet or numbers to indicate synthetic parameters of the representation of the categories of the proposed classification.

Extrait sur la Composition des Machines, par M. J. Lanz & A. B. Betancourt.

Les Machines se divisent en quatre classes principales, savoir : les Machines à mouvement continu, les Machines à mouvement alternatif, les Machines à mouvement rectiligne, et les Machines à mouvement circulaire. Ces classes se subdivisent en plusieurs sous-classes, qui sont désignées par des lettres et des chiffres. Les Machines à mouvement continu se divisent en Machines à mouvement continu simple et en Machines à mouvement continu composé. Les Machines à mouvement alternatif se divisent en Machines à mouvement alternatif simple et en Machines à mouvement alternatif composé. Les Machines à mouvement rectiligne se divisent en Machines à mouvement rectiligne simple et en Machines à mouvement rectiligne composé. Les Machines à mouvement circulaire se divisent en Machines à mouvement circulaire simple et en Machines à mouvement circulaire composé.

(a)

varie d'après une loi donnée.

	A	B
1	On transforme le mouvement rectiligne en circulaire par le 3, et inversement le mouvement circulaire en rectiligne par le 3.	
2	On transforme le mouvement rectiligne en circulaire par le 3, et inversement le mouvement circulaire en rectiligne par le 3.	
3	On transforme le mouvement rectiligne en circulaire par le 3, et inversement le mouvement circulaire en rectiligne par le 3.	

(b)

Figure 4. Classification of machines and mechanisms in (Lanz & Betancourt, 1808): (a) The summary table; (b) Zoomed view.

This classification was considered and expanded by several authors, especially

in terms of the number of machines and mechanisms, while in many cases still maintaining the formal representation of the synthetic classification in [Figure 4](#), as for example in the encyclopedic work of Gian Antonio Borgnis ([Borgnis, 1818-1823](#)).

In the example ([Figure 5](#)), in 1841, letter labels are used for a more general and compact mechanism classification as proposed by Robert Willis ([Willis, 1841](#)).

SYNOPTICAL TABLE			
OF THE			
ELEMENTARY COMBINATIONS OF PURE MECHANISM.			
	DIRECTIONAL RELATION CONSTANT.		DIRECTIONAL RELATION CHANGING PERIODICALLY.
	<i>Velocity-Ratio Constant.</i>	<i>Velocity-Ratio Varying.</i>	<i>Velocity-Ratio Constant or Varying.</i>
	CLASS A.	CLASS B.	CLASS C.
DIVISION A. By Rolling Contact.	Rolling cylinders, cones, and hyperboloids. General arrangement and form of toothed wheels. Pitch.	Rolling curves and rolling curve-wheels. Roëmer's & Huyghens' wheels, &c. Wheels with intermitted teeth. Rolling-curve levers.	Mangle-wheels. Mangle-racks. Escaping gearings.
DIVISION B. By Sliding Contact.	Forms of the individual teeth of wheels. Cams. Screws. Endless screws or worms and their wheels.	Pin and slit lever. Cams. Unequal worm. Geneva stop and other intermittent motions.	Pin and slit lever. Cams in general. Swash plate. Double screw. Spiral and solid cams. Escapements.
DIVISION C. By Wrapping Connectors.	Arrangement and material of bands. Form of their pulleys. Guide pulleys. Geering chains. Arrangements for limited motions.	Curvilinear pulley. Fusees.	Curvilinear pulley and lever.
DIVISION D. By Link-work.	Cranks and link-work for equal rotations. Cranks for limited motions. Bell crank-work.	Link-work. Hooke's joints.	Cranks, excentrics, and other link-work. Ratchet wheels and clicks. Intermittent link-work.
DIVISION E. By Reduplication.	Tackle of all kinds, with parallel cords and in trains.	Tackle with unparallel cords.	

Figure 5. Classification of machines and mechanisms by Robert Willis ([Willis, 1841](#)).

Even in modern times, this solution of representing typologies and functions with parametric label notations is used in summary tables, as for example in the encyclopedic work of I. Ivan Artobolevsky ([Artobolevsky, 1975-1980](#)), of which [Figure 6](#) shows examples. It is worth noting the use of letter and number labels to indicate different typologies and also to catalogue individual machines and mechanisms.

Group No.	I			
Group name	Elements of Mechanisms			
Group index	EM			
	No.	Name	Subgroup index	Mechanism No.
	1. 2.	Kinematic pairs Movable joints	KP MJ	1 through 54 55 through 119
Group No.	II			
Group name	Simple Lever Mechanisms			
Group index	SL			
	No.	Name	Subgroup index	Mechanism No.
	1. 2.	Lever mechanisms Gripping, clamping and expanding mechanisms	L	120 through 162
	3.	Balance mechanisms	GC	163 through 245
	4.	Brake mechanisms	B	246 through 251
	5.	Stop, detent and locking mechanisms	Br	252 through 257
			SD	258 through 334

(a)

index							
SC	LC	GL	LR	FL	EL	WL	LS
1660 through 1664							
				2017 and 2018			
				1975 through 1985		2194	2267
	1699 and 1700				2128 and 2129	2189	
				1939 through 1970			
1645 through 1648	1701 through 1703	1750 through 1760	1835	1921 through 1938		2179	
1672 through 1683	1729 through 1732	1783 through 1788	1846 through 1856	2105 through 2108	2114 through 2117	2200	2282 through 2288
1634 through 1639	1808 and 1809	1761			2142		2247 and 2248

(b)

Figure 6. Classification of machines and mechanisms (Artobolevsky, 1975-1980) as a function of: (a) Structure characteristics; (b) Functional features.

4. Early Symbolic Notations

Charles Babbage (1791-1871) can be considered the first to attempt an articulated notation with mathematical features to represent the structure and operational performance of mechanisms. He proposed the invention of the first modern symbolic notation in a publication specifically on the subject in 1826 (Babbage, 1826) that he further republished in the years that followed (Babbage, 1830; 1851). Babbage proposed his symbolic notation because he considered significant and of practical implementation the power of symbols in notations with mathematical features as able to express complex ideas and to facilitate the discovery of new ones as the historical investigation (Dutz & Schlimm, 2021) has explained. The primary goal of the proposed Babbage notation was to provide the most complete information about mechanical structure and the operation features in a synthetic symbolic form. Considering these targets, Babbage introduced specific symbols for mechanism structures and operations integrated by labeling notes to represent a machine structure, using different letters for frame, movable parts, working points, and component order (Babbage, 1826; 1830; 1851).

More specifically, in his 1826 publication (Babbage, 1826), Babbage gives grammar rules and a letter dictionary on how to indicate or name the mechanism structure by addressing components. The sign dictionary was quite complex in terms

of the number of signs and sign interpretation, and then the grammar of the representation of relationships between the signs was also quite difficult to implement and indeed to understand in analyzing the mechanism features. **Figure 7** shows basic notation signs and procedure steps in **Figure 8** as proposed in the Babbage notation. An example of its application is reported in **Figure 9** as provided by **Babbage (1826)**, in which one can note the above-mentioned limits of the proposed label notation.

Following Charles Babbage, Robert Willis (1800-1875) proposed a limited re-elaboration of the Babbage notation in a specific chapter of his book of 1841 (**Willis, 1841**). The notation consists of a set of motion lines to represent the motion of bodies, which included solid and dotted, straight and curved, parallel and not parallel, and so on. Special lines were also defined to indicate the transmission flow of the motion among the mechanism components, as illustrated in the example he reported for a saw-mill mechanism as in **Figure 10** (**Willis, 1841**). As a result, although the table representation of the mechanism and its operation was reduced with some less complicated sign complexity, as can be noted by comparing **Figure 9(b)** and **Figure 10(b)**, the machine schematic presentation turned out to be still complicated and not of practical use and understanding.

Babbage’s notation and its re-elaboration by Willis are an attempt to provide a synthetic graphical representation of the structure and functionality of mechanisms. The aim was to facilitate the identification of characteristic parameters of both structural design and functionality using a graphical dictionary of drawings in a graphical algebra of relationships between parameters. However, the attempt was unsuccessful, primarily due to the complexity of both the dictionary of signs and the algebra of relationships, which produced a representative table of the mechanism under examination that was difficult to manipulate and interpret, so that it was quickly forgotten. However, the attempt highlighted the need for a synthetic representation of the structure and functionality of mechanisms that was distinct from the mechanical structure of the mechanism under examination, generating a great interest in researching further methods of notation and labeling of mechanisms.

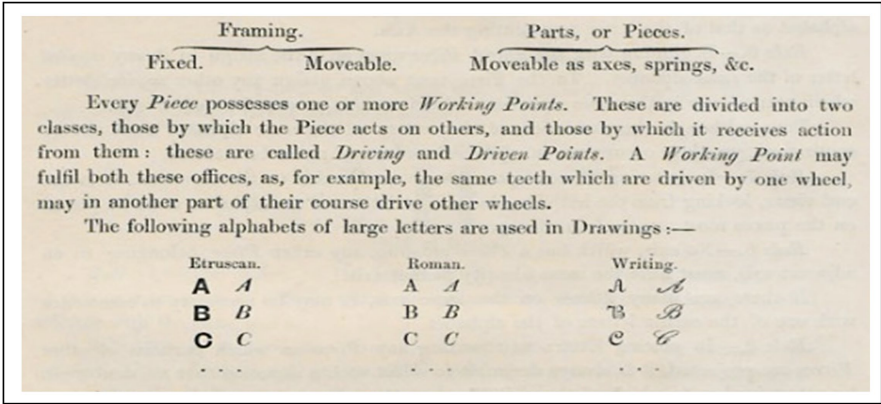
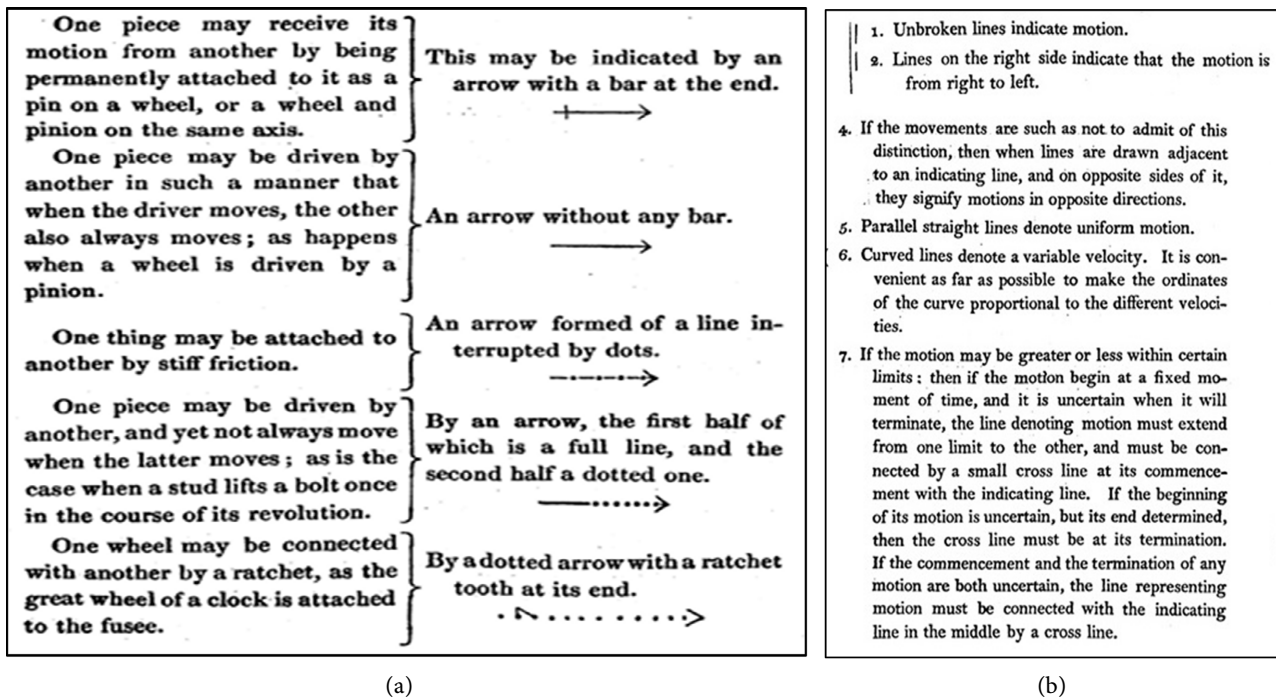


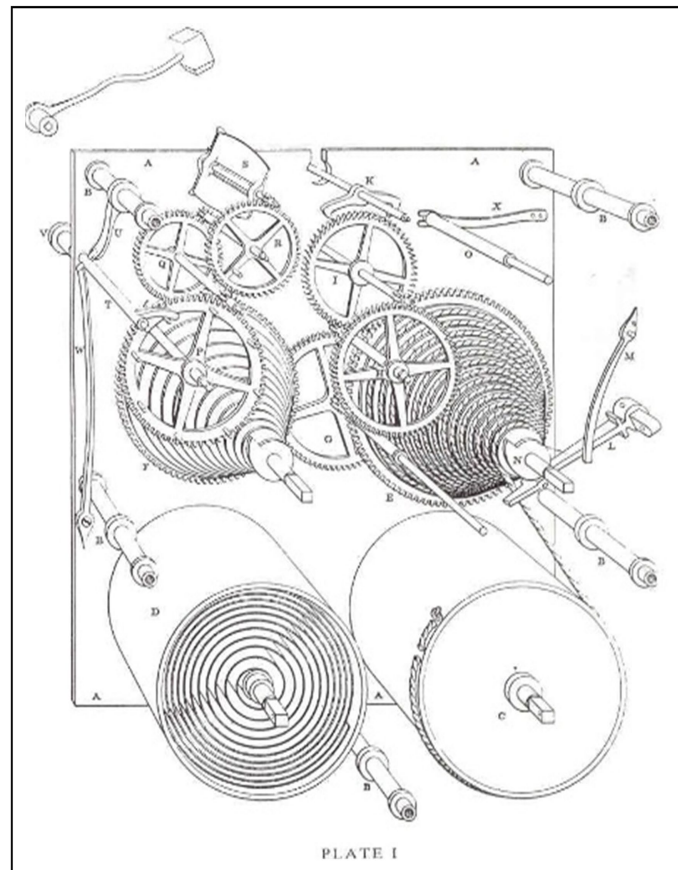
Figure 7. Babbage’s alphabet of machine parts from his publication (**Babbage, 1851**).



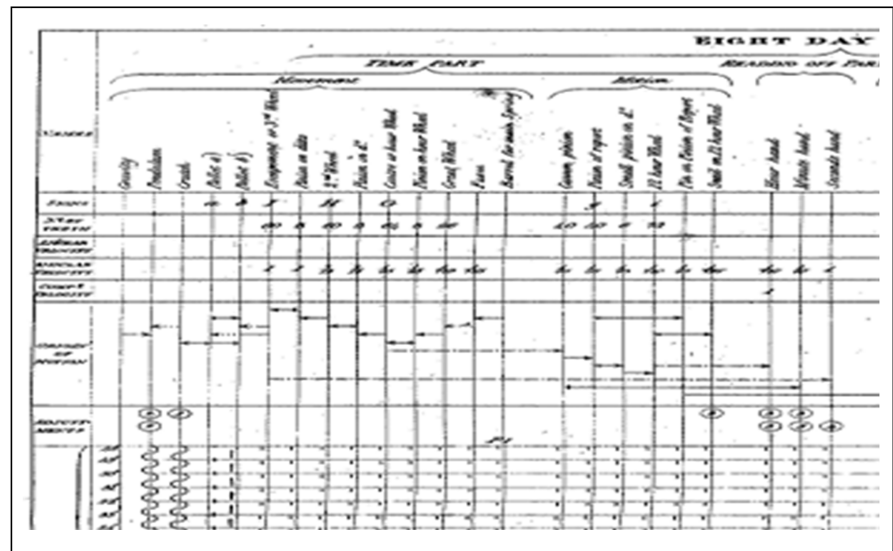
(a)

(b)

Figure 8. Symbolic notation proposed by Charles Babbage in 1826 (Babbage, 1826): (a) Basic signs for motion representation; (b) Procedure steps for mechanism representation.

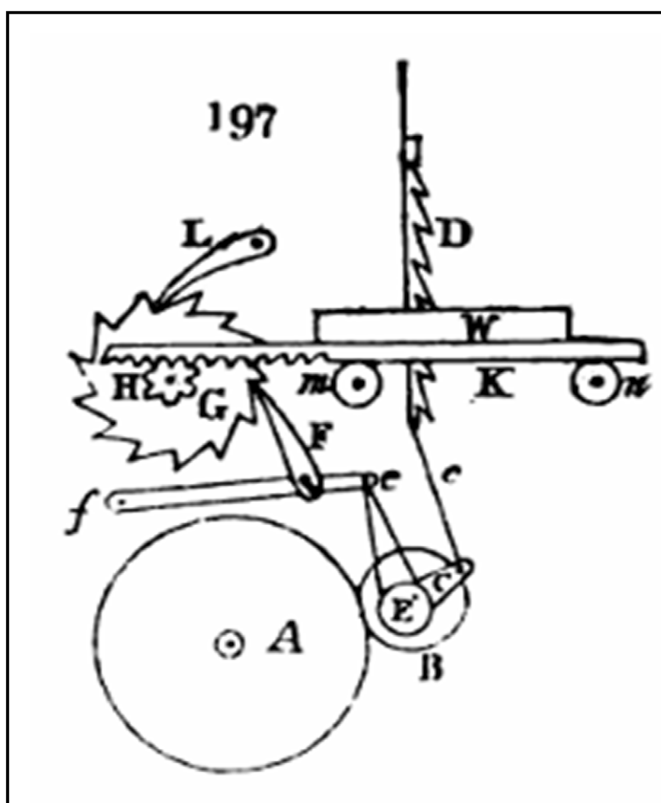


(a)

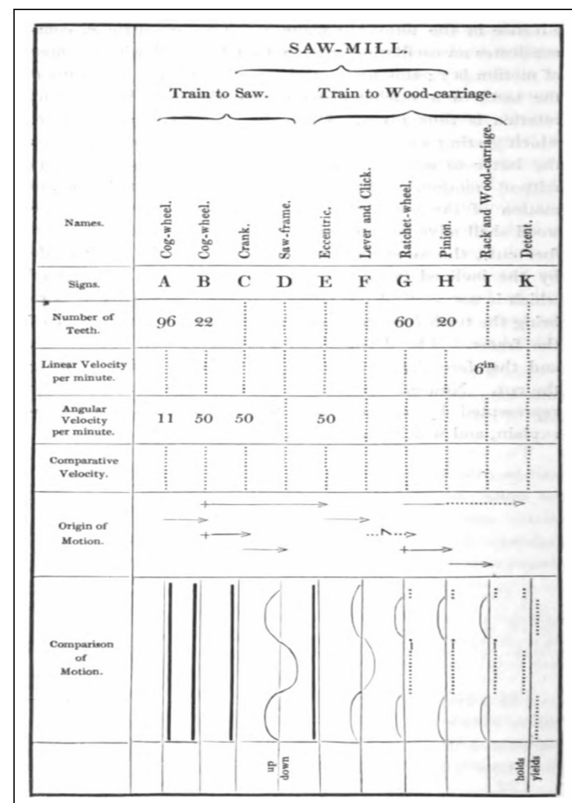


(b)

Figure 9. Example of symbolic notation (Babbage, 1826) for a clock mechanism: (a) The mechanical design; (b) Part of the Babbage notation reported in a large table.



(a)



(b)

Figure 10. Example of symbolic notation (Willis, 1841): (a) Saw-mill mechanism, (b) The re-elaborated Babbage notation.

5. Formula-Oriented Symbolic Notation

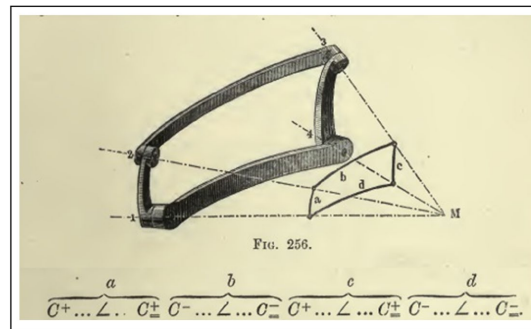
In his 1875 book (Reuleaux, 1875), Franz Reuleaux (1829-1905) proposed a revo-

lutionary approach to analyzing the construction and operation of mechanisms, leading to a new symbolic notation. Reuleaux considered a mechanism as a network of constrained links and suggested a way to distinguish one mechanism from another by looking at the assembly of joint pairs in the mechanism structure. In a specific chapter of his book, he proposed that each kinematic pair can be modelled by a symbol and an entire mechanism can be represented by a sequence of those symbols. He modelled the structure of a body link with corresponding letter-symbols indicating the connected pairs with letters and the link shape with graphical symbols for the geometric aspects, as reported in the examples in **Figure 11(a)**. For example, C will represent cylinders rolling together, whereas C– and C+ indicate the cylindrical pair with its two surfaces matching each other, with C– as one full and C+ one open. Thus, lower pairs (formed by bodies with parallel axes) are named as $R_{=}^+ R^-$ and $P_{=}^+ P^-$, for revolute and prismatic pairs, respectively, in which the symbol = indicates that the axis of the two surfaces R+ and R– coincides. Using this graphical symbolic notation, Reuleaux proposed a formula to represent a mechanism with its kinematic structure by indicating the pair types. **Figure 11(b)** shows his proposed illustrative example referring to a four-bar linkage (Reuleaux, 1875), by using the letters for the pair surfaces and graphical signs for the geometric configurations of the axis assembly. It is to be noted that each link is represented by a short sequence of symbols of letters and graphical signs, making the notation compact and easily understood. Thus, the first application of the proposed notation was understood as a design tool, being a formalism for synthesizing mechanisms in a more general way and topology. Reuleaux proposed topology design of mechanism structures using a manipulation of those formulas, such as to get kinematic inversion and increase the number of link elements to generate new mechanisms. In addition, the formula manipulation of the components in the link representation and in the mechanism link sequence permitted generation of mechanisms from coplanar to spatial chains, reduction of kinematic chain elements, increase of kinematic chains, and generation of compound chains. Nevertheless, Reuleaux did not use his new notation in analyzing the mechanisms in his book extensively and left the notation unexplored for further practical applications, considering it an elegant mathematical-oriented representation of mechanism structure.

Relevant is the development that Francesco Masi (1852-1944) proposed in 1883 in his book (Masi, 1883) for the mechanism classification by using a re-elaboration of Reuleaux's approach and formalism in (Reuleaux, 1875). Masi revised and completed the mechanism symbolic notation by expressing the formalism in clearer technical signs; for example, C+ is for a bulk cylinder (pivot), C– is for a hole cylinder (bearing), and the sign = is for parallel axes. Thus, he expressed a formula for a mechanism with additional signs, such as for fixed bodies using a segment with a dashed line, and for moving bodies using a straight segment, as shown in **Figure 12**. It is worthful to highlight a comparison of the Masi notation in **Figure 13** with Reuleaux notation in **Figure 11** referring to the symbolic notation for a space four-bar linkage showing a more compact expression.

C^+ Full cylinder, C^- Open cylinder, S^+ Screw spindle, S^- Nut, K^+ Full cone, K^- open cone, K^0 Plane cone (Cone having a vertex angle of 180°). $\tilde{C}$ Cylinder upon a general curvilinear base, $\hat{C}$ the same cylinder full, $\hat{C}^-$ the same open,	$=$ equal, $>$ greater than, $<$ less than, ∞ infinite; $ $ coaxial, $\parallel$ parallel, $\angle$ oblique, $\perp$ normal; $\diagup$ crossed obliquely; $\perp$ crossed at right angles; $\neq$ equal and con-axial, $\#$ equal and parallel; $\cong$ coincident; $\square$ conplane,— lying in the same plane, ∇ anti-parallel (in a quadrilateral),
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(a)



(b)

Figure 11. The proposed symbolic notation I (Reuleaux, 1875): (a) Basic mechanism elements and structure types; (b) Example of notation for four-bar mechanisms.

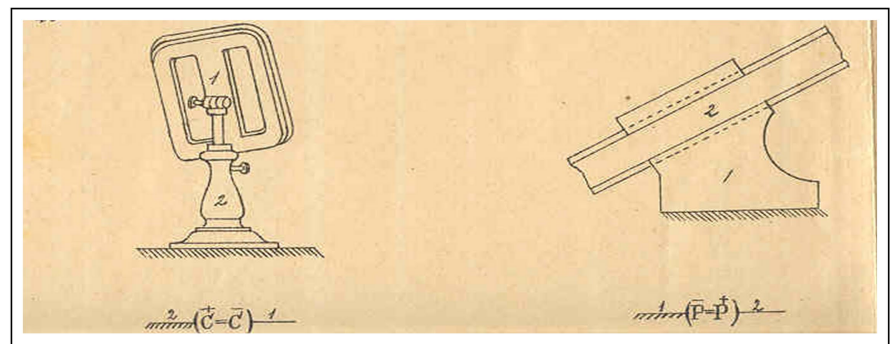


Figure 12. Examples of symbolic notation and formalism for an analytical representation of basic pairs in mechanisms by Francesco Masi in 1883 (Masi, 1883).

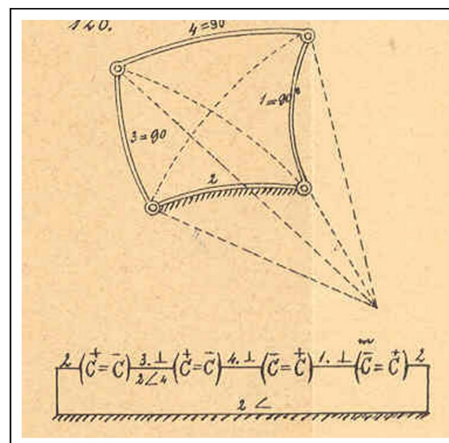


Figure 13. The proposed symbolic notation of Francesco Masi in 1883 (Masi, 1883), for four-bar mechanisms.

In addition, the proposed symbolic notation was useful as an analytical representation of mechanisms, avoiding/replacing large mechanism drawings, as in the case in **Figure 14**. The Masi symbolic notation can be efficiently used also to develop a procedure for type synthesis, as in the example in **Figure 15** for the case of slider-crank mechanisms and its inversions by using permutation of the order of the mechanism components in choosing the frame link for the mechanism. The notation is also very efficient in showing the variety of possibilities of a kinematic chain by easily manipulating the expression by reordering the place of the components or by adding other elements.

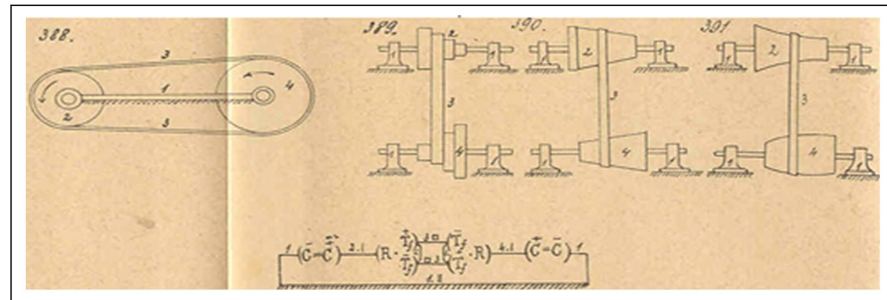


Figure 14. The proposed symbolic notation of Francesco Masi in 1883 (Masi, 1883), representing several belt transmission mechanisms.

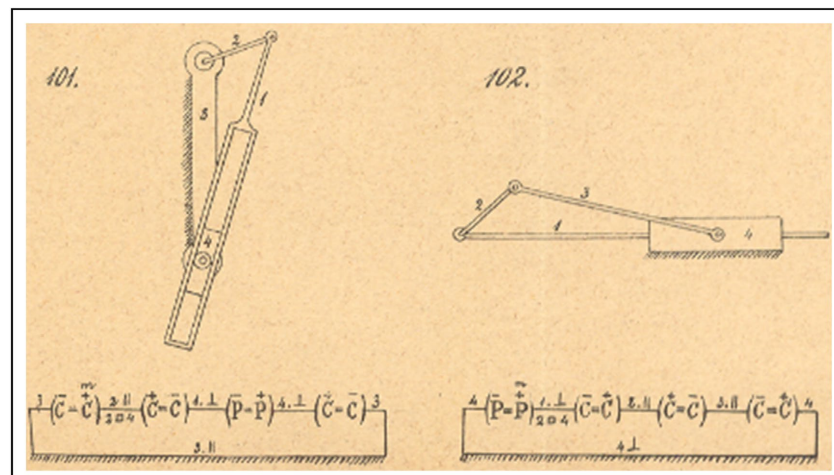


Figure 15. Notations and schemes of mechanisms from the slider-crank chain by Francesco Masi in 1883 (Masi, 1883).

In addition, the formalism of symbolic notation can be useful as a tool for classification rules, when one considers that the characters/rules of a mechanism classification can be expressed as a function of the parameter labels or symbols for the kinematic design of mechanisms. Following the great interest of the time on mechanism classification as in the works by Wills (1841) and Reuleaux (1875) that were also developed from early work by Monge and his pupils (Ceccarelli, 2004; Lanz & Betancourt, 1808; Bognis, 1818-1823), Masi has proposed his view and formulation of mechanism classification, as an extension also of an Italian tradition started

by a textbook (Giulio, 1846) of Carlo Giulio in 1846 in Turin.

Francesco Masi proposed his classification of mechanisms (Masi, 1883) as a function of the concepts of simple and compound mechanisms with kinematic chains and their inversions in categories and classes of mechanisms. Simple mechanisms are composed of one circuit of links, while compound mechanisms are modelled with more circuits of links with even different natures. Following Reuleaux's concept, a kinematic chain is defined as the kinematic structure of a mechanism with no frame link assigned, so that the kinematic inversion consists in assigning a frame role to different links of the chain. Thus, Masi could apply his notation in a theory for mechanism classification by giving a unifying view of mechanism designs as a systematic organization of the large variety of mechanism designs as potential solutions at that time. Using his notation, Masi also formulated a computation of the design possibilities that permitted him to identify 25,200 binary mechanisms and 10,362,600 quaternary mechanisms. The proposed notation was also implemented by Masi in teaching activities in his 1897 textbook (Masi, 1897), with a proper organization of the variety of mechanisms that was a reference for the Italian community for a long time, also inspiring modern textbooks.

Indeed, the above developments and proposals for symbolic notations of the mechanism structure produced great attention and inspired further modeling and formalism throughout the 20th century, even using different mathematical approaches and formalisms in activities with research, design, and teaching goals of applications.

Much of the above-mentioned notation formalisms was and is still used also in the teaching literature supporting the formation of engineers (Krylov et al., 2024).

6. Conclusion

In this paper, the evolution of early label notation for mechanisms is outlined, with main contributions from pioneering proposals during the 19th century that may still have an impact in today's field of mechanical engineering. Although the proposed notations were considered to have great potential even at the time of their proposal, they were limited by the difficulties of their practical application. In particular, the Babbage notation and its slight modification by Willis resulted in too much complication in being formulated with a very complex and large table and even in using them for a practical analysis of mechanism performance. The next analytical formulation by Reuleaux and its modification by Masi was not completely handsome for mathematical treatment, although it was used both for analysis and design procedures, since the main disadvantage was in its not complete mathematical form suitable for mathematical manipulations. As a result of those proposals, immediately during the 19th century, symbolic notation for machines and mechanisms was addressed as a research problem with practical aims in both analysis and design of mechanisms, so that further developments gave achievements in theory and design of mechanisms that were expanded in the 20th century with an ongoing impact still in the early years of the 21st century. In ad-

dition, the significance of those symbolic notations can be highlighted by the fact that they were used and are still used for educational aims by providing students with graphical representations and mathematical formulations as structured methods for knowledge explanation and fixation, making the operations of analysis and synthesis clear, unambiguous, and understandable for practical professional activities.

Author Contributions

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix: Biographical Notes

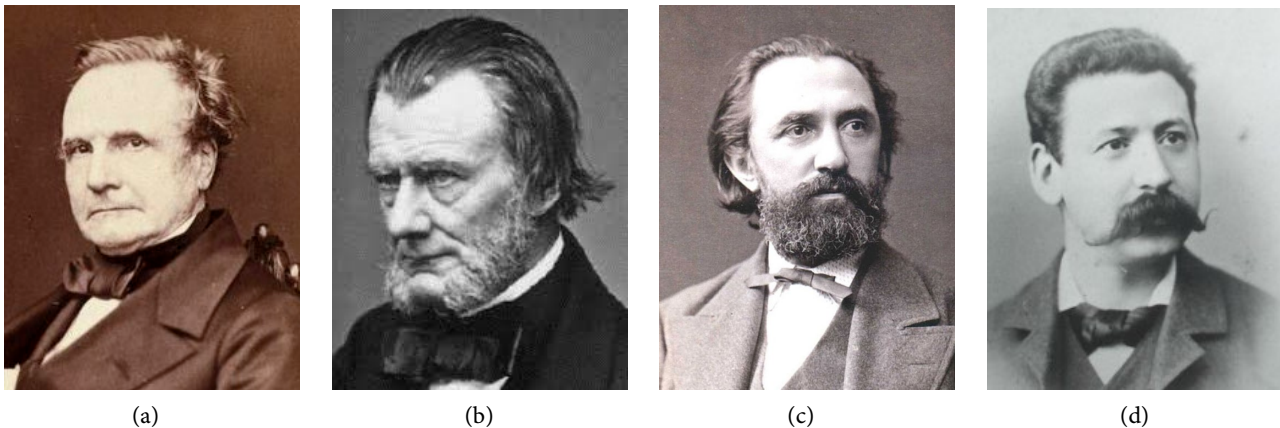


Figure A1. Portraits of main figures in the historical evolution of labelling and symbolic notation of mechanisms: (a) Charles Babbage (1791-1871); (b) Robert Willis (1800-1875); (c) Franz Reuleaux (1829-1905); (d) Francesco Masi (1852-1944).

A1—Charles Babbage (1791-1871)

Charles Babbage was born on December 26, 1791, in London, where he died on October 18, 1871. He served as Lucasian Professor of Mathematics at the University of Cambridge. In 1816, he was elected a fellow of the Royal Society of London. In 1831, he designed a mechanical digital computer known as the Difference Engine that is based on mechanical transmissions by gears. Only later was part of this room-sized calculator built. He contributed to establishing the modern postal system in England, inventing the postage stamp for prepaid postal service. He also invented several devices such as a speedometer and the locomotive cowcatcher.

A2—Robert Willis (1800-1875)

Robert Willis was born in London on 27 February 1800 and died in Cambridge on 28 February 1875. In 1822, he started his studies at Gonville and Caius College in Cambridge, from which he received his B.A. degree in 1826. In 1827, he was ordained deacon and priest with residence in Cambridge. From 1837 to 1875, Willis served as Jacksonian Professor of Natural Philosophy at Cambridge, and from 1853 onward he was a lecturer in applied mechanics at the government school of mines. He devoted great attention and activity to the history of architecture, including many studies of medieval cathedrals. In 1844, Willis was an early member of the British Archaeological Association (later Royal Institute).

A3—Franz Reuleaux (1829-1905)

Franz Reuleaux was born in Eschweiler (near Aachen, Germany) on 30 September 1829. He received practical education under Professor Ferdinand Redtenbacher at Karlsruhe Polytechnic School from 1850 to 1852. He completed his studies at the Universities of Berlin and Bonn in 1854. After a brief work period in industry, Reuleaux got a position at the newly founded Federal Polytechnic Institute (now ETH Zurich) in 1856, where he worked in the Department of Mechanics and Technology with Gustav Zeuner until 1864. In 1864, he got a chair at the Berlin Gewerbeakademie (later the University of Applied Sciences), where he remained until 1896. Reu-

leaux served on several international juries and commissions and was considerably involved in the formation of a patent system, as he was active in German politics. Franz Reuleaux died on 20 May 1905 in Berlin.

A4—Francesco Masi (1852-1944)

Francesco Masi was born on 28 February 1852 in Guastalla (Reggio Emilia). He got his engineering degree in 1875 at the Royal School of Engineering in Turin. His first employment in 1875 was as professor of Physics and Mechanics at the Technical College in Cagliari, but in 1877 he went to Bologna at the Royal School of Engineering as assistant professor in the field of Mechanics of Machinery. In 1891, he was appointed full professor at Bologna University, where he was active until his death on 30 November 1944 in Bologna. He was married to Teresa, the daughter of the renowned Italian poet Giosuè Carducci, but they never had children. Francesco Masi directed his attention to TMM but also to the fields of Hydraulics, Technical Drawing, and Mechanics of Agricultural Machines by developing teaching activity in Technical Colleges and at Bologna University.